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Numerical Analysis of Oil-Water Transport in Horizontal Pipes Using 1D Transient Mathematical Model of Thermal Two-Phase Flows

Evgeniy Burlutskiy

***Abstract*—**The paper presents a one-dimensional transient mathematical model of thermal oil-water two-phase emulsion flows in pipes. The set of the mass, momentum and enthalpy conservation equations for the continuous fluid and droplet phases are solved. Two friction correlations for the continuous fluid phase to wall friction are accounted for in the model and tested. The aerodynamic drag force between the continuous fluid phase and droplets is modeled, too. The density and viscosity of both phases are assumed to be constant due to adiabatic experimental conditions. The proposed mathematical model is validated on the experimental measurements of oil-water emulsion flows in horizontal pipe [1,2]. Numerical analysis on single- and two-phase oil-water flows in a pipe is presented in the paper. The continuous oil flow having water droplets is simulated. Predictions, which are performed by using the presented model, show excellent agreement with the experimental data if the water fraction is equal or less than 10%. Disagreement between simulations and measurements is increased if the water fraction is larger than 10%.

***Keywords*—**Mathematical model, Oil-Water, Pipe flows.

I. INTRODUCTION

A development of new technologies on conventional and heavy oil production require more intensive oil and gas transmission from one place to another one. Water is used usually for the pipeline transport of heavy oil. Multiphase oil-water flows in pipes is much more complex process compare to single phase flows due to phases interaction and re-distribution within the cross-sectional area of the pipeline. The overall pressure drop in oil-water two-phase emulsion flow regime is much less comparing to single phase water or single phase oil flows. However, the pressure loss in multiphase flow regime depends on many parameters such as the fluid density, viscosity, temperature, pipe inner diameter and water fraction. It is difficult to estimate the pressure drop at multiphase flow regime. A transient mathematical model of thermal multiphase oil-water flows in pipes is highly desirable. A local distribution of basic flow parameters, which are obtained by using the mathematical modeling, may significantly help in the pipeline design and flow assurance. The information on the mathematical modeling and experimental study of oil and water two-phase transport in pipes is limited in the open source literature, especially if the viscosity of oil is very high compare to the same value of water. However, intensive experimental measurements of pressure losses at different oil-water mixture velocities and water fraction are conducted during last time [1-5]. The influence of the pipe inner diameter, oil and water density and viscosity ratios, at different multiphase flow regimes is carefully examined experimentally. Viscosity values are varied in those studies in the range between $1 \cdot 10^{-3}\ Pa \cdot s$ and $3400 \cdot 10^{-3}\ Pa \cdot s$. Most of measurements are made by using small-diameter shock tubes, where the friction force influences on the flow behavior much stronger compare to large-diameter pipes.

Most part of oil and gas flow assurance software, which are based on one-dimensional mathematical models of two-phase flows in pipes, are laboratory codes. The ability of such codes is limited on the application area due to various simplified assumptions. A one-dimensional thermal transient two-fluid mathematical model of oil-gas-water three-phase bubbly flows in vertical pipe is presented in [6]. A one-dimensional OLGA code [7], which is developed by SPT-group, is the most well-known and frequently used commercial software in the field of oil and gas flow assurance. Simulations of oil and gas multiphase flows in pipes and wells having a complex topology, which are made by using this code, usually have a high level of accuracy comparing to available measured data. However, this code does not simulate the process of rapid decompression in natural gas mixtures correctly. All predictions, which are made by using OLGA [8], show a significant over-prediction of the pressure time history values and a poor comparison with the experimental data [9].

The paper presents a one-dimensional transient mathematical model of thermal oil-water two-phase emulsion flows in pipes. The proposed mathematical model is validated on the experimental measurements of oil-water two-phase flows in horizontal pipe [1,2]. Numerical analysis on single- and two-phase oil-water flows in a pipe is presented in the paper. Single phase oil and single phase water flows in a pipe are simulated first in order to validate the continuous phase to wall friction correlation. The Blasius [10] and Taitel & Dukler [11] friction correlations are tested on the experimental data. The continuous oil flow having water droplets is simulated, too. Predictions, which are performed by using the presented model, are presented in the paper.

