

# CT Reconstruction from a Limited Number of X-Ray Projections

Tao Quang Bang, and Insu Jeon

**Abstract**—Most CT reconstruction system x-ray computed tomography (CT) is a well established visualization technique in medicine and nondestructive testing. However, since CT scanning requires sampling of radiographic projections from different viewing angles, common CT systems with mechanically moving parts are too slow for dynamic imaging, for instance of multiphase flows or live animals. A large number of X-ray projections are needed to reconstruct CT images, so the collection and calculation of the projection data consume too much time and harmful for patient. For the purpose of solving the problem, in this study, we proposed a method for tomographic reconstruction of a sample from a limited number of x-ray projections by using linear interpolation method. In simulation, we presented reconstruction from an experimental x-ray CT scan of a Aluminum phantom that follows to two steps: X-ray projections will be interpolated using linear interpolation method and using it for CT reconstruction based upon Ordered Subsets Expectation Maximization (OSEM) method.

**Keywords**—CT reconstruction, X-ray projections, Interpolation technique, OSEM

## I. INTRODUCTION

XRAY Computer tomography (CT) has provided a unique window into the anatomy of the human body since the invention of the CT scanner by Hounsfield in 1972. Nowadays, X-ray CT is widely used in many applications, can reconstruct the cross-section of object from its projections, not only a well recognized imaging modality in medicine [1] but also an essential one in industrial quality control and nondestructive evaluation [2, 3]. For the reason that many possible density values within the object being reconstructed may occur, a large number of projections are necessary to ensure the accurate reconstruction of density distribution for the CT method. However, for some simple images comprising only several parts and the density value is constant within each part, it might be possible to reconstruct the images with fewer projections for the decrease of density variable [4].

In most CT system, a complete projection data set for the image reconstruction is obtained by rotating either the radiation sources of the detectors or both of them around the object. However, a major limitation in such CT systems is the long time taken to obtain sufficient projection data, they are

therefore unsuitable for imaging temporal variations of phase distribution in multi-phase flow. In order to meet the requirement of high temporal resolution, only limited projection data can be obtained from a few-view CT system. The challenge lies in how to accurately reconstruct images from under-sampled projection data. It is an ill-posed problem in mathematics and regularization techniques are needed to produce reasonable reconstruction [3]. The traditional image reconstruction technique (ART) methods [5], [6] and [7]. Compared with these methods, statistical techniques can obtain a non-negative solution and lower image distortion [8]. Typical examples of statistical methods involve the maximum entropy (ME), expectation maximization (EM), maximum a posteriori (MAP), maximum likelihood (ML), and minimum cross-entropy (MCE) reconstruction algorithms [9], [10], [11], [12] and [13].

For most of the existing methods, at least 40 projections are usually needed to get a satisfactory reconstruction. The major clinical concern for CT is the radiation dose ill posed to the patients during imaging procedure [14]. The imaging dose can be controlled by reducing the number of projections or decreasing the tube current and pulse duration. However, all these methods of restricting imaging dose lead to degradations of image quality due to insufficiency of information in the data collected. In order to keep radiation dose as minimal as possible, while increase the quality of the reconstructed images, one needs to enhance the resolution of the projected image in the Radon domain without increasing the total number of projections.

We developed a linear interpolation (LI) technique to reconstruct images from a limited number of X-ray projections. This paper is organized as follows. The next section provides brief description of the CT data we use in the experiments. We describe a procedure for X-ray CT reconstruction and LI method for X-ray projections in section III. An image reconstruction techniques based on Ordered Subsets Expectation Maximization (OSEM) method using our proposed linear interpolation method is explained in section IV. We show and discuss some experiment results in section V. Conclusion are given at last.

## II. X-RAY PROJECTIONS

In this study, we used an Aluminum phantom for taking X-ray projections with X-ray voltage is 160kV, X-ray current is 50mA, detector size is 9.0 inch, distance from X-ray source to detector and from X-ray source to object is 579.3 mm and 163.9

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mm, respectively. The size of each projection is 1016 x 2024 pixels as shown in Fig. 1.



Fig. 1 The specimen (a) and X-ray projection (b)

### III. PROCEDURE FOR X-RAY CT RECONSTRUCTION & INTERPOLATION METHOD

#### A. Procedure for X-ray CT reconstruction

The method to obtain the CT images from limited projections is shown in Figure 2. First, the specimen will be took X-ray projections depending on shooting angles. Then we developed interpolation method for decreasing number of X-ray projections. Finally, CT images will be reconstructed by using Ordered Subsets Expectation Maximization (OSEM) method.

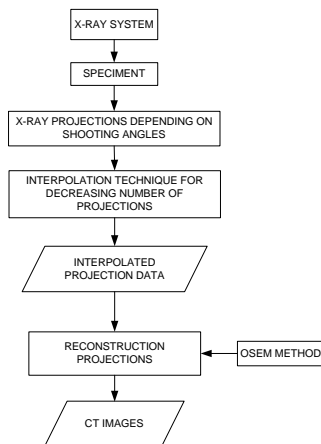


Fig. 2 Procedure for X-ray CT reconstruction

#### B. Linear interpolation method

Interpolation is commonly used in medical image processing and is required, whenever the acquired image data is not at the same level of discretization as desired or whenever geometric transformations of the image data are necessary.

