

# A Simulation Study of Bullwhip Effect in a Closed-Loop Supply Chain with Fuzzy Demand and Fuzzy Collection Rate under Possibility Constraints

Debabrata Das, Pankaj Dutta

**Abstract**—Along with forward supply chain organization needs to consider the impact of reverse logistics due to its economic advantage, social awareness and strict legislations. In this paper, we develop a system dynamics framework for a closed-loop supply chain with fuzzy demand and fuzzy collection rate by incorporating product exchange policy in forward channel and various recovery options in reverse channel. The uncertainty issues associated with acquisition and collection of used product have been quantified using possibility measures. In the simulation study, we analyze order variation at both retailer and distributor level and compare bullwhip effects of different logistics participants over time between the traditional forward supply chain and the closed-loop supply chain. Our results suggest that the integration of reverse logistics can reduce order variation and bullwhip effect of a closed-loop system. Finally, sensitivity analysis is performed to examine the impact of various parameters on recovery process and bullwhip effect.

**Keywords**—Bullwhip Effect, Fuzzy Possibility Measures, Reverse Supply Chain, System Dynamics.

## I. INTRODUCTION

WITH the technological advancements and rapid changes in demand pattern, diverse ranges of products are entering into the market with reduced lifecycle which leads to the environmental disasters. The awareness of product take-back and recovery has been increasing in various supply chains not only due to the obligation imposed by legislation but also competitive economics worldwide [1]. Pagell et al. [2] pointed out that product remanufacturing is the most desirable option for end-of-life product management than a scrap or spares recovery since it minimizes the environmental impacts, results in lower loss of value, and can create new market opportunities. Remanufactured products are often offered as an alternative to the original products to the customers those are attracted by the brand, but do not wish to pay the price of a new product [3]. For example, there are number of industries such as electronics and automobiles in which the remanufactured product is priced lower than original products in order to capture the demand [4]. In the

present study, we address the benefits of employing product exchange (PE) policy in customer's outlet to increase the collection rate of used products and consequently selling of remanufactured products.

Fleischmann et al. [5] provided a review of the quantitative models for reverse logistics (RL) in which they reported that most of the papers in the area of integrated RL are confined to single issues while comprehensive approaches are rare as variety of factors are involved in a general framework and the complexity of their interdependencies. Furthermore, long-term strategic management problems of a closed-loop system have not been studied extensively. System dynamics (SD) is a powerful methodology for obtaining the insights of these kinds of problems having dynamic complexity; but there are very few literatures which modeled the integrated aspects of forward and reverse supply chain using SD. Spengler and Schroter [6] modeled an integrated production and recovery system for supplying spare parts using SD to evaluate various strategies. Georgiadis and Vlachos [7] developed a SD model to evaluate the effect of environmental issues on long-term decision making in collection and remanufacturing activities. In the present study, we develop a simulation model using SD framework for a closed-loop supply chain in which the uncertainty issues associated with acquisition and collection of used product have been quantified using possibility measures.

An important observation in supply chain, known as bullwhip effect refers to the phenomenon where orders to the supplier tend to have larger variance than actual sales to end consumer (e.g., demand distortion), and the distortion propagates upstream in an amplified form (e.g., variance amplification) [8]. Carlsson and Fuller [9] summarized the negative effects of amplification which encourage researchers to study on how to avoid and lessen bullwhip effects. As the quality and quantity of used products return to collection points are uncertain in reverse channel, systematic distortion is inevitable and bullwhip effect may occur at retailer, distributor and manufacturer level. There are very few literatures who talked about order variations and bullwhip effect in a closed-loop supply chain ([10]-[11]), but it is yet to receive attention in the context of SD framework ([12]-[13]).

In this paper, we propose a SD framework for a closed-loop supply chain with fuzzy demand and fuzzy collection rate to analyze the forward as well as backward movements of product through different stages of supply chain network with three way recovery, namely; product remanufacturing, component reuse & remanufacturing, and raw material

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recovery. We bring in the concept of PE policy to make the collection and recycling process faster and better. We simulate the order variation of different logistics participants over time and compare the bullwhip effects of the traditional forward

## II. PROBLEM DESCRIPTION

In this work, we focus on an integrated forward-reverse supply chain (see Fig. 1). Due to the high complexity of the problem; we divide the whole system into two parts:

### A. Forward Supply Chain with Product Exchange Policy

In forward supply chain, generally, the finished products are first transferred from the producer/manufacturer to the distributor then to the retailer and finally sold to the customer to satisfy the demands. Due to the increasing standards of living, the concept of product exchange is getting popularity in India, especially for automobile and electronics products. In this model we categorize the demand as “demand with exchange” and “demand without exchange”. The customer can exchange their old used product with a fresh new product in a retail market at a discounted price which will effectively increase the product collection rate, market share and satisfy the customer’s need. Hence, the incorporation of PE policy plays an important role in the process of RL.