Dr. Evgeniy Burlutskiy was with PETROSOFT-DC, Singapore. He is now with the A*STAR Institute of High Performance Computing (IHPC), 1 Fusionopolis Way, 138632, Singapore (phone: +65 6419 1386; e-mail: burlutskiye@ihpc.a-star.edu.sg and e.burlutskiy@petrosoft-dc.com).

II. One-Dimensional Mathematical Model of Transient Thermal Two-Phase Flows in Pipes

The set of the mass, momentum and enthalpy conservation equations for continues fluid phase and droplet phase is solved in the mathematical model. This set of equations for the case of two-phase emulsion flows in a pipe in general form is written as [12]:

$$\frac{\partial \alpha_C \rho_C}{\partial t} + \frac{\partial \alpha_C \rho_C U_C}{\partial z} = 0 \tag{1}$$

$$\frac{\partial \alpha_D \rho_D}{\partial t} + \frac{\partial \alpha_D \rho_D U_D}{\partial z} = 0 \tag{2}$$

$$\frac{\partial \alpha_C \rho_C U_C}{\partial t} + \frac{\partial \alpha_C \rho_C U_C^2}{\partial z} = -\alpha_C \frac{\partial P}{\partial z} - R_{C-D} - R_{C-Wall} \tag{3}$$

$$\frac{\partial \alpha_D \rho_D U_D}{\partial t} + \frac{\partial \alpha_D \rho_D U_D^2}{\partial z} = -\alpha_D \frac{\partial P}{\partial z} + R_{C-D} \tag{4}$$

$$\frac{\partial \alpha_C \rho_C h_C}{\partial t} + \frac{\partial \alpha_C \rho_C U_C h_C}{\partial z} = \alpha_C \left(\frac{\partial P}{\partial t} + U_C \frac{\partial P}{\partial z} \right) \tag{5}$$

$$\frac{\partial \alpha_D \rho_D h_D}{\partial t} + \frac{\partial \alpha_D \rho_D U_D h_D}{\partial z} = \alpha_D \left(\frac{\partial P}{\partial t} + U_D \frac{\partial P}{\partial z} \right) \tag{6}$$

$$\alpha_C + \alpha_D = 1 \tag{7}$$

Here, indexes C, D denotes the continuous (C) and droplet (D) phases; ρ_C, ρ_D are densities of the continuous phase and droplets, correspondently; α_C, α_D are the volume fraction of the continuous phase and droplets; U_C, U_D are the velocities of the continuous phase and droplets; P is the total pressure; R_{C-Wall} is the friction between the continuous fluid phase and the wall; R_{C-D} is the friction term, which represents the friction between the continuous phase and droplets; h_C, h_D are the enthalpy of the continuous phase and droplets, t is the time, z is the axial co-ordinate. The water is considered as the continuous fluid phase if the oil phase is represented by droplets in the model and opposite.

The friction between the core phase and the wall, which is considered in the present paper, is written in Blasius form (8) and in Taitel & Dukler form (9) here [10,11] as:

$$R_{C-Wall} = \frac{\Pi}{S} \frac{\xi_{C-Wall}\, \rho_C U_C^2}{8}$$
$$\begin{cases} \xi_{C-Wall} = 64 / Re_C \quad , \quad Re_C < 1600 \\ \xi_{C-Wall} = 0.316 / Re_C^{0.25} \quad , \quad Re_C > 1600 \end{cases} \tag{8}$$

$$R_{C-Wall} = \frac{\Pi}{S} \xi_{C-Wall}\, \rho_C U_C^2$$
$$\begin{cases} \xi_{C-Wall} = 64 / Re_C \quad , \quad Re_C < 1600 \\ \xi_{C-Wall} = 0.046 / Re_C^{0.2} \quad , \quad Re_C > 1600 \end{cases} \tag{9}$$

$$Re_C = \rho_C U_C D_{pipe} / \mu_C \tag{10}$$

Here, Π is the perimeter of the pipe; S is the cross-sectional area of the pipe; ξ_{C-Wall} is the friction coefficient, D_{pipe} is the diameter of the pipe; μ_C is the continuous phase viscosity.

