Simplex Method for Fuzzy Variable Linear Programming Problems

S.H. Nasseri, and E. Ardil

Abstract—Fuzzy linear programming is an application of fuzzy set theory in linear decision making problems and most of these problems are related to linear programming with fuzzy variables. A convenient method for solving these problems is based on using of auxiliary problem. In this paper a new method for solving fuzzy variable linear programming problems directly using linear ranking functions is proposed. This method uses simplex tableau which is used for solving linear programming problems in crisp environment before.

Keywords—Fuzzy variable linear programming, fuzzy number, ranking function, simplex method.

I. INTRODUCTION

ZIMMERMANN [10] proposed the first formulation of fuzzy linear programming. Fang and Hu [4] considered linear programming with fuzzy constraint coefficients. Vasant and et al [9] applied linear programming with fuzzy parameters for decision making in industrial production planning. Maleki and et al [6, 7] introduced a linear programming problem with fuzzy variables and proposed a new method for solving these problems using an auxiliary problem. Mahdavi-Amiri and Nasseri [5] described duality theory for the fuzzy variable linear programming (FVLP) problems. This study focuses on FVLP problems. Hence, first some important concepts of fuzzy theory are reviewed and concept of the comparison of fuzzy numbers by introducing a linear ranking function is described. Moreover, fuzzy basic feasible solution for the FVLP problems and also optimality conditions along with fuzzy simplex algorithm for solving the fuzzy variable linear programming problems is proposed.

II. DEFINITIONS AND NOTATIONS

In this section, some of the fundamental definitions and concepts of fuzzy sets theory initiated by Zadeh [2] (taken from Bezdek [3]) are reviewed.

Definition 2.1. A fuzzy set $\tilde{a}$ in R is a set of ordered pairs:

$$\tilde{a} = \{(x, \mu_{\tilde{a}}(x)) \mid x \in R\}$$

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$\mu_{\tilde{a}}(x)$ is called the membership function of x in $\tilde{a}$ which maps R to a subset of the nonnegative real numbers whose supremum is finite. If $\sup_x \mu_{\tilde{a}}(x) = 1$ the fuzzy set $\tilde{a}$ is called normal.

Definition 2.2. The support of a fuzzy set $\tilde{a}$ on R is the crisp set of all $x \in R$ such that $\mu_{\tilde{a}}(x) > 0$.

Definition 2.3. The set of elements that belong to the fuzzy set $\tilde{a}$ on R at least to the degree α is called the $\alpha - cut$ set:

$$\overline{a}_\alpha = \{x \in R \mid \mu_{\tilde{a}}(x) \geq \alpha\}.$$

Definition 2.4. A fuzzy set $\tilde{a}$ on R is convex if

$$\mu_{\tilde{a}}(\lambda x + (1 - \lambda) y) \geq \min\{\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)\},$$

$x, y \in R$, and $\lambda \in [0,1]$.

Note that, a fuzzy set is convex if all $\alpha - cuts$ are convex.

Definition 2.5. A fuzzy number $\tilde{a}$ is a convex normalized fuzzy set on the real line R such that

1) It exists at least one $x_0 \in R$ with $\mu_{\tilde{a}}(x_0) = 1$.

2) $\mu_{\tilde{a}}(x)$ is piecewise continuous.

A fuzzy number $\tilde{a}$ is a trapezoidal fuzzy number if the membership function of it be in the following form:
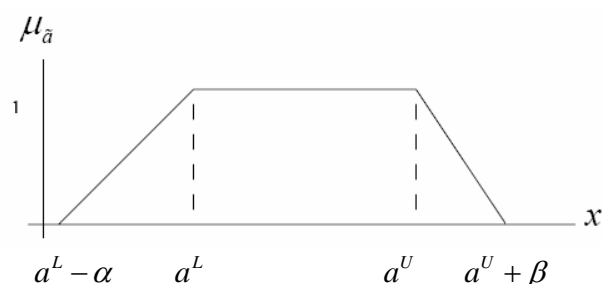

Fig. 1 Trapezoidal Fuzzy Number

We may show any trapezoidal fuzzy number by $\tilde{a} = (a^L, a^U, \alpha, \beta)$, where the support of $\tilde{a}$ is $(a^L - \alpha, a^U + \beta)$, and the core of $\tilde{a}$ is $[a^L, a^U]$. Let $F(R)$ be the set of trapezoidal fuzzy numbers. Note that, we consider $F(R)$ throughout this paper.