### B. Reverse Supply Chain with Three Way Recovery

In the reverse channel, we address the recovery process in three distinct ways, namely; product remanufacturing, component reuse and remanufacturing, and raw material recovery. We assume that remanufacturing activity can bring the products and components back into an “as good as new” condition by carrying out the necessary disassembly, overhaul and replacement operations. The products sold at the end of their life-cycle turn into used products, which are collected for reuse. In the simulation study, it is assumed that there is no constraint on the capacity of collection, inspection, sorting and restoring. After initial inspection, if the collected products are

supply chain with that of closed-loop supply chain. Also, sensitivity analysis is performed to examine the impact of various parameters on recovery process and bullwhip effect.

accepted for remanufacturing, then with some reprocessing, the *remanufactured products* can be sold in the retail market. If the products are not in a condition to remanufacture, then it is disassembled into various components. During the process of product remanufacturing, if new replacement is required for some components, then the old components are sent to reprocessing center for further recovery. In this model, we assume that the derived components can have three categories: one is *direct reusable components* that can be directly used without any further processing to increase the inventory of component in the forward channel; the second one is the *remanufactured component* which require some reprocessing before adding it to the component inventory in the forward channel; the rest of the components can be used either to *recover raw material* which effectively increase the raw materials inventory in the forward channel or can be sent directly for controllable disposal as shown in Fig. 1.

## III. FUZZY SET THEORY

A decision situation related to human aspects, in fact, has only a little to do with the absolute attributes – certainty and precision – which are not present in our cognition, perception, reasoning and thinking. It has been argued in a large body of recent literature that fuzzy sets theory could provide an appropriate framework for dealing with uncertainties in areas where intuition and subjective judgment play an important role. In such cases uncertainty is caused by the imprecision of natural language description rather than the existence of statistical frequency of the occurrence of events. Zadeh [14] has propounded the fuzzy set theory but detail reference for fuzzy sets and its applications can be found in [15].

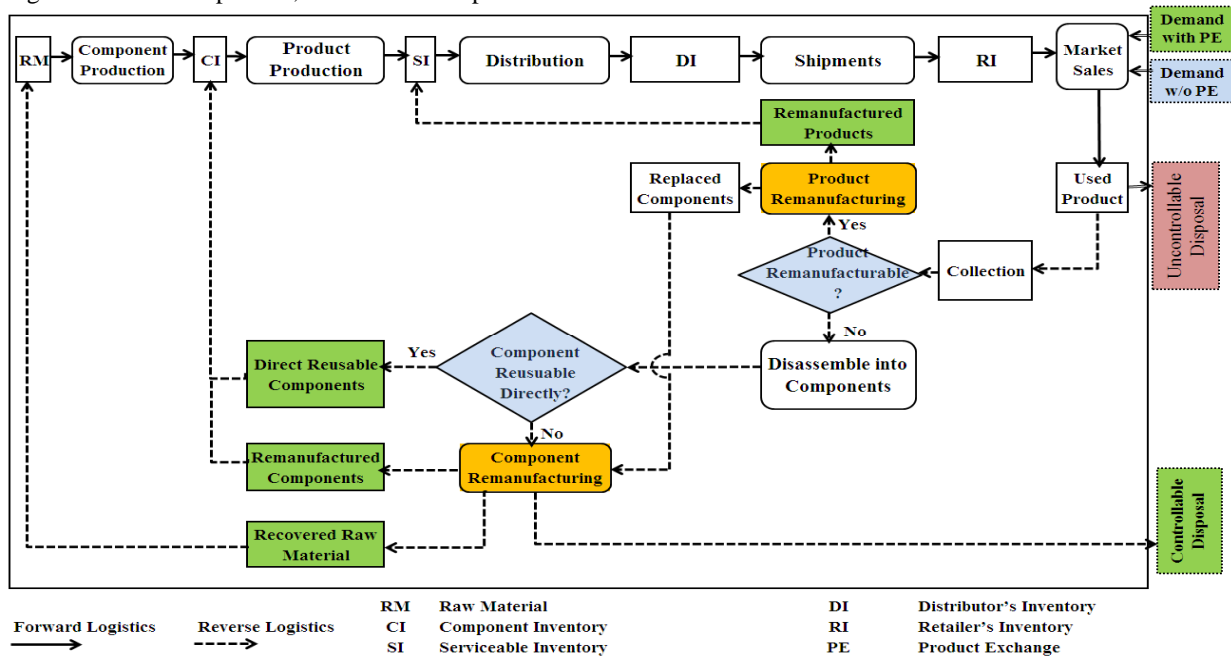


Fig. 1 Flow chart of the closed-loop supply chain

### A. Triangular Fuzzy Number

Triangular fuzzy number (TFN)  $\tilde{a}$  (see Fig.2) is the fuzzy number with the membership function  $\mu_{\tilde{a}}(x)$ , a continuous mapping:  $\mu_{\tilde{a}}(x): \mathbb{R} \rightarrow [0,1]$  such that

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & \text{for } -\infty < x < a_1 \\ \frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{for } a_3 < x < \infty \end{cases}$$

