Stability analysis of linear fractional order neutral system with multiple delays by algebraic approach

Lianglin Xiong, Yun Zhao, and Tao Jiang

Abstract—In this paper, we study the stability of n-dimensional linear fractional neutral differential equation with time delays. By using the Laplace transform, we introduce a characteristic equation for the above system with multiple time delays. We discover that if all roots of the characteristic equation have negative parts, then the equilibrium of the above linear system with fractional order is Lyapunov globally asymptotical stable if the equilibrium exist that is almost the same as that of classical differential equations. An example is provided to show the effectiveness of the approach presented in this paper.

Keywords—Fractional neutral differential equation, laplace transform, characteristic equation.

I. INTRODUCTION

FRACTIONAL Fractional differential equations have gained considerable importance due to their application in various sciences, such as viscoelasticity, electroanalytical chemistry, electric conductance of biological systems, modeling of neurons, diffusion processes, damping laws, rheology, etc. Fractional order differential equation is represented in continuous-time domain by differential equations of non integer-order. Moreover, time delay is often present in real processes due to transportation of materials or energy. Therefore, most fractional systems may contain a delay term, such as fractional order neutral systems or some other fractional order delay systems.

Analysis of stability is fundamental to any control system. Recently, considerable attention has given to the stability problems arising from neutral systems. And various analysis techniques have been utilized to derive stability criteria for the systems by many researchers [1]-[8], and the references therein. On the other hand, although the problem of stability is a very essential and crucial issue for control systems including fractional order systems, due to the complexity of the relations, it has been discussed and investigated only in some recent literature [9]-[20], and the references therein. In the last five years, considerable attention has also been paid to obtain analytical robust stability conditions for fractional order linear time invariant (FO-LTI) systems, and the Cauchy initial value problem for various kinds of fractional order systems. The Cauchy initial value problem were discussed for various type fractional neutral functional differential equations and many criteria on existence and uniqueness were obtained. However, there is not much work on the subject of stability for fractional order neutral system, besides [21]-[29]. More recently, used

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the characteristic equation of the neutral system with single delay, stability criteria were derived in terms of the spectral radius of modulus matrices in [30]-[31], and the examples were showed that the conditions obtained in those paper are less conservative than some existing results. However, the stability for the fractional neutral functional differential equations hasn't been get attention by many researchers. All of those have motivated our research.

In this paper, we are interested in the stability of n-dimensional linear fractional neutral differential equation with multiple time delays. Similar to the approach of [9], making use of the Laplace transform, a characteristic equation for the above system with multiple time delays is introduced. We discover that if all roots of the characteristic equation have negative parts, then the equilibrium of the above linear system with fractional order is Lyapunov globally asymptotical stable if the equilibrium exist that is almost the same as that of classical differential equations. Finally, one special numerical example is given to illustrate the effectiveness of the obtained results.

II. PROBLEM STATEMENT

This section start with recalling the essentials of the fractional calculus. The idea of fractional calculus has been known since the development of the regular calculus, with the first reference probably being associated with Leibniz and L'Hospital in 1695 where half-order derivative was mentioned.

The fractional calculus is a name for the theory of integrals and derivatives of arbitrary order, which unifies and generalizes the notions of integer-order differentiation and n-fold integration. There are three main uesed definitions of fractional integration and differentiation, such as Grunwald-Letnikov's definition, Riemann-Liouville's definition, Caputo's fractional derivative. The former two definitions are often used by pure mathematicians, while the last one is adopted by applied scientists, since it is more convenient in engineering applications. Here we only discuss Caputo derivative:

$$ {}_0^C D_t^\alpha x(t) = \frac{d^q x(t)}{dt^q} = J^{m-q} x^{(m)}(t), \quad \alpha > 0 $$

where $m = [q]$, i.e., m is the first integer that is not less than q, $x^{(m)}$ is a conventional mth order derivative, J^β is the βth order Riemann-Liouville integral operator, which is expressed as follows:

$$ J^\beta x(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-s)^{\beta-1} x(s) ds, \quad \beta > 0. $$

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In engineering, the fractional order q often lies in $(0,1)$, so we always suppose that the 'order' q is a positive number but less than 1 in this paper.