The friction between continuous core phase and droplets is written [13] as:

$$R_{C-D} = \frac{6\,\alpha_D}{d_{droplet}} \frac{\rho_D \left(U_C - U_D \right) d_{droplet}}{6\,\tau_u} \tag{11}$$

$$\tau_u = \frac{4}{3} \frac{\rho_D}{\rho_C} \frac{d_{droplet}}{C_D \left| U_C - U_D \right|} \tag{12}$$

$$C_D = \begin{cases} \frac{24}{Re_D} \left(1 + 0.15 \cdot Re_D^{0.687} \right) \quad , \quad Re_D \le 1000 \\ 0.44 \quad , \quad Re_D > 1000 \end{cases} \tag{13}$$

$$Re_D = \rho_C \left| U_C - U_D \right| d_{droplet} / \mu_C \tag{14}$$

Here, C_D is the aerodynamic friction coefficient. Continuous fluid phase viscosity is considered to be constant due to adiabatic boundary conditions in the experimental data [1,2]. However, the model is progress to account for the viscosity as a function of the density and temperature.

The algorithm of solving of the set of One- Dimensional transient governing equations of the fluid mixture flow in a pipe is based on the Tri-Diagonal Matrix Algorithm (TDMA), also known as the Thomas algorithm [14]. It is a simplified form of Gaussian elimination that can be used to solve tri-diagonal systems of equations. The set of unsteady governing equations is transformed into the standard form of the discrete analog of the tri-diagonal system [14] by using the fully implicit numerical scheme. In this case the equation is reduces to the steady state discretization equation if the time step goes to infinity.

III. Transient Multiphase Flow Program - Dead Oil (TMPD-pipe code)

A one-dimensional transient mathematical model of thermal multiphase flows in pipes with constant fluid density was developed under the research project “Oil-Gas-Water Multi-Phase flows in pipelines and wells” in PETROSOFT-DC. The mathematical model was implemented into the FORTRAN computer code and was named the “Transient Multiphase flow Program for Dead oil - pipe flows” (TMPD-pipe code). More information is available on www.petrosoft-dc.com.

IV. Numerical Analysis on Single- and Two-Phase Emulsion Flows of Oil and Water in a Pipe

The presented mathematical model is validated on the experimental data on oil-water two-phase flows in pipes [1,2]. The measurements are made at University College London, UK. The main test section of the facility is horizontal stainless steel pipe having a total length of 8 m and the inner diameter of 38 mm. Water and oil are used as the test fluids. The density and viscosity of oil are 828 kg/m^3 and $6 \cdot 10^{-3}\ Pa \cdot s$, correspondently. Measured oil-water mixture velocity at the test section is varied in the range from 0.8 m/s to 3 m/s. The water fraction is varied from 0% (i.e. pure oil) to 100% (i.e. pure water) in the experiments.

The computational pipe, which has a length of 8 m and the inner diameter of 38 mm, is simulated by using the proposed model. The set of governing equations is solved on the mesh having 40 grid nodes. Predictions are started from setting up the input velocity, temperature and volume fraction values of each phase at pipe inlet and the pressure drop values over the pipe. New values of the velocity, temperature and volume fraction of each phase (i.e. oil and water) are re-calculated at every time step. The mass flow rate of each phase is kept constant at each computational node over the pipe as well as the total mass flow rate. The upper limit of the time step is selected from the point of view of the numerical stability of calculations. The time step is chosen to be of $1 \cdot 10^{-3}$ sec in those calculations.