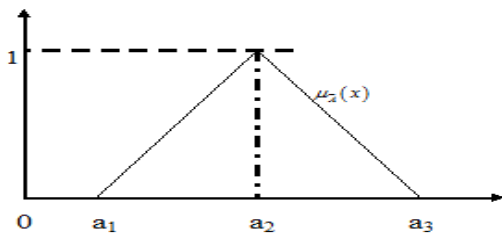


Fig. 2 Membership function of TFN  $\tilde{a}$

### B. Possibility and Necessity measures

There are several representations of fuzzy constraints. Here we use possibility/necessity measure concept in which fuzzy numbers are interpreted by the degree of uncertainty. Possibility (optimistic sense) and necessity (pessimistic sense) means the maximum and minimum chance (at least) to be selected by the decision maker (DM), respectively. Analogous to chance constrained programming with stochastic parameters, in a fuzzy environment; it is assumed that some constraints may be satisfied with a least possibility,  $\eta_1$  ( $0 \leq \eta_1 \leq 1$ ) (i.e. here the 'chance' is represented by the 'possibility'). Again, some constraints may be satisfied with some predefined necessity,  $\eta_2$  ( $0 \leq \eta_2 \leq 1$ ) [16]. Therefore, if a DM desires to impose the resource constraint in possibility sense, he/she is optimistic and for necessity constraint, it is pessimistic (i.e.; conservative imposition). According to Dubois and Prade [16], if  $\tilde{A}$  and  $\tilde{B}$  be two fuzzy numbers with membership function  $\mu_{\tilde{A}}(x)$  and  $\mu_{\tilde{B}}(y)$  respectively, then

$$Pos(\tilde{A} * \tilde{B}) = \{\sup(\min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \mathfrak{R}, x * y)\}$$

$$Nes(\tilde{A} * \tilde{B}) = \{\inf(\max(1 - \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)), x, y \in \mathfrak{R}, x * y)\}$$

where the abbreviation *Pos* represents possibility, *Nes* represents necessity and  $*$  is any of the relations  $<, >, =, \leq, \geq$ . The dual relationship of possibility and necessity requires that  $Nes(\tilde{A} * \tilde{B}) = 1 - Pos(\overline{\tilde{A} * \tilde{B}})$ .

Let  $\tilde{a} = (a_1, a_2, a_3)$  be a triangular fuzzy number and  $b$  is a crisp number then the following lemma holds [17]:

*Lemma: When  $b$  is a crisp number,  $Pos(\tilde{a} \geq b) \geq \eta$  if and*

*only if  $\frac{a_3 - b}{a_3 - a_2} \geq \eta$  (see Fig.3)*

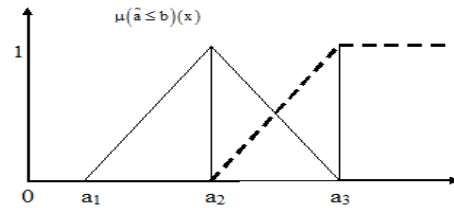


Fig. 3 Measures of  $Pos(\tilde{a} \geq b)$

## IV. SYSTEM DYNAMICS MODEL

System dynamics is a modeling and simulation methodology for framing, understanding, and discussing complex issues and problems. The structure of a system in SD methodology is described by causal-loop diagrams. A causal loop diagram is a visual representation of the feedback loops in a system. A negative or balancing feedback loop exhibits goal-seeking behavior and the system seeks to return to an equilibrium situation. In a positive, or reinforcing, feedback loop an initial disturbance leads to further change, suggesting the presence of an unstable equilibrium. The structure of a SD model contains stock (state), flow (rate) and auxiliary/constant variables. Stock variables are the accumulations (e.g. inventories) within the system. The flow variables represent the flows in the system (e.g. remanufacturing rate) from one stock to another. With a causal loop diagram, the stock and flow diagram shows relationships among variables which have the potential to change over time.

The mathematical formulation consists of a system of differential equations, which is numerically solved via simulation. Nowadays, high-level graphical simulation programs support the analysis and study of these systems. These programs include Vensim, i-think and Powersim etc. Here, we choose Vensim (version: windows 5.10 e) as a tool to simulate the model.

The *stock variables* in order that they appear in the forward and reverse channel are the following: Raw Materials, Components Inventory, Serviceable Inventory, Distributor Orders Backlog, Distributors Inventory, Retail Orders Backlog, Retail Inventory, Collected Products, Uncontrollable Disposal, Product Accepted For Remanufacturing, Products Rejected For Remanufacturing, Inventory Of Components From Rejected Products, Components Accepted For Direct Reuse, Components Rejected For Direct Reuse, Remanufactured Components, Recovered Raw Material, Controllable Disposal.