In the present article, we study the following $n-$dimensional linear fractional order neutral systems with multiple time delays:

$$
\begin{cases}
{}^C_0 D_t^{\alpha_1} x_1(t) - \sum_{i=1}^n c_{1i}\, {}^C_0 D_t^{\alpha_1} x_i(t - r_{1i}) = \\
\qquad \sum_{i=1}^n [a_{1i} x_i(t) + b_{1i} x(t - \tau_{1i})] \\
{}^C_0 D_t^{\alpha_2} x_2(t) - \sum_{i=1}^n c_{2i}\, {}^C_0 D_t^{\alpha_2} x_i(t - r_{2i}) = \\
\qquad \sum_{i=1}^n [a_{2i} x_i(t) + b_{2i} x(t - \tau_{2i})] \\
\qquad\qquad \vdots \\
{}^C_0 D_t^{\alpha_n} x_n(t) - \sum_{i=1}^n c_{ni}\, {}^C_0 D_t^{\alpha_n} x_i(t - r_{ni}) = \\
\qquad \sum_{i=1}^n [a_{ni} x_i(t) + b_{ni} x(t - \tau_{ni})]
\end{cases}
\tag{1}
$$

where α_j is real and lies in $(0,1)$. the initial values $x_j(t) = \phi_j(t)$ are given for $-\max_{i,j}(\tau_{ji}, r_{ji}) = -\rho_{\max} \le 0$ and $j = 1, \cdots, n$.

Next, we study the stability of system (1). Taking Laplace transform [32] on both sides of (1) gives

$$
\begin{cases}
s^{\alpha_1} X_1(s) - \sum_{i=1}^n c_{1i}[s^{\alpha_1} e^{-sr_{1i}} X_i(s) - e^{-sr_{1i}} s^{\alpha_1-1}\phi_i(0) \\
\qquad + \int_{-r_{1i}}^0 e^{-s(r_{1i}+t)}\, {}^C_0 D_t^{\alpha_1}\phi_i(t)dt] \\
= s^{\alpha_1-1}\phi_1(0) + \sum_{i=1}^n [a_{1i} X_i(s) + b_{1i} e^{-s\tau_{1i}}(X_i(s) \\
\qquad + \int_{-\tau_{1i}}^0 e^{-st}\phi_i(t)dt)] \\
s^{\alpha_2} X_2(s) - \sum_{i=1}^n c_{2i}[s^{\alpha_2} e^{-sr_{2i}} X_i(s) - e^{-sr_{2i}} s^{\alpha_2-1}\phi_i(0) \\
\qquad + \int_{-r_{2i}}^0 e^{-s(r_{2i}+t)}\, {}^C_0 D_t^{\alpha_2}\phi_i(t)dt] \\
= s^{\alpha_2-1}\phi_2(0) + \sum_{i=1}^n [a_{2i} X_i(s) + b_{2i} e^{-s\tau_{2i}}(X_i(s) \\
\qquad + \int_{-\tau_{2i}}^0 e^{-st}\phi_i(t)dt)] \\
\qquad\qquad \vdots \\
s^{\alpha_n} X_n(s) - \sum_{i=1}^n c_{ni}[s^{\alpha_n} e^{-sr_{ni}} X_i(s) - e^{-sr_{ni}} s^{\alpha_n-1}\phi_i(0) \\
\qquad + \int_{-r_{ni}}^0 e^{-s(r_{ni}+t)}\, {}^C_0 D_t^{\alpha_n}\phi_i(t)dt] \\
= s^{\alpha_n-1}\phi_n(0) + \sum_{i=1}^n [a_{ni} X_i(s) + b_{ni} e^{-s\tau_{ni}}(X_i(s) \\
\qquad + \int_{-\tau_{ni}}^0 e^{-st}\phi_i(t)dt)]
\end{cases}
$$

where $X_i(s)$ is the Laplace transform of $x_i(t)$ with $X_i(s) = \mathcal{L}(x_i(t)), 1 \le i \le n$. We can rewrite (2) as follows:

$$
\Delta(s) \bullet \begin{pmatrix} X_1(s) \\ X_2(s) \\ \vdots \\ X_n(s) \end{pmatrix} = \begin{pmatrix} d_1(s) \\ d_2(s) \\ \vdots \\ d_n(s) \end{pmatrix},
\tag{2}
$$