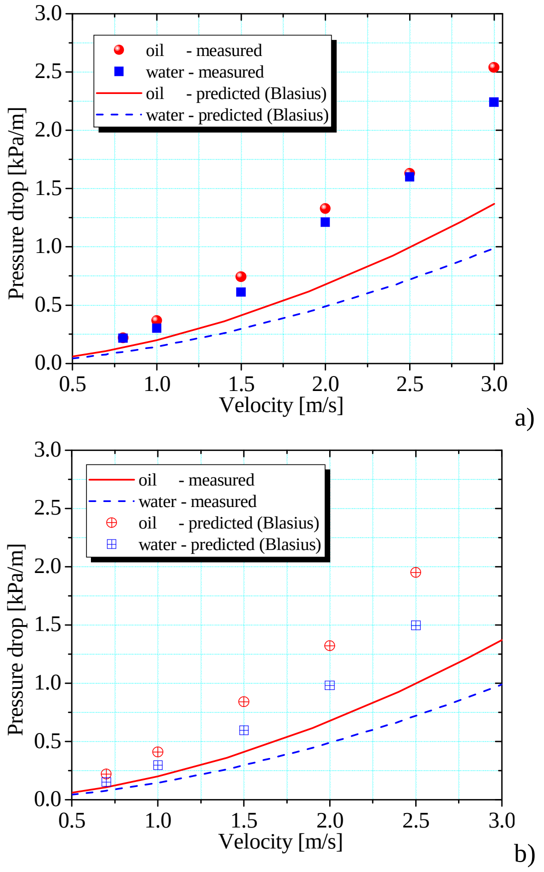

Fig. 1 Measured and predicted pressure drop as a function of velocity (a) [1] and (b) [2]

Predictions are started from single-phase flow simulations. The continuous oil and continuous water flows are calculated by using the proposed mathematical model (i.e. TMPD-pipe code) in the experimental conditions of [1,2]. The Blasius friction correlation (8), which accounts for the continuous fluid to wall friction, is tested on the selected set of experimental data. Fig. 1 shows measured and predicted values of the pressure drop as a function of fluid velocity. Symbols represent the experimental measurements in all figures here. Predictions, which are performed by using the proposed model, are shown as curves. The overall pressure drop is significantly under-predicted by the proposed model if the Blasius friction correlation is employed in the model (fig. 1).

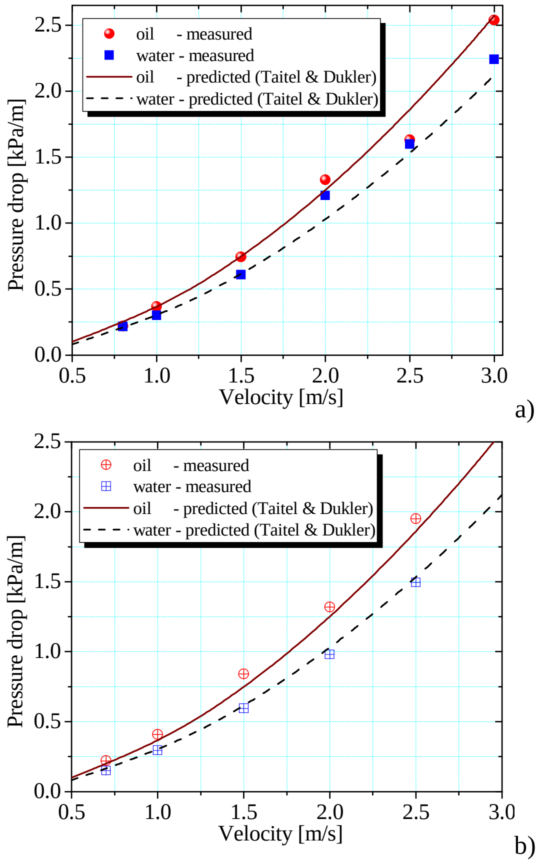

Fig. 2 Measured and predicted pressure drop as a function of the velocity (a) [1] and (b) [2]

The Taitel & Dukler [11] friction correlation (9) is tested on the same experimental data set. Fig. 2 shows measured and predicted pressure drop values for different velocities. Predicted values show a better fit to the experimental data compare to previous one if the Taitel & Dukler friction correlation is employed in the model. Therefore, the Taitel & Dukler friction correlation is selected to be accounted for in the proposed mathematical model due to a better agreement between predicted and measured values of the pressure drop in this case.