The *flow variables* accordingly the orders of their associated stock variables are following: Components Production Rate, Products Production Rate, Components Used For Product Production, Shipments To Distributor, Distributor Orders Backlog Reduction Rate, Distributors Order, Shipments To Retailer, Retail Order Backlog Reduction Rate, Retail Order, Retail Sale, Total Collection Rate, Product Acceptance Rate For Remanufacturing, Product Rejection Rate For Remanufacturing, Product Remanufacturing Rate, Component Replacement Rate, Components From Rejected Products, Components Rejection Rate For Direct Reuse,

Components Acceptance Rate For Direct Reuse, Component Remanufacturing Rate, Raw Material Recovery Rate, Disposal Rate, Uncontrollable Disposal Rate.

The first step of the analysis is to capture the relationships among the system operations in a SD manner. Fig.4 depicts the stock-flow diagram of the closed-loop supply chain with PE policy in customer outlets. Because of the high complexity of the closed-loop supply chain, it can be divided into the following subsystems:

1. Forward Supply Chain
2. Demand with Product Exchange Policy
3. Reverse Supply Chain
4. Possibility Constraints on Collection and Satisfaction Rate

**A. Forward Supply Chain**

The forward supply chain begins from the upper left corner of Fig. 4. Raw Materials are furnished by external suppliers and recycling the used products (Raw Material Recovery Rate) from the reverse channel as well. Components Production Rate depletes raw materials and increase Components Inventory.

The equations related to component production rate are

following:

$$\text{Components Production Rate} = \text{MAX} (\text{MIN} (\text{MIN} (\text{Raw Materials} / \text{Component Production Time}, (\text{Expected Distributors Orders} * \text{Components Per Product} - \text{Expected Reusable Component} + \text{CI Discrepancy} / \text{CI Adj Time})), \text{Component Production Capacity}), 0)$$

$$\text{Expected Reusable Components} = \text{SMOOTH} (\text{Component Remanufacturing Rate} + \text{Components Acceptance Rate for Direct Reuse}, 1)$$

$$\text{CI Discrepancy} = \text{MAX} (\text{Desired CI} - \text{Components Inventory}, 0)$$

$$\text{Components Inventory} = \text{Integration} (\text{Components Production Rate} + \text{Component Remanufacturing Rate} + \text{Components Acceptance Rate for Direct Reuse} - \text{Components used for Product Production})$$

One important term of Components Production Rate is CI Adj time that represents how quickly the firm tries to correct the discrepancy. The remanufacturing process supplements the production process. Producer's requirement for components is satisfied with a mix of new components produced by firm, and remanufactured components derived from used products.

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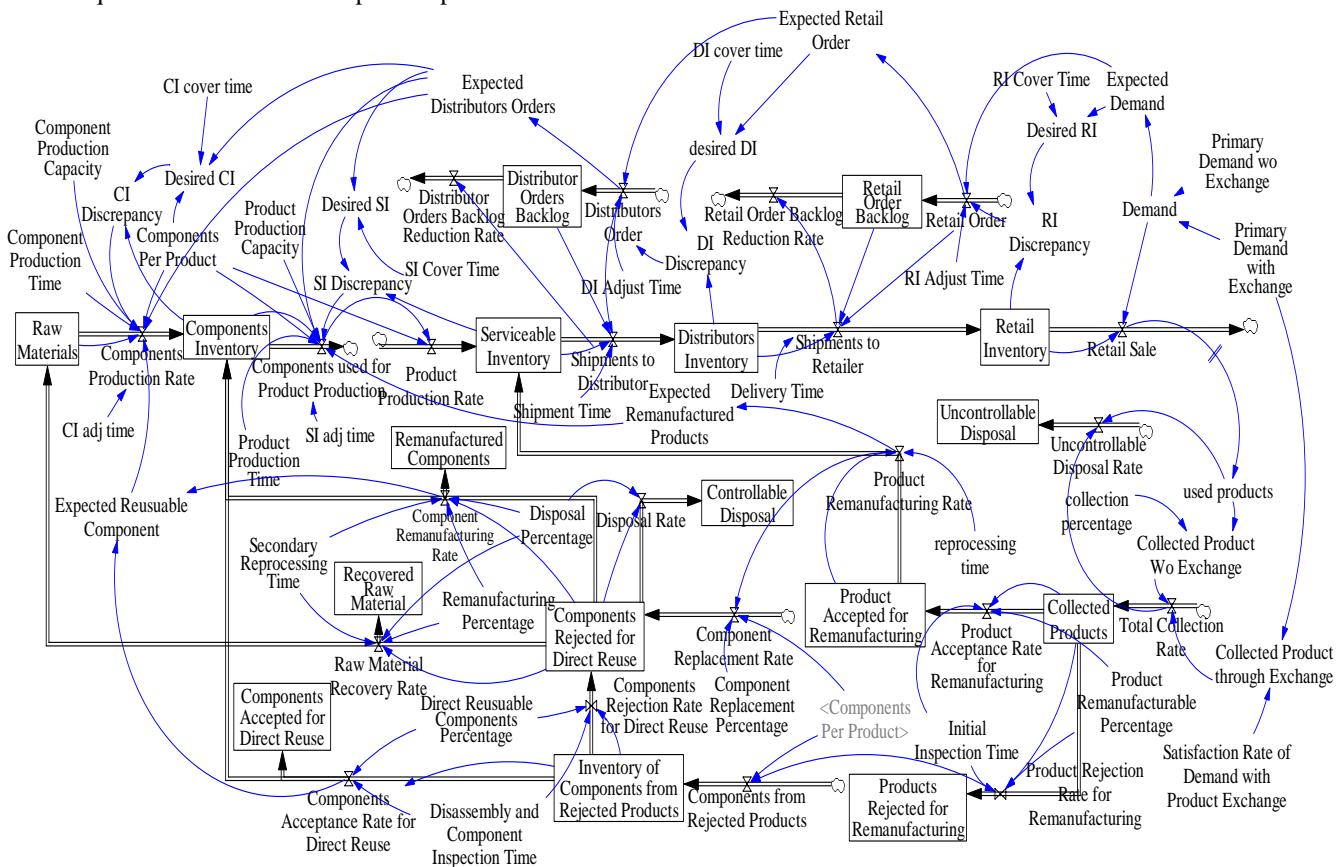