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A. Arithmetic on Fuzzy Numbers

Since in this paper we only consider the trapezoidal fuzzy numbers therefore we define arithmetic on the elements of $F(R)$. Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers and $x \in R$. Then, we define

$$x > 0, x\tilde{a} = (xa^L, xa^U, x\alpha, x\beta)$$
$$x < 0, x\tilde{a} = (xa^U, xa^L, -x\beta, -x\alpha)$$
$$\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta)$$
$$\tilde{a} - \tilde{b} = (a^L - b^U, a^U - b^L, \alpha + \theta, \beta + \gamma)$$

III. RANKING FUNCTIONS

A convenient method for comparing of the fuzzy numbers is by use of ranking functions. We define a ranking function $\Re : F(R) \to R$, which maps each fuzzy number into the real line. Now, suppose that $\tilde{a}$ and $\tilde{b}$ be two trapezoidal fuzzy numbers. Therefore, we define orders on $F(R)$ as following:

$$\tilde{a} \underset{\Re}{\geq} \tilde{b} \text{ if and only if } \Re(\tilde{a}) \geq \Re(\tilde{b}) \qquad (1)$$

$$\tilde{a} \underset{\Re}{>} \tilde{b} \text{ if and only if } \Re(\tilde{a}) > \Re(\tilde{b}) \qquad (2)$$

$$\tilde{a} \underset{\Re}{=} \tilde{b} \text{ if and only if } \Re(\tilde{a}) = \Re(\tilde{b}) \qquad (3)$$

where $\tilde{a}$ and $\tilde{b}$ are in $F(R)$. Also we write $\tilde{a} \underset{\Re}{\leq} \tilde{b}$ if and only if $\tilde{b} \underset{\Re}{\geq} \tilde{a}$.

Lemma 3.1. Let $\Re$ be any linear ranking function. Then

i) $\tilde{a} \underset{\Re}{\geq} \tilde{b}$ if and only if $\tilde{a} - \tilde{b} \underset{\Re}{\geq} 0$ if and only if $-\tilde{b} \underset{\Re}{\geq} -\tilde{a}$

ii) If $\tilde{a} \underset{\Re}{\geq} \tilde{b}$ and $\tilde{c} \underset{\Re}{\geq} \tilde{d}$, then $\tilde{a} + \tilde{c} \underset{\Re}{\geq} \tilde{b} + \tilde{d}$.

These are many numbers ranking function for comparing fuzzy numbers. Here, we use from linear ranking functions, that is, a ranking function $\Re$ such that

$$\Re(k\tilde{a} + \tilde{b}) = k\Re(\tilde{a}) + \Re(\tilde{b}). \qquad (4)$$

One suggestion for a linear ranking function as following:

$$\Re(\tilde{a}) = a^L + a^U + \frac{1}{2}(\beta - \alpha). \qquad (5)$$

where $\tilde{a} = (a^L, a^U, \alpha, \beta) \in F(R)$, and adopted by Maleki and et al [6, 7].

Then, for trapezoidal fuzzy numbers $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$, we have

$$\tilde{a} \underset{\Re}{\geq} \tilde{b} \text{ if and only if } a^L + a^U + \frac{1}{2}(\beta - \alpha) \geq b^L + b^U + \frac{1}{2}(\theta - \gamma).$$

IV. FUZZY LINEAR PROGRAMMING

In this section, we introduce fuzzy linear programming (FLP) problems. In order to the definition of fuzzy linear programming problems it is necessary to introduce linear programming problems.

A. Linear Programming

A linear programming (LP) problem is defined as:
$$\begin{aligned} \text{Max} \quad & z = cx \\ \text{s.t.} \quad & Ax = b \qquad (6) \\ & x \geq 0 \end{aligned}$$

where $c = (c_1, ..., c_n), b = (b_1, ..., b_m)^T$, and $A = [a_{ij}]_{m \times n}$.