Two-phase oil-water flows in a pipe are simulated by using the presented mathematical model when the continuous fluid phase to wall friction correlation is selected (i.e. Taitel & Dukler). The proposed model is validated on oil-water two-phase emulsion flows in a pipe [1]. The continuous oil flow having water droplets is considered. Fig. 3(a) shows measured and predicted pressure drop values as a function of oil and water mixture velocity. The flow having water fraction (WF) of 10% is simulated. The friction between the continuous oil phase and the wall as well as the aerodynamic drag force between water droplets and the continuous oil phase is accounted for in the presented model, too. Fig. 3(a) shows a good comparison between predictions and measurements. This fact indicates that total friction balance is modeled correctly in the presented mathematical model.

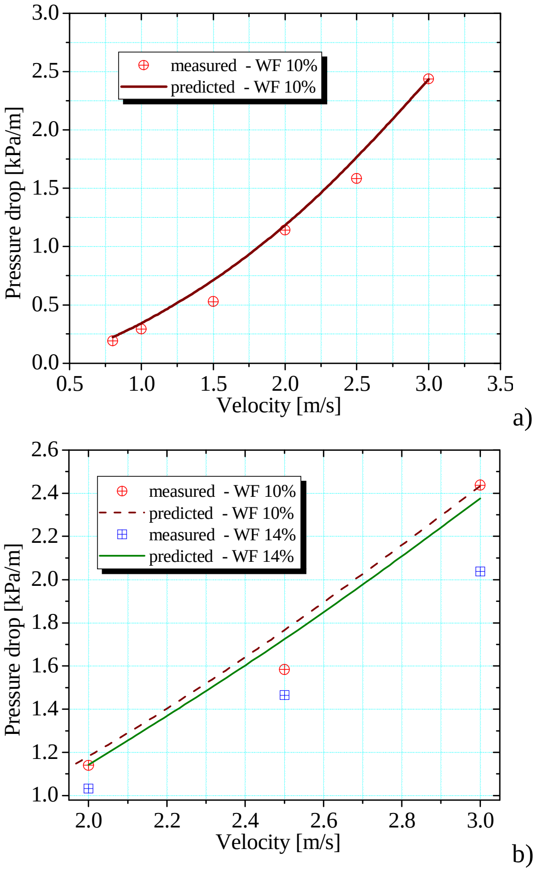

Fig. 3 Measured and predicted pressure drop as a function of mixture velocity (a) WF of 10% and (b) WF of 14%; [1]

The continuous oil flows having the water fraction of 14% (figure 3(b)) and 20% (figure 4(a)) are simulated by using TMPD-pipe code, too. Both, fig. 3(b) and 4(a) contain measured and predicted values of the pressure drop, when the water fraction is equal to 10%, as well. Those values are used in those figures as the reference. Figures show a small decrease in predicted values for water fraction of 14% and 20% compare to previous case (i.e. the water fraction of 10%). The numerical study shows that slow decrease in pressure drop values, when the water fraction is less that 10%, is caused due to better transport of water droplets compare to single phase flows. The proposed model simulates oil-water two-phase emulsion flow correctly if the water fraction is equal or less than 10-12%.