in which

$$
\begin{aligned}
d_j(s) &= \sum_{i=1}^n b_{ji} e^{-s\tau_{ji}} \int_{-\tau_{ji}}^0 e^{-st}\phi_i(t)dt \\
&+ \sum_{i=1}^n c_{ji} e^{-sr_{ji}} \int_{-r_{ji}}^0 e^{-st}\, {}^C_0 D_t^{\alpha_j}\phi_i(t)dt \\
&+ \sum_{i=1}^n c_{ji} e^{-sr_{ji}} s^{\alpha_j-1}\phi_i(0) + s^{\alpha_j-1}\phi_j(0), \\
& \qquad j = 1, \cdots, n.
\end{aligned}
\tag{3}
$$

$$
\Delta(s) = \begin{pmatrix}
\Delta_{11}(s) & \Delta_{12}(s) & \cdots & \Delta_{1n}(s) \\
0 & \Delta_{22}(s) & \cdots & \Delta_{2n}(s) \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \Delta_{nn}(s)
\end{pmatrix}
\tag{4}
$$

where

$$
\Delta_{11}(s) = s^{\alpha_1} - c_{11} s^{\alpha_1} e^{-sr_{11}} - a_{11} - b_{11} e^{-s\tau_{11}},
$$
$$
\Delta_{22}(s) = s^{\alpha_2} - c_{22} s^{\alpha_2} e^{-sr_{22}} - a_{22} - b_{22} e^{-s\tau_{22}},
$$
$$
\Delta_{nn}(s) = s^{\alpha_n} - c_{nn} s^{\alpha_n} e^{-sr_{nn}} - a_{nn} - b_{nn} e^{-s\tau_{nn}},
$$
$$
\Delta_{ji}(s) = s^{\alpha_j} - c_{ji} s^{\alpha_j} e^{-sr_{ji}} - a_{ji} - b_{ji} e^{-s\tau_{ji}}.
$$

We call $\Delta(s)$ a characteristic matrix of system (1) for simplicity and $\det(\Delta(s))$ a characteristic polynomial of (1). The distribution of $\det(\Delta(s))'s$ eigenvalues totally determines the stability of system (1). This can be seen from the following discussion.

III. MAIN RESULTS

In this section, we establish several stability condition for fractional order neutral systems.

Obviously, if a linear fractional differential equation has a non-zero equilibrium, we can move this equilibrium to the origin by the translation transform. Throughout the paper, we always suppose that (1) has a zero solution and all complex computations are done in the branch of the principle value of argument.

Theorem 1: If all the roots of the characteristic equation $\det(\Delta(s)) = 0$ have negative real parts, then the zero solution of system (1) is Lyapunov globally asymptotically stable.

Proof: Multiplying s on both sides of (2) gives

$$
\Delta(s) \bullet \begin{pmatrix} sX1(s) \\ sX_2(s) \\ \vdots \\ sX_n(s) \end{pmatrix} = \begin{pmatrix} sd_1(s) \\ sd_2(s) \\ \vdots \\ sd_n(s) \end{pmatrix}.
\tag{5}
$$

If all roots of the transcendental equation $\det(\Delta(s)) = 0$ lie in open left half complex plane,i.e., $Re(s) < 0$, then we consider

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(5) in $Re(s) \geq 0$. In this restricted area, (5) has a unique solution $\left(\begin{array}{cccc} sX_1(s) & sX_2(s) & \cdots & sX_n(s) \end{array} \right)$ So, we have

$$\lim_{s \to 0, Re(s) \geq 0} sX_i(s) = 0, i = 1, 2, \cdots, n.$$

From the assumption of all roots of the characteristic equation $\det(\Delta(s)) = 0$ and the final-value theorem of Laplace transform [32], we get

$$\lim_{t \to +\infty} x_i(t) = \lim_{s \to 0, Re(s) \geq 0} sX_i(s) = 0, i = 1, 2, \cdots, n.$$

It implies that the fractional order neutral system is Lyapunov globally asymptotically stable. It completes the proof. ∎

Remark 1. This result contains that of Theorem 1 in [9]. In fact, when $C = 0$, the neutral system is typical fractional order system with multiple time delay system, the result is obviously consist with that of [9]. Although this theorem is an extension of [9] in some sense, this results will be important to the stability analysis for fractional order neutral systems.

