Elliptical Features Extraction Using Eigen Values of Covariance Matrices, Hough Transform and Raster Scan Algorithms

J. Prakash, and K. Rajesh

Abstract—In this paper, we introduce a new method for elliptical object identification. The proposed method adopts a hybrid scheme which consists of Eigen values of covariance matrices, Circular Hough transform and Bresenham's raster scan algorithms. In this approach we use the fact that the large Eigen values and small Eigen values of covariance matrices are associated with the major and minor axial lengths of the ellipse. The centre location of the ellipse can be identified using circular Hough transform (CHT). Sparse matrix technique is used to perform CHT. Since sparse matrices squeeze zero elements and contain a small number of nonzero elements they provide an advantage of matrix storage space and computational time. Neighborhood suppression scheme is used to find the valid Hough peaks. The accurate position of circumference pixels is identified using raster scan algorithm which uses the geometrical symmetry property. This method does not require the evaluation of tangents or curvature of edge contours, which are generally very sensitive to noise working conditions. The proposed method has the advantages of small storage, high speed and accuracy in identifying the feature. The new method has been tested on both synthetic and real images. Several experiments have been conducted on various images with considerable background noise to reveal the efficacy and robustness. Experimental results about the accuracy of the proposed method, comparisons with Hough transform and its variants and other tangential based methods are reported.

Keywords—Circular Hough transform, covariance matrix, Eigen values, ellipse detection, raster scan algorithm.

I. INTRODUCTION

THE most significant problems in object recognition is to find the features of the image. Ellipse detection is one of the key problems in image processing. In machine vision applications, the detection of an ellipse is very important. Successful identification of the object can be helpful in various fields including industrial and biomedical applications.

A variety of approaches have been suggested for detecting the ellipse and estimating the associated parameters. The Hough transform (HT) [1] has been recognized as a robust

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K. Rajesh is with Philips Software Centre, Manyata Tech Park, Nagavara, Bangalore-560 045, Karnataka state, India (phone: +91-80-25658369, e-mail: rajesh.nair@philips.com). technique for the detection of analytically defined shapes in images. However, direct applications of HT are limited since the memory space and computing time grow exponentially with the number of parameters. For example the detection of ellipse is characterized with five parameters viz., central position coordinates (x_0, y_0) length of major and minor axis (a,b) and the orientation of the major axis (θ) requires five dimensional arrays for accumulation. In standard HT the long computation times are caused due to the fact that the HT increment the cells in the accumulator array corresponding to all curves pass through all points in the image. Since the size of the accumulator array is exponential to the number of parameters, the storage space requirements become excessive. In order to solve these problems, the generalized Hough transform (GHT) [2] [3] method reduces the dimensions of the parameter space by breaking it down into several lower order ones.

The randomized Hough transform (RHT) [4] [5] accumulate points in a parameter space by randomly choosing n-tuples of pixels from an image and computing the parameters of the object which passes through these pixels. It is useful for the detection of objects which are defined as a linear function of a set of parameters. For curves expressed by equations which are nonlinear with respect to the parameters the RHT cannot be directly used. Generally the HT based methods, Probabilistic Hough transform (PHT), Progressive probabilistic HT (PPHT), Combinatorial HT (CHT) and Geometric symmetry Hough transform (GSHT) requires large storage and high processing time. Since they take 5D accumulator arrays in standard Hough transform (SHT) parameter space they are not suitable for ellipse identification. Furthermore, it is not easy to choose a suitable peak finding rule to the multiple peaks problem.

On the other hand, some Non-HT methods for ellipse detection were proposed by other researchers. In particular Wen-Yen Wu et al [6] proposed a simple algorithm to detect ellipses using geometric properties. This method detects the centre of the ellipse by its symmetrical property. It finds the edge points on the ellipse by an intuitive geometric property and estimates the five parameters (x_0, y_0, a, b, θ) of the ellipse by a least sum of squares fitting method. A fast ellipse detector using geometric symmetry developed by Chen et al [7] used the global geometric symmetry to locate all possible symmetric

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centers of ellipses in an image and then all feature points are classified into several sub images according to these points. Finally, the accumulative concept of the HT is used to extract all ellipses in the image. Tsuji et al [8] have recommended the use of the gradient operator to obtain two opposite points with the same tangent and determine the centre of the ellipse. The lengths of the major and minor axes and the angle of orientation are then computed by using the least mean square method in curve fitting. Consistent symmetric axis method developed by H. T. Sheu et al [9] used the geometrical properties of the ellipse enclosed by a rectangle will be employed to derive a procedure for the estimation of the parameters. In this method the parameters of the ellipse are calculated without resorting to gradient operations.