Fig. 4 Stock-flow diagram of the closed-loop supply chain with PE policy

Similarly, Producer's requirement for products is satisfied with a mix of new products produced by firm, and remanufactured products derived from collected products. The equations related to product production rate are following:

$$\text{Product Production Rate} = \text{Components used for Product Production} / \text{Components Per Product}$$

$$\text{Components used for Product Production} = \text{MAX} (\text{MIN} (\text{MIN} (\text{Components Inventory} / \text{Product Production Time}, \text{Product Production Capacity} * \text{Components Per Product}), (\text{Expected Distributors Orders} - \text{Expected Remanufactured Products} + \text{SI discrepancy} / \text{SI Adj Time}) * \text{Components Per Product}), 0)$$

$$\text{Expected Remanufactured Products} = \text{SMOOTH} (\text{Product Remanufacturing Rate}, 1)$$

$$\text{SI discrepancy} = \text{MAX} (\text{Desired SI-Serviceable Inventory}, 0)$$

$$\text{Serviceable Inventory} = \text{Integration} (\text{Product Production Rate} + \text{Product Remanufacturing Rate} - \text{Shipments to Distributor})$$

Product production rate depletes Component Inventory and increase Serviceable Inventory. Shipments to Distributor deplete Serviceable Inventory and increase Distributors Inventory. In the same way, products delivered from the upper stream increase the inventory of retailer, which can satisfy the demand of end-users. The equations related to distributor's inventory, transportation and order are presented below:

$$\text{Distributors Inventory} = \text{Integration} (\text{Shipments to Distributor} - \text{Shipments to Retailer})$$

$$\text{Shipments to Distributor} = \text{IF THEN ELSE} (\text{Serviceable Inventory} - \text{Distributors Order} - \text{Distributor Orders Backlog} >= 0, \text{Distributors Order} + \text{Distributor Orders Backlog}, \text{Serviceable Inventory}) / \text{Shipment Time}$$

$$\text{Distributor Orders Backlog} = \text{Integration} (\text{Distributors Order} - \text{Distributor Orders Backlog Reduction Rate})$$

$$\text{Distributors Order} = \text{Expected Retail Order} + \text{DI discrepancy} / \text{DI Adjust Time}$$

$$\text{Distributor Orders Backlog Reduction Rate} = \text{Shipments to Distributor}$$

#### B. Demand with Product Exchange Policy

The demand in the retail market has been categorized as "demand with exchange" and "demand without exchange". The exchange policy of old used products with a remanufactured or a fresh new product in the retail market at a discounted price has a great impact in product recovery. In this paper, we assume that demand is lost if it is not satisfied in the current period. Although, *Distributor Orders Backlog* and *Retailer Orders Backlog* are satisfied in a future period.

TABLE I

DEMAND WITH AND WITHOUT PE POLICY

Policy	Demand (Products/week)
Without PE Policy	Random Fuzzy (400,500,600)
With PE Policy	Random Fuzzy (180,200,220)

In the proposed SD model, we assume that all the demands are triangular fuzzy number which is shown in TABLE I and

those demands are represented into the SD framework as follows:

$$\text{Demand wo Exchange} = \text{RANDOM TRIANGULAR} (400, 600, 400, 500, 600, 1)$$

$$\text{Demand with Exchange} = \text{RANDOM TRIANGULAR} (180, 220, 180, 200, 220, 1)$$

#### C. Reverse Supply Chain

The recycling process which we incorporate into our SD framework consists of collection, product recovery, and component and material recovery.