Remark 2. If $\tau_{ji} = \tau > 0, r_{ji} = r > 0$ for $i, j = 1, 2, \cdots, n$ and $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 1$, then the characteristic matrix and characteristic equation of (1) are reduced to $sI - se^{-sr}C - A - e^{-s\tau}B$ and $\det(sI - se^{-sr}C - A - e^{-s\tau}B) = 0$ respectively. They coincide with the usual definitions of the characteristic matrix and characteristic equation of neutral delayed equations. Especially, if $B = C = 0$, then the characteristic matrix and characteristic equation of (1) are respectively reduced to $sI - A$ and $\det(sI - A) = 0$, which agree with the typical definitions for typical differential equations.

Further more, If $\tau_{ji} = r_{ji} = \tau = r > 0$ for $i, j = 1, 2, \cdots, n$ and $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$, then systems (1) become as

$$^C_0 D^\alpha_t x_j(t) - \sum_{i=1}^{n} c_{ji} {}^C_0 D^\alpha_t x_i(t-\tau) = \sum_{i=1}^{n} [a_{ji}x_i(t) + b_{ji}x(t-\tau)]$$
(6)

In short, equation (6) can be written as

$$^C_0 D^\alpha_t x(t) - C \, {}^C_0 D^\alpha_t x(t-\tau) = Ax(t) + Bx(t-\tau) \quad (7)$$

where $A = (a_{ij})_{n \times n}$, $B = (b_{ij})_{n \times n}$, $C = (c_{ij})_{n \times n}$, $x^T(t) = \left(\begin{array}{cccc} x_1(t) & x_2(t) & \cdots & x_n(t) \end{array} \right)^T$, and the characteristic matrix of (1) is reduced to

$$s^\alpha I - s^\alpha e^{-s\tau}C - A - e^{-s\tau}B$$

and the characteristic equation is reduced to

$$\det(s^\alpha I - s^\alpha e^{-s\tau}C - A - e^{-s\tau}B) = 0 \quad (8)$$

Moreover, If $\tau_{ji} = \tau, r_{ji} = r > 0$ for $i, j = 1, 2, \cdots, n$ and $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$, then systems (1) return to

$$^C_0 D^\alpha_t x_j(t) - \sum_{i=1}^{n} c_{ji} {}^C_0 D^\alpha_t x_i(t-r) = \sum_{i=1}^{n} [a_{ji}x_i(t) + b_{ji}x(t-\tau)]$$
(9)

Similarly, equation (9) can be given as

$$^C_0 D^\alpha_t x(t) - C \, {}^C_0 D^\alpha_t x(t-r) = Ax(t) + Bx(t-\tau) \quad (10)$$

and the characteristic matrix of (1) is reduced to

$$s^\alpha I - s^\alpha e^{-sr}C - A - e^{-s\tau}B$$

and the characteristic equation is reduced to

$$\det(s^\alpha I - s^\alpha e^{-sr}C - A - e^{-s\tau}B) = 0 \quad (11)$$

Therefore, based on these characteristic equations (8) and (11), one can obtain the stability analysis for the fractional order neutral systems in different case, similar to some existing results for neutral systems.

IV. EXAMPLE

This section will list one example to show the effectiveness of our new criteria for asymptotic stability of fractional order neutral systems.

Example 1. Consider system (7) with

$$A = \left(\begin{array}{cc} 4 & 1 \\ -2 & 5 \end{array} \right), \quad B = \left(\begin{array}{cc} 1.4 & -0.2 \\ -0.5 & 2.1 \end{array} \right),$$

$$C = \left(\begin{array}{cc} -0.71 & 0.44 \\ -0.64 & 0.32 \end{array} \right), \quad \tau = 0.9874, \quad \alpha = 0.18$$

Clearly, the characteristic equation of this systems is

$$\det(s^{0.18}I - s^{0.18}e^{-0.9874s}C - A - e^{-0.9874s}B) = 0 \quad (12)$$

With a simple calculation in the Matlab toolbox, all the roots of the characteristic equation have negative real parts. According to Theorem 1, the system is asymptotically stable.

V. CONCLUSION

Some new stability conditions for of a class of fractional order neutral systems are achieved in this paper. By using the Laplace's transformation the characteristic equation is introduced for the fractional order neutral systems. All the roots of the characteristic equation have negative real parts implies the asymptotically stable for the corresponding systems. An Illustrative example is given to demonstrate the effectiveness of the main results presented in this paper.

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