From the above discussion, it is evident that, ellipse detection based on HT methods requires large storage space and high computational time. Methods based on gradient vectors are very sensitive to noise. Techniques based on geometric symmetric properties exhibits less accuracy in object identification. In our proposed method we use the statistical parameters such as large and small Eigen values to find the major and minor axial lengths, CHT to detect the centre location of the ellipse and Bresenham's raster scan algorithm to obtain the boundary points accurately.

II. PROPOSED METHOD

The proposed method for detecting ellipses in images consists of six steps.

Step-I For the given gray scale image, find out the edge image using suitable edge detection operators.

Step-II Obtain the large and small Eigen values for the covariance matrices of the edge image.

Step-III Obtain the ratio of large Eigen values to small Eigen values for different angles.

Step-IV Find out the major and minor axial lengths from the Eigen values.

Step-V Perform CHT using sparse matrices technique to find the centre of the ellipse.

Step-VI Find out the boundary points of the ellipse using Bresenham's raster scan algorithm.

III. COVARIANCE MATRICES AND THEIR EIGEN VALUES

In this section, we derive the major and minor axial lengths of the ellipse using the statistical and geometric properties associated with Eigen values of the covariance matrix of data points on a digital boundary over a region of support.

Let the sequence of n digital points describe the boundary of an object p

 $p = \{p_i = (x_i, y_i), i = 1, 2, 3, \dots, n\}$, where p_{i+1} is a neighbor of p_i (modulo n), and (x_i, y_i) are the Cartesian coordinates of p_i in the image. Denote $s_k(p_i)$ as a small curve segment of p, which is defined by the region of support between points p_{i-k} and p_{i+k} for some integer k, that is

$$s_k(p_i) = \{p_j, j = i - k, i - k + 1, \dots, i + k - 1, i + k\}.$$

The covariance matrix c of a curve segment $s_k(p_i)$ is given by

$$c = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix},$$

where

$$c_{11} = \left[\frac{1}{2k+1} \sum_{j=i-k}^{i+k} x_j^2\right] - c_x^2,$$

$$c_{12} = c_{21} = \left[\frac{1}{2k+1} \sum_{j=i-k}^{i+k} x_j \cdot y_j\right] - c_x \cdot c_y$$

$$c_{22} = \left[\frac{1}{2k+1} \sum_{j=i-k}^{i+k} y_j^2\right] - c_y^2$$

 c_x and c_y are the geometrical centre of the curve segment $s_k(p_i)$, that is

$$c_{x} = \left[\frac{1}{2k+1}\sum_{j=i-k}^{i+k} x_{j}\right],$$
$$c_{y} = \left[\frac{1}{2k+1}\sum_{j=i-k}^{i+k} y_{j}\right],$$

The covariance matrix is $2x^2$ symmetric and positive semi definite. There are two Eigen values λ_l and λ_s for the matrix c, which are

$$\lambda_{l} = \frac{1}{2} \left[a_{11} + a_{22} + \sqrt{(a_{11} - a_{22})^{2} + 4a_{12}^{2}} \right],$$

$$\lambda_{s} = \frac{1}{2} \left[a_{11} + a_{22} - \sqrt{(a_{11} - a_{22})^{2} + 4a_{12}^{2}} \right].$$

The Eigen values of the matrix c can be used to extract the shape information about a curve. It can be shown that when the shape s is a straight line segment, the smaller Eigen values λ_s for the line segment in the continuous domain will be zero, regardless the length and orientation of the line segment [10][11]. The two Eigen values will be equal if the shape s is a full circle [12]. If the shape s is an ellipse, then $\lambda_l > \lambda_s$ and $\sqrt{\lambda_l}$, $\sqrt{\lambda_s}$ are the semimajor and semiminor axial lengths of the ellipse [13]. Therefore the large Eigen values λ_l and small Eigen values λ_s of the covariance matrix c can be utilized to measure the length of the major axis and minor axis of an ellipse respectively.