In this study, it is assumed that incorporation of PE policy in retail market influence the collection process. Sold products after their uses turn into used products. Then, *Used Products* are either uncontrollably disposed (*Uncontrollable Disposal*) or collected for reuse (*Collected Products*). *Total Collection Rate* depends upon the collection of used products directly from the end-user plus the rate at which the used products get collected from the customer through the PE policy in the retail market. The equations related to collection rate are following:

$$\text{Total Collection Rate} = \text{Collected Product through Exchange} + \text{Collected Product Wo Exchange}$$

$$\text{Collected Product Wo Exchange} = \text{Used Products} * \text{Collection Percentage}$$

$$\text{Collected Product through Exchange} = \text{Demand with Exchange} * \text{Satisfaction Rate of Demand with Product Exchange}$$

Product recovery includes performance testing, sorting, cleaning, and replacing components as necessary, and, in some cases, upgrading the product since it was initially sold. Out of the *Product Accepted for Remanufacturing*, remanufactured products (*Product Remanufacturing Rate*) are added to the serviceable inventory in the forward channel and the components that are replaced (*Component Replacement Rate*) during product remanufacturing by new components are processed further for raw material recovery and component remanufacturing.

$$\text{Product Accepted for Remanufacturing} = \text{Integration} (\text{Product Acceptance Rate for Remanufacturing} - \text{Product Remanufacturing Rate})$$

$$\text{Product Remanufacturing Rate} = \text{Product Accepted for Remanufacturing} / \text{Reprocessing Time}$$

In reprocessing, recovered components and materials from discarded products undergo special or additional processing before reuse. In the model, it is assumed that the disassembled components can have three categories: one is direct reusable components (*Components Accepted for Direct Reuse*) that can be directly used to increase the *Components Inventory* in the forward channel; the second is the part of *Components Rejected for Direct Reuse* which requires further reprocessing. After reprocessing, the *Remanufactured Components* can be used to increase the *Components Inventory* in the forward channel.

$$\text{Component Remanufacturing Rate} = (\text{Components Rejected for Direct Reuse}) * (1 - \text{Disposal Percentage}) * \text{Remanufacturing Percentage} / \text{Secondary Reprocessing Time}$$

The third is rejected components that does not survive the first two screening levels but can be used either for raw material recovery (*Recovered Raw Material*) to increase the *Raw Materials* inventory in the forward channel or sent directly for *Controllable Disposal*.

*Recovered Raw Material = Integration (Raw Material Recovery Rate)*

*Raw Material Recovery Rate = Components Rejected for Direct Reuse\*(1-Disposal Percentage)\*(1-Remanufacturing Percentage) / Secondary Reprocessing Time*

*Controllable Disposal = Integration (Disposal Rate)*  
*Disposal Rate = Components Rejected for Direct Reuse\*Disposal Percentage*

#### D. Possibility Constraints on Collection and Satisfaction Rate

In the proposed SD framework, the uncertainty issues associated with *collection of used products* and *satisfaction of customers willing to exchange their used products for a fresh new product* have been quantified using the possibility constraint programming approach.

*Collection* is a process that involves taking back discarded products from different consumer sources. The quality and quantity of used products return to the collection points are uncertain in the reverse channel. It is always expected that the DM would like to maintain a predefined threshold value of collection rate in every period to increase profitability in remanufacturing and to satisfy the legislations requirements. In the proposed SD model, it is assumed that the collection rate ( $\tilde{C}_R$ ) is a triangular fuzzy distribution,  $\tilde{C}_R = (a_1, a_2, a_3)$  with a constant deviation of 0.20 from the central value i.e.  $a_2 - a_1 = a_3 - a_2 = 0.2$  and that the  $\tilde{C}_R$  should be more than or equal to 50% of used product with at least 95% probability. But, as  $\tilde{C}_R$  is a fuzzy number, the following possibility constraint has to be satisfied to fulfill DM's requirement:

$$Pos(\tilde{C}_R \geq 0.5) \geq 0.95$$

Now, from the lemma of section (3), it is clear that

$$\frac{a_3 - 0.5}{a_3 - a_2} \geq 0.95 \text{ i.e. } \frac{a_3 - 0.5}{0.2} \geq 0.95 \text{ i.e. } a_3 \geq 0.69$$

So,  $a_2 \geq 0.49$  and  $a_1 \geq 0.29$ .

Hence, from the above calculation we can say with 95% possibility that collection percentage will be more than or equal to 50% if  $\tilde{C}_R = (0.29, 0.49, 0.69)$ . In the proposed SD model,  $\tilde{C}_R$  is described as *Collection Percentage = RANDOM TRIANGULAR (0, 1, 0.29, 0.49, 0.69, 1)*

The *incorporation of PE policy* plays an important role in the process of closed-loop supply chain as the customer can exchange their old used product with a fresh new product in a retail market at a discounted price which will effectively increase the product collection rate, market share and satisfy the customer's need. Therefore, it is always expected that the DM would like satisfy almost all the customers who are

interested to exchange their used products to buy a fresh new product. In the proposed model, it is assumed that the satisfaction rate ( $\tilde{S}_R$ ) is a triangular fuzzy distribution  $\tilde{S}_R = (c_1, c_2, c_3)$  with a constant deviation of 0.10 from the central value i.e.  $c_2 - c_1 = c_3 - c_2 = 0.1$  and that the  $\tilde{S}_R$  should be than or equal to 85% of used product in that period with at least 95% probability. But, as  $\tilde{S}_R$  is a fuzzy number, the following possibility constraint has to be satisfied to fulfill DM's requirement:

$$Pos(\tilde{S}_R \geq 0.85) \geq 0.95$$

Now, from the lemma of section (3), it is clear that

$$\frac{c_3 - 0.85}{c_3 - c_2} \geq 0.95 \text{ i.e. } \frac{c_3 - 0.85}{0.1} \geq 0.95 \text{ i.e. } c_3 \geq 0.945$$

So,  $c_2 \geq 0.845$  and  $c_1 \geq 0.745$ .