In the above problem the all of parameters are crisp [1]. Now, if the some of parameters be fuzzy numbers we obtain a fuzzy linear programming which is defined in the next subsection.

B. Fuzzy Linear Programming

Suppose that in the linear programming problem some parameters be fuzzy number. Then, we have a fuzzy linear programming problem. Hence, it is possible the some coefficients of the problem in the objective function, technical coefficients, the right-hand side coefficients or decision making variables be fuzzy number [5], [6], [7], [8]. Here, we focus on the linear programming problems with fuzzy variables which is defined in the next section

V. FUZZY VARIABLE LINEAR PROGRAMMING

A fuzzy variable linear programming (FVLP) problem is defined as follows:
$$\begin{aligned} \text{Max} \quad & \tilde{z} \underset{\Re}{=} c\tilde{x} \\ \text{s.t.} \quad & A\tilde{x} \underset{\Re}{=} \tilde{b} \qquad (7) \\ & \tilde{x} \underset{\Re}{\geq} 0 \end{aligned}$$

where $\tilde{b} \in (F(R))^m, \tilde{x} \in (F(R))^n, A \in R^{m \times n}, c^T \in R^n$, and $\Re$ is a linear ranking function.

Definition 5.1. We say that fuzzy vector $\tilde{x} \in (F(R))^n$ is a fuzzy feasible solution to (7) if and only if $\tilde{x}$ satisfies the constraints of the problem.

Definition 5.2. A fuzzy feasible solution $\tilde{x}_*$ is a fuzzy optimal solution for (7), if for all fuzzy feasible solution $\tilde{x}$ for (7), we have $c\tilde{x}_* \underset{\Re}{\geq} c\tilde{x}$.

A. Fuzzy Basic Feasible Solution

Here, we describe fuzzy basic feasible solution (FBFS) for the FVLP problems which established by Mahdavi-Amiri and Nasseri [5].

For the FVLP problem is defined in (7), consider the system $A\tilde{x} \underset{\Re}{=} \tilde{b}$ and $\tilde{x} \underset{\Re}{\geq} 0$. Let $A = [a_{ij}]_{m \times n}$. Assume

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$rank(A) = m$. Partition A as $[\,B \quad N\,]$ where B, $m \times m$, is nonsingular. It is obvious that $rank(B) = m$. Let y_j be the solution to $By = a_j$. It is apparent that the basic solution

$$\widetilde{x}_B = (\widetilde{x}_{B_1}, ..., \widetilde{x}_{B_m})^T \underset{\Re}{=} B^{-1}\widetilde{b}, \widetilde{x}_N \underset{\Re}{=} 0 \qquad (8)$$

is a solution of $A\widetilde{x} \underset{\Re}{=} \widetilde{b}$. We call $\widetilde{x}$, accordingly partitioned as $(\widetilde{x}_B^T \quad \widetilde{x}_N^T)^T$, a *fuzzy basic solution* corresponding to the basis B. If $\widetilde{x}_B \underset{\Re}{\geq} 0$, then the fuzzy basic solution is feasible and the corresponding fuzzy objective value is $\widetilde{z} \underset{\Re}{=} c_B \widetilde{x}_B$, where $c_B = (c_{B_1}, ..., c_{B_m})$. Now, corresponding to every fuzzy nonbasic variable $\widetilde{x}_j$, $1 \leq j \leq n$, $j \neq B_i$, and $i = 1, ..., m$, define

$$z_j = c_B y_j = c_B B^{-1} a_j. \qquad (9)$$

If $\widetilde{x}_B \underset{\Re}{>} 0$, then is $\widetilde{x}$ called a *nondegenerate fuzzy basic feasible solution*, and if at least one component of $\widetilde{x}_B$ is zero, then $\widetilde{x}$ is called a *degenerate fuzzy basic feasible solution.*

The following theorem characterizes optimal solutions. The result corresponds to the so-called nondegenerate problems, where all fuzzy basic variables corresponding to every basis B are nonzero (and hence positive) [5].