Fig. 1 shows an ellipse with major axis length of 60 pixels and minor axis length of 40 pixels with different orientations.

Table I summarizes the calculated Eigen values λ_l and λ_s for different region of support (w) and orientations (θ) for

the ellipses shown in Fig. 1. From Table I we note that the Eigen values λ_l and λ_s are associated with major and minor axial lengths of an ellipse and their ratio $r = (\lambda_l / \lambda_s)$ is constant. The actual ratio is 1.5 and the calculated ratio is 1.47. The size of the region of support will affect the Eigen values of a given boundary. In our method we consider the region of support *w* as 5X5 and 7X7. The region of support for the computation of the covariance matrix can also be selected adaptively.



Fig. 1 Ellipse with different orientations

TABLE I

The Eigen Values λ_l and λ_s for Different Orientations and Region

OF SUPPORT								
	Region of support							
θ	W=5X5			W=7X7				
	λ_l	λ_{s}	r	λ_l	λ_{s}	r		
0^0	1.0921	0.7432	1.4694	1.0897	0.7398	1.4729		
30 ⁰	1.0919	0.7429	1.4697	1.0893	0.7396	1.4728		
60^{0}	1.0917	0.7426	1.4701	1.0890	0.7393	1.4730		
90 ⁰	1.0914	0.7422	1.4704	1.0887	0.7391	1.4730		
120^{0}	1.0916	0.7425	1.4701	1.0889	0.7392	1.4730		
150^{0}	1.0919	0.7428	1.4699	1.0894	0.7395	1.4732		
180^{0}	1.0922	0.7433	1.4693	1.0898	0.7399	1.4729		

IV. CIRCULAR HOUGH TRANSFORM

Finding the centre is an important step in ellipse detection. In this section, after finding the major and minor axial lengths the centre location of the ellipse can be identified using CHT. In our approach, before performing CHT we removed all the small Eigen values λ_s whose values are zero or approximately zero using suitable threshold value, since they represent linear segment. The large Eigen values λ_l and small Eigen values λ_s whose values are same are removed because they represent circular feature. Finally, we have only the Eigen values which represent the major and minor axes of an ellipse. Hence the storage requirement and computational time to perform CHT can be reduced considerably and also 5D accumulator arrays for the ellipse can be reduced to 3D arrays as a circle. Perform CHT using the duality between centers of circles formed by major and minor axes which are shown in Fig. 2(a). The CHT using sparse matrix technique is shown in Fig. 2(b). To find the valid Hough peaks, we used the neighborhood suppression scheme [14] [15]. The cross sectional view of Hough transform with peak identified is as shown in Fig. 2(c). From Fig. 2(c), we observed that the Hough peak for major and minor axial circles is same and it represents the centre location of the ellipse.



V. FINDING THE ELLIPTICAL POINTS

After the center, major axis and minor axis have been detected; the next step is to locate all the points on the ellipse boundary. In order to find the elliptical points we use the Bresenham's raster scan algorithm [16]. In this algorithm the ellipse equations are greatly simplified if the major and minor axes are originated to align with the coordinated axis (x, y). The equation of an ellipse in terms of centre (x_0, y_0) and semi major and semiminor axes parameters (a,b) is given by

$$\left(\frac{x-x_0}{a}\right)^2 + \left(\frac{y-y_0}{b}\right)^2 = 1.$$

Using polar coordinates the boundary points can be described with the parametric equations $x = x_0 + a\cos\theta$ and $y = y_0 + b\sin\theta$. Symmetry conditions can be used to further reduce computations. An ellipse in standard position is symmetric between quadrants. Thus we calculated point positions along the elliptical arc throughout one quadrant, and then we obtain the positions in the remaining three quadrants by symmetry. This method of ellipse boundary points detection is highly robust against partial occlusions and defected ellipses.