Hence, from the above calculation we can say with 95% possibility that collection percentage will be more than or equal to 85% if  $\tilde{S}_R = (0.745, 0.845, 0.945)$ . In the proposed SD model,  $\tilde{S}_R$  is described as: *Satisfaction Rate of Demand with Product Exchange = RANDOM TRIANGULAR (0, 1, 0.745, 0.845, 0.945, 1)*.

## V. RESULT AND DISCUSSION

In this section, we demonstrate the behavior analysis of the closed-loop supply chain and discuss some of the important results. Before analyzing the performance of the integrated system, we set the important parameters as follows: *components per product=3, component production time=1.2 weeks, product production time=2 weeks, shipment time from producer to distributor=1.5 weeks, delivery time from distributor to retailer=1.5 weeks, cycle life of product=50 weeks*. We assume that 80 % of collected products are accepted for remanufacturing after initial inspection, 15 % of the components get replaced by the new ones from the product which accepted for remanufacturing, 65% of the components are reusable immediately from the collected products which rejected for remanufacturing. The length of the time horizon is 300 weeks for the simulation.

### A. Bullwhip Effects and Order Variations

We use the corresponding stock-flow diagram (Fig. 4) of the integrated forward-reverse supply chain to simulate its system performance and compare the bullwhip effect of the closed-loop supply chain with that of traditional (i.e. only forward) supply chain. Fig. 5 (Fig. 6) compares the actual demand at retailer level and order placed by retailer (distributor) over time in closed loop supply chain (SC) and traditional supply chain.

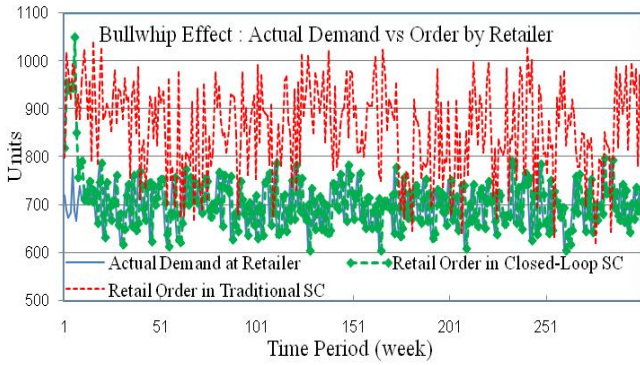


Fig. 5 Bullwhip effect at retailer

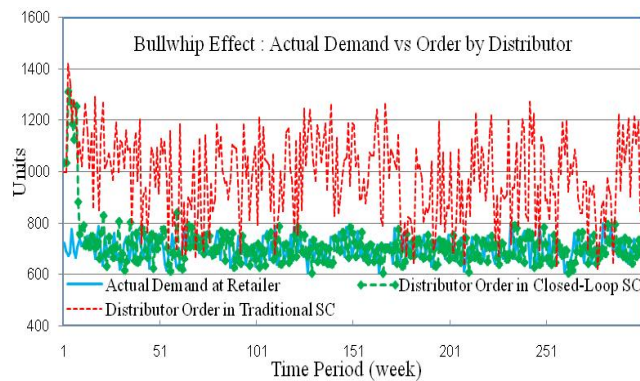


Fig. 6 Bullwhip effect at distributor

It is very clear from the two graphs that variation of orders at both retailer and distributor is much higher in traditional supply chain compared to that of in closed-loop supply chain.

We compute the bullwhip effect of the systems using the following formulation given by [18]:

$$\text{Bullwhip Effect} = \frac{\text{Var}(\text{OrderRate})}{\text{Var}(\text{Demand})}$$

and make a comparison of bullwhip effect at retailer and distributor level for closed loop supply chain and traditional supply chains and presented below.

TABLE II  
 COMPARISON OF BULLWHIP EFFECTS IN TWO CASES

	Retailer		Distributor	
	Closed-Loop SC	Traditional SC	Closed-Loop SC	Traditional SC
Bullwhip Effect	1.74	5.16	4.19	15.81

Since the remanufactured products, reusable components and remanufactured components supplement the product inventory and component inventory of producer in the forward channel; the inventory discrepancy at various stages of the forward supply chain decreases. Our result in TABLE II shows that the bullwhip effect of the retailer and distributor in the closed-loop supply chain is much less than that of traditional supply chain.