Theorem 5.1. Assume the FVLP problem is nondegenerate. A fuzzy basic feasible solution $\widetilde{x}_B \underset{\Re}{=} B^{-1}\widetilde{b}, \widetilde{x}_N \underset{\Re}{=} 0$ is optimal to (7) if and only if $z_j \geq c_j$ for all $1 \leq j \leq n$.

Maleki and et al [6, 7] was proposed a method for solving FVLP problems by use of solving an auxiliary problem. They discuss on the some relations between the FVLP problem and the auxiliary problem. Then, they used from these results for solving the FVLP problems. Here, we propose simplex method for solving FVLP problems.

VI. Simplex Method for the FVLP Problems

A. Fuzzy Simplex Method in Tableau Format

Consider the FVLP problem as is defined in (7).

$$\text{Max } \widetilde{z} \underset{\Re}{=} c_B \widetilde{x}_B + c_N \widetilde{x}_N$$

$$\text{s.t. } B\widetilde{x}_B + N\widetilde{x}_N \underset{\Re}{=} \widetilde{b} \qquad (10)$$

$$\widetilde{x}_B, \widetilde{x}_N \underset{\Re}{\geq} 0$$

Then, it is possible we write $\widetilde{x}_B \underset{\Re}{=} B^{-1}\widetilde{b} - B^{-1}N\widetilde{x}_N$ and $\widetilde{z} \underset{\Re}{=} c_B(B^{-1}\widetilde{b} + B^{-1}N\widetilde{x}_N) + c_N\widetilde{x}_N$. Also, we may rewrite

$\widetilde{x}_B + B^{-1}N\widetilde{x}_N \underset{\Re}{=} B^{-1}\widetilde{b}$, and also for objective function

$$\widetilde{z} + (c_B B^{-1}N - c_N)\widetilde{x}_N \underset{\Re}{=} c_B B^{-1}\widetilde{b}.$$

Currently $\widetilde{x}_N \underset{\Re}{=} 0$, and then $\widetilde{x}_B \underset{\Re}{=} B^{-1}\widetilde{b}$, and $\widetilde{z} \underset{\Re}{=} c_B B^{-1}\widetilde{b}$. Then, we may rewrite the above FVLP problem in the following tableau format:

TABLE I
FUZZY SIMPLEX TABLEAU

	$\widetilde{z}$	$\widetilde{x}_B$	$\widetilde{x}_N$	R.H.S.
$\widetilde{z}$	1	0	$c_B B^{-1}N - c_N$	$c_B B^{-1}\widetilde{b}$
$\widetilde{x}_B$	0	I	$B^{-1}N$	$B^{-1}\widetilde{b}$

The above tableau gives us all the information we need to proceed with the simplex method. The cost row in the above tableau is $(\gamma_j)_{j \neq B_i} = (c_B B^{-1} a_j - c_j)_{j \neq B_i} = (z_j - c_j)_{j \neq B_i}$. According to the optimality condition for these problems we are at the optimal solution if $\gamma_j \geq 0$ for all $j \neq B_i$. On the other hand, if $\gamma_l < 0$, for a $l \neq B_i$ then we may exchange $\widetilde{x}_{B_r}$ with $\widetilde{x}_l$. Then we compute the vector $y_l = B^{-1} a_l$. If $y_l \leq 0$, then $\widetilde{x}_l$ can be increase indefinitely, and then the optimal objective is unbounded. On the other hand, if y_l has at least one positive component, then the increase in will be blocked by one of the current basic variables, which drops to zero.

B. Pivoting

If $\widetilde{x}_l$ enters the basis and $\widetilde{x}_{B_r}$ leaves the basis, then pivoting on y_{rl} can be stated as follows:

1) Divide row r by y_{rl}.

2) For $i = 1, ..., m$ and $i \neq r$, update the i th row by adding to it $-y_{il}$ times the new r th row.

3) Update row zero by adding to it γ_l times the new r th row.