VI. EXPERIMENTAL RESULTS

This section presented the result of experiments in which the proposed method was applied to several images. To test the proposed approach we used both synthetic and real world images with additive impulse and Gaussian noise. In addition, we considered disjoint, defective and occluded ellipses also. The use of synthetic images for which ground truth is known enables us to give some quantitative estimate of the accuracy obtained. The correctness of the output can be measured by the error factor, which is the ratio of real parameters to the identified parameters.



Fig. 3 Extraction of ellipses (a) original image (b) Noisy image (c) CHT image (d) ellipses detected (e) Overlapped with noisy image (f) Overlapped with original image

Fig. 3(a) shows a typical synthetic image including good, disjoint and occluded ellipses. For the testing of the method under noise condition, an additive impulse with density 0.05 and Gaussian noise with mean m=0 and variance v=0.01are added and is shown in Fig. 3(b). To calculate the Eigen values, we consider the window size w as 7X7 and the ratio of λ_l to λ_s as 3.4538 for upper bound and 1.9725 for lower bound, since they represent lengths of major and minor axes of larger and smaller ellipses respectively. The CHT used to find the centre locations is shown in Fig. 3(c). Once the axes parameters and centre location are identified, the boundary parameters are evaluated using raster scan algorithm. The resulting ellipses extracted are shown in Fig. 3(d). To test the correctness and accuracy, the extracted ellipses are overlapped with noisy image and original image shown in Fig 3(e) and 3(f) respectively. Table II shows the true values and estimated values for five parameters of different ellipses. From Table II we observe that, the absolute difference between the true values and the estimated values for the five parameters of perfect ellipse is only one pixel. For broken ellipse it is 2 to 4 pixels and for occluded ellipse it is 1 to 2 pixels. Hence the proposed method has best performance for the detection of the perfect ellipses. The broken ellipses have larger errors than that

of the occluded ellipses. This is due to the fact that the degree of breakage of the broken ellipses is higher than the degree of occlusion of the occluded ellipses.

 TABLE II

 THE REAL AND ESTIMATED PARAMETERS WITH MEASURED ERROR FACTOR

Type of ellipse	Original Parameters		Estimated parameters	% Error
Perfect ellipse	(x_c, y_c)	(118,174)	(118,175)	1.0000
	(a,b)	(164,46)	(164,47)	0.9893
	θ	0^0	0	1.0000
	(x_c,y_c)	(115,90)	(112,87)	1.0306
ellinse	(a,b)	(82.37)	(78,34)	1.0697
empse	θ	90 ⁰	88 ⁰	1.0227
Occluded ellipse	(x_c, y_c)	(46,46)	(45,45)	1.0223
	(a,b)	(73,37)	(71,35)	1.0426
	θ	00	00	1.0000

In order to evaluate the consistency of the proposed method, an image containing different forms of ellipses as well as some shaded objects was tested. This is to evaluate the ability of the proposed method for detecting the elliptical objects in the mixed conditions. Fig. 4(a) shows an image with different forms of ellipses with other objects. The proposed method detected all the ellipses properly and all other shape objects were discarded. The CHT and result of the proposed method are shown in Fig. 4(b) and 4(c) respectively.



Fig. 4 Extraction of ellipses from mixed objects image (a) Mixed object image (b) CHT image (c) Extracted ellipses

The proposed method has been tested on a real world 256X256 gray scale image shown in Fig. 5(a). Canny edge operators are used to obtain the edge image shown in Fig. 5(b). To calculate the Eigen values, we use the window size 'w' as threshold (t) as $1.258 \le t \le 1.2$ 7X7 with for λ_i and λ_s respectively. They represent lengths of major and minor axes of larger and smaller ellipses. The CHT of the Eigen value image is very complex, because it consists of all linear and nonlinear points. Sparse matrix technique is used to reduce storage space and computational complexity. the Neighborhood suppression scheme is applied to find the valid Hough peaks which represent the centre location of the ellipses. The CHT with valid Hough peaks identified is shown in Fig. 5(c). The identified ellipses from the CHT are shown in Fig. 5(d). To reveal the correctness and accuracy, the detected ellipses are overlapped with edge image and original image as shown in Fig. 5(e) and 5(f) respectively.