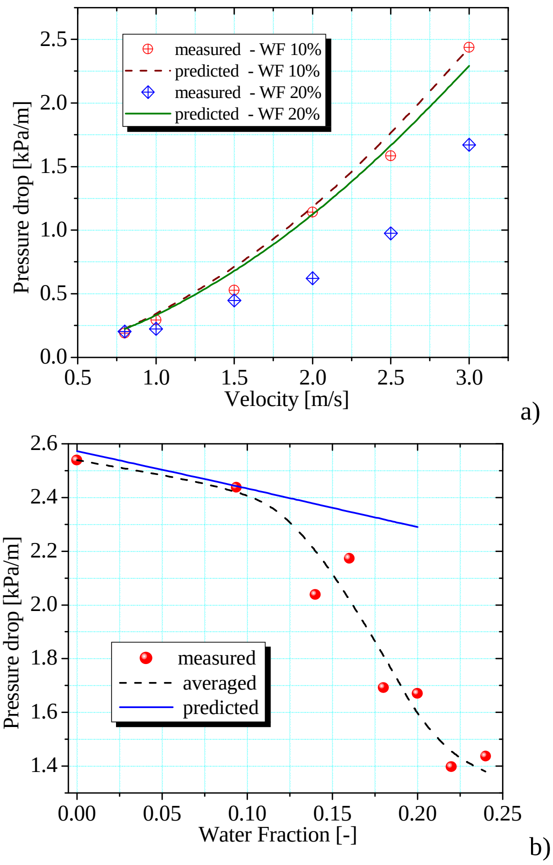

Fig. 4 Measured and predicted pressure drop as a function of (a) mixture velocity, WF of 20% and (b) water fraction, mixture velocity of 3 m/s; [1]

Fig. 4(b) explains the disagreement between measured and predicted data for the case of large water fraction values (i.e. more than 10-12%). The pressure drop of oil-water mixture as a function of water fraction is shown on fig. 4(b). Those values correspond to oil-water mixture velocity of 3m/s. The average curve here (fig. 4(b)) is created by the paper author himself in order to show the average behavior of the experimental data in this range due to high oscillation of measured points. This average curve is the “averaged” experimental reference line here. The decrease in measured pressure drop is very strong when the water fraction is larger than 10% (fig. 4(b)). The minimum in pressure drop values is reached when the water fraction is about 25-30% [1]. Those values are even lower than pressure drop values in the case of single phase oil and single phase water flows (fig. 2). A rapid strong decrease in pressure drop values, when the water fraction is larger than 10-12%, is not caused due to better transport of water droplets (i.e. not due to the droplet drag force contribution). Numerical analysis shows that the contribution of the droplet drag force itself is not strong enough to reduce the overall pressure drop in the pipe so dramatically. The pressure drop strong decrease is caused due to the phase’s re-distribution within the cross-sectional area of the pipe. The proposed mathematical model of transient thermal two-phase flows in a pipe predicts the velocity of both phases and the overall pressure drop in oil-water emulsion flows in a pipe much better than other analytical and mathematical models, which are available from the open source literature. Simulations, which are performed by using the presented mathematical model, are quick in time also.

V. CONCLUSION

The one-dimensional transient mathematical model of thermal oil-water two-phase emulsion flows in pipes is presented in the paper. The set of the mass, momentum and enthalpy conservation equations for the continuous fluid and droplet phases are solved in the model. Blasius and Taitel & Dukler friction correlations for the continuous phase to wall friction are considered and validated in the model. The aerodynamic drag force between the continuous fluid phase and droplets in the form of Schiller-Naumann is accounted for. Numerical analysis on single- and two-phase oil-water flows in a pipe is presented in the paper.

Proposed mathematical model is validated on the experimental measurements of oil-water two-phase flows in horizontal pipe [1,2]. Blasius correlation for the continuous phase to wall friction shows a poor comparison with the experimental data for the case of single phase oil and single phase water flows in a pipe. The Taitel & Dukler friction correlation is chosen to be the basic friction correlation in the model due to better performance and agreement with experimental data. Numerical analysis on single- and two-phase oil-water flows in a pipe is presented in the paper. The continuous oil flow having water droplets is simulated. Predictions, which are made by using the presented model, show excellent agreement with the experimental data if the water fraction is equal or less than 10%. Disagreement between simulations and measurements is increased if the water fraction is larger than 10%.

The presented model is very useful in pipeline design and flow assurance. The model is successfully approved on the experimental data on oil-water two-phase emulsion flows in a pipe. The influence of the density, viscosity, pressure, temperature, water fraction, mixture velocity, and inner pipe diameter may be examined by using the presented model.

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