### B. Sensitivity Analysis

Product remanufacturing plays an important role in the recovery process of used product. *Product Remanufacturable*

*Percentage* assumes how much percentage of collected products is accepted for remanufacturing after initial inspection. Here, for the sensitivity analysis, we vary the above parameter in our SD model to see changes in recovery process. From Fig. 7, it is clear that if we increase the product remanufacturable percentage then the average remanufactured products per week increases but the component and raw material recovery rate decreases.

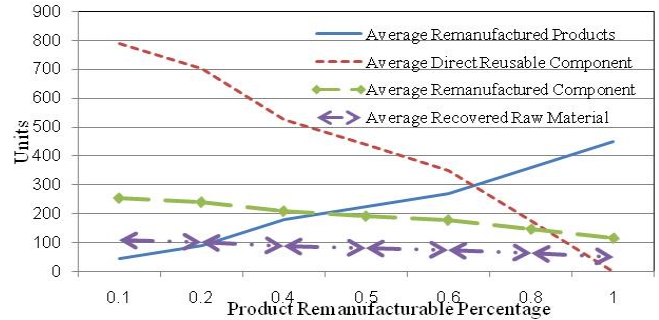


Fig. 7 Sensitivity analysis of recovery process

Inventory cover time and inventory adjustment time are the important parameters in determining the bullwhip effect at both retailer and distributor level. Inventory cover time determines the safety stock for the inventory. It describes a level of extra stock that is maintained to mitigate risk of stock outs due to uncertainties in supply and demand. Inventory adjustment time represents how quickly a firm tries to correct the discrepancy between desired serviceable inventory and actual serviceable inventory. Here, we make an attempt to see how the bullwhip effect changes at retailer and distributor level due to changes in inventory cover time and inventory adjustment time. From Fig. 8, it is clear that bullwhip effect increases both at retailer and distributor level as the retail inventory (RI) cover time increases; but the bullwhip effect decreases at both levels as the retail inventory (RI) adjustment time increases. The main reason is that if a firm adjusts the discrepancy between desired serviceable inventory and actual serviceable inventory very quickly, then the variations in order increases. From Fig. 9, it can be seen that the changes of cover time and adjustment time in distributor level has almost no impact in determining the bullwhip effect at retailer level. But the bullwhip effect increases at distributor level with the increment of distributor inventory (DI) cover time and decreases with the increment of distributor inventory (DI) adjustment time.

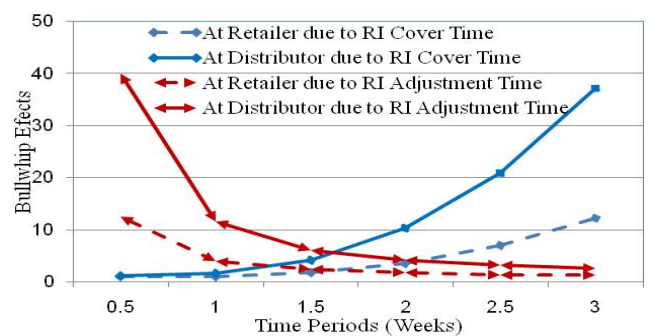


Fig. 8 Sensitivity analysis of RI cover and adjustment time

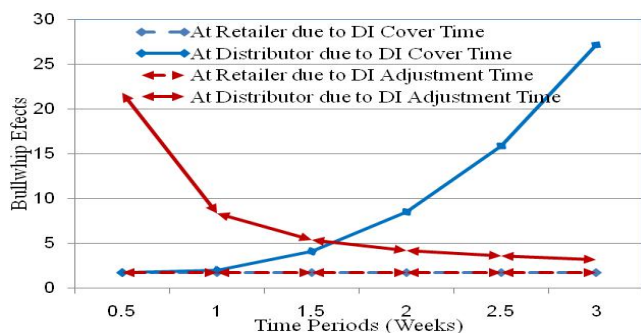


Fig. 9 Sensitivity analysis of DI cover and adjustment time

## VI. CONCLUSION

In this paper, we proposed a SD framework to analyze the long-term behavior of a multi-echelon closed-loop supply chain with fuzzy demand and fuzzy collection rate by incorporating various recycling activities, namely; collection, product remanufacturing, component reuse and remanufacturing, and raw material recovery. We also brought in the concept of employing product exchange policy in customer's outlet to enhance the collection and recycling process. The uncertainty issues associated with acquisition and collection of used product have been quantified using possibility measures. In the simulation study, we analyzed the order variation at both retailer and distributor level and compare the bullwhip effects of different logistics participants over time between the traditional forward supply chain and the closed-loop supply chain. Our results showed that the integration of reverse logistics can reduce the order variation and bullwhip effect of the closed-loop system. Also, sensitivity analysis is performed to examine the impact of inventory adjustment time, cover time on the order variance and bullwhip effect. The developed model can be used to conduct various "what-if" analyses thus identifying efficient policies and further to answer questions about the long-term operation of the integrated forward-reverse supply chains. The proposed SD framework can be extended by including the associated costs which helps to measure the economic performance of the integrated supply chain.

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