Fig. 5 Detection of ellipses from real image (a) Original image (b) Edge image (c) CHT image (d) Extracted ellipses (e) Overlapped with edge image (f) Overlapped with original image

VII. PERFORMANCE EVALUATION

In order to illustrate the effectiveness of the proposed method, we compare it with several HT and tangential based methods. The domain of interest for this experimental comparison is computational time and percentage of accuracy. The proposed method is compared with standard Hough transform (SHT), randomized Hough transform (RHT), Probabilistic Hough transform (PHT), Progressive probabilistic HT(PPHT), Combinatorial HT(CHT), Geometric symmetry Hough transform (GSHT) and tangential method. The correctness of output is measured by the error rate which is the ratio of real primitives to the number of spurious or mismatched primitives. To test the accuracy and performance we used a 64X64 binary synthetic ellipse image shown in Fig. 6(a) with xc=32, Y_c=32, a=68,b=43 and $\theta = 0^{0}$. Fig. 6(b) shows the HT with peak identified. The result of different HT methods overlapped with original image is shown in Fig. 6.



Fig. 6 Results of different HT methods (a) Original image (b) Circular HT (c) SHT (d) RHT (e) PHT (f) PPHT (g) Combinatorial HT (h) GSHT (i) Tangential Method (j) Our method

Table III shows the error factor and computational time required to identify the ellipses after performing the HT and valid peak identification for different HT methods and our proposed method. From Table III it is evident that, our method detects the elliptical features correctly. Hence the error factor is 1.0125 i.e. the accuracy is 98.76% which is more than any other HT based methods. The graphical representation of the same is as shown in Fig. 7. The computational time of our method is approximately equal to GSHT and tangential methods but the accuracy in ellipse identification is considerably good when compared to other HT based methods. In our work the execution time is concerned with only detection of ellipses. The propagation time is not considered. From Table III it is apparent that our proposed method takes less computational time and it is shown in Fig. 7.

TABLE III Comparison of Different HT Methods and our Proposed Method									
Sl.No	HT methods	Computational time (Sec)	Error factor	% of Accuracy					
1	SHT	73.52	1.9852	50.37					
2	RHT	69.74	1.8491	54.80					
3	PHT	53.79	1.6283	61.41					
4	PPHT	42.51	1.4236	70.24					
5	CHT	37.89	1.1253	88.87					
6	GSHT	32.42	1.0353	96.59					
7	Tangential method	32.38	1.0241	97.64					
8	Our Method	32.27	1.0125	98.76					







Fig. 8 Performance evaluation on noisy image (a) Original image (b) Noisy image (c) Edge image (d) Ellipses extracted (e) Overlapped with edge image (f) Overlapped with original image

Another evaluation is done to measure the resistance of our method against noise. In this evaluation the density of salt and pepper noise was increased from 0 to 70 percent and the error rate was evaluated. Results of this experiment show that the noise with less than 22% density has no effect on the accuracy of the ellipse identification.

In contrast the HT and its variants lose robustness against noise from 9%. After this threshold, the error rate increases exponentially. When the noise density exceeds 32%, the SHT totally fails to identify the features. But our method works satisfactorily up to 70% of the noise density. Fig. 8 shows the result of applying the proposed method on a noisy image. It detected two ellipses correctly when 40% salt and pepper noise was applied to original image. Fig. 9 draws the error rate versus noise density. From this it is evident that our proposed method can withstand noise up to 70% and being efficient than other HT methods which are very sensitive to noise.



VIII. CONCLUDING REMARKS

In this paper we discussed an efficient and accurate method for ellipse detection in both simulated and real world images. The distinct features of our proposed method from HT based methods are, it is a hybrid system consisting of Eigen values approach, CHT and raster scan algorithm. It does not use the tangents of edge points which are difficult to be correctly estimated. It avoids false alarms. The amount of data required for ellipse detection and parameter estimation using the proposed method is minimal. Performance evaluation is done by comparing different HT methods with our method on error factor, computational time and noise withstanding capability. The algorithms based on HT require more memory and high computational time. As compared with conventional HT methods, the main strengths of our method are its low computational time, less memory requirement and accuracy of detection. Since, Bresenham's raster scan algorithm is used to identify the positions of the geometric primitive, it works well when the boundary is distorted and disconnected due to noise or occlusions.

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