There are many ways of calculating the motion vector by this [15], [16]. We shall discuss one of the more popular techniques [17]. In general, pixels in non-occluded regions can be related to each other by means of a general transformation of the form  $P(t+1)_{x+\delta x, y+\delta y} = P(t)_{x, y} + H(t)_{x, y}$

where the function  $H(t)_{x, y}$  compensates for intensity differences between the images, and  $(\delta x, \delta y)$  defines the displacement vector of the pixel at time  $t+1$ .

### IV. CT RECONSTRUCTION ALGORITHM

After interpolation method is performed, we provide brief description of OSEM reconstruction method because it is well-known and widely available in literature. OSEM is an approach to accelerate the iterative reconstruction process. With OSEM the projection data are grouped into several ordered subsets and the pixel in a reconstructed image is modified with the projection data in the subset. The following is definition of the ordered subsets image reconstruction by EM algorithm.

Iterative image reconstruction methods with expectation-maximization (EM) algorithm typically update the value of pixel  $x_j$  at  $n$ th iteration according to the following multiplication scheme

$$x_j^{n+1} = \frac{x_j^n}{\sum_{j=1}^N p(i/j)} \sum_{i=1}^M \frac{y_i p(i/j)}{\sum_{j=1}^N x_j^n p(i/j)}, \quad j=1, 2, \dots, L$$

$x_j^n$ : value of pixel  $x_j$  after  $n$ th iteration

$x_j^{n+1}$ : value of pixel  $x_j$  after  $(n+1)$ th iteration

$M \times N$ : projection matrix

Let  $S_1, S_2, \dots, S_L$  denote the  $L$  sorted subsets  $\Rightarrow$  the  $N_0$  of projection data in every subset  $J=M/L$

1. Set  $j=0$ , initialize the image  $X=X^0$

2. Iterative the following steps until image  $X_i$  satisfies the convergence request:

a) Set  $X=X_i, i=i+1$

b) For each subset  $S_l, l=1, 2, \dots, L$  process the following iteration

b1) Project: For every detector  $i(i=1, 2, \dots, M)$  in subset  $S_l$ , assume  $a_{ij}$  denote the  $x_j$  contribution to detector  $i$ , calculate its mathematical expectation

$$y_i^l = \sum_{j=1}^N a_{ij} x_j^l$$

b2) Back Project: Update the image at the sub iteration  $l$

$$x_j^{l+1} = \frac{x_j^l}{\sum_{j=1}^N a_{ij}} \times \sum_{i=1}^N \frac{y_i^l a_{ij}}{\sum_{j=1}^N a_{ij} x_j^l}, \quad j=1, 2, \dots, N$$

b3)  $l=l+1$

c) If  $l=L+1$  then  $X^l=X^1$ , repeat steps (a) and (b) to continue the next iteration

3. End

### V. EXPERIMENT RESULTS AND DISCUSSIONS

Using X-ray projection data that we described in section II for our proposed method, we used only 30 X-ray projections to obtain CT images. First, by using LI method for previous and current projections, we made an interpolated projection as shown in Figure 3.

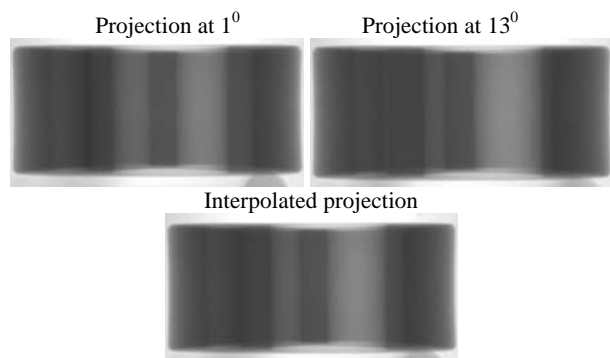


Fig. 3 The interpolated image

After using LI method, we used OSEM method to reconstruct CT images. Figure 4 shows the CT image using only 30 X-ray projections with LI method and Figure 5 indicates the difference between CT image using 30 X-ray projections (with LI method) and using 60 X-ray projections (without LI method).

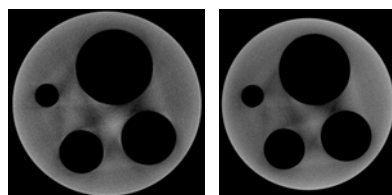


Fig. 4 The CT images using 30 X-ray projections with LI method (a) and using 60 projections (b)

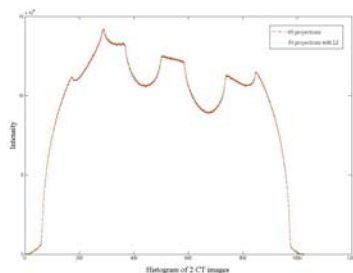


Fig. 5 The difference between CT image using 30 (with LI method) and using 60 projections

## VI. CONCLUSION

Using linear interpolation technique, we can decrease number of X-ray projections. It will be used to decrease radiation dosage for patients and construct CT images fastly. In the future, we will decrease number of X-ray projections more by developing the interpolation using multi-image to obtain CT images. Furthermore, we will try to apply our method for a complex specimen.

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