Theorem 6.1. If in a fuzzy simplex tableau, an l exists such that $z_l - c_l < 0$ and there exists a basic index i such that $y_{il} > 0$, then a pivoting row r can be found so that pivoting on y_{rl} will yield a fuzzy feasible tableau with a corresponding nondecreasing objective value.
Proof. We need a criterion for choosing a fuzzy basic variable to leave the basis so that the new simplex tableau will remain feasible and the new objective value is nondecreasing. Assume column l is the pivot column. Also, suppose that

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$\tilde{x}_{\Re} = (\tilde{x}_B^T, \tilde{x}_N^T)^T$ is a fuzzy basic feasible solution to the FVLP problem, where $\tilde{x}_B \underset{\Re}{=} B^{-1}\tilde{b}$, and $\tilde{x}_N \underset{\Re}{=} 0$. Then, the corresponding fuzzy objective value is $\tilde{z} \underset{\Re}{=} c_B B^{-1}\tilde{b} \underset{\Re}{=} c_B \tilde{y}_0$.

On the other hand, for any fuzzy basic feasible solution to the FVLP problem, we have

$$\tilde{x}_B + \sum_{j \neq B_i} y_j \tilde{x}_j \underset{\Re}{=} \tilde{y}_0 \qquad (11)$$

where $y_j = B^{-1}a_j$.

So, if $\tilde{x}_l$ enters into the basis we may write

$$\tilde{x}_B \underset{\Re}{=} \tilde{y}_0 - y_l \tilde{x}_l \qquad (12)$$

Since, we want $\tilde{x}_B$ be feasible, hence

$\tilde{y}_{i0} - y_{il}\tilde{x}_l \underset{\Re}{\geq} 0$, for all $i = 1,...,m$.

If $y_{il} \leq 0$, then it is obvious that the above condition is hold. Hence, for all $y_{il} > 0$, we need to have

$$\tilde{x}_l \underset{\Re}{\leq} \frac{\tilde{y}_{i0}}{y_{il}} \qquad (13)$$

To satisfy (5) it is sufficient to let

$$\frac{\tilde{y}_{r0}}{y_{rl}} \underset{\Re}{=} \min\{\frac{\tilde{y}_{i0}}{y_{il}} \mid y_{il} > 0\} \qquad (14)$$

Also, for any fuzzy basic feasible solution to the FVLP problem, we have

$$\tilde{z} \underset{\Re}{=} c_B \tilde{y}_0 - \sum_{j \neq B_i} (z_j - c_j)\tilde{x}_j \qquad (15)$$

So, if we enter $\tilde{x}_l$ into the basis we have

$$\tilde{z} \underset{\Re}{=} c_B \tilde{y}_0 - (z_l - c_l)\tilde{x}_l \qquad (16)$$

We note that the new objective value is nondecreasing, since

$$\tilde{z} \underset{\Re}{=} c_B \tilde{y}_0 - (z_l - c_l)\tilde{x}_l \underset{\Re}{\geq} c_B \tilde{y}_0 \qquad (17)$$

Using the fact that $(z_l - c_l)\tilde{x}_l \underset{\Re}{\leq} 0$.

Theorem 6.2. If for any fuzzy basic feasible solution to the FVLP problem there is some column not in basis for which $z_l - c_l < 0$ and $y_{il} \leq 0, i = 1,...,m$, then the FVLP problem has an unbounded solution.

Proof. Suppose that $\tilde{x}_B$ is a fuzzy basic solution to the FVLP problem, so

$$\tilde{x}_{B_i} + \sum_{j \neq B_i} y_{ij}\tilde{x}_j \underset{\Re}{=} \tilde{y}_{i0}, \ i = 1,...,m, \ j = 1,...,n, \quad (18)$$

or

$$\tilde{x}_{B_i} \underset{\Re}{=} \tilde{y}_{i0} - \sum_{j \neq B_i} y_{ij}\tilde{x}_j, \quad i = 1,...,m, \ j = 1,...,n. \qquad (19)$$

Now, if we enter $\tilde{x}_l$ into the basis, then we have $\tilde{x}_l \underset{\Re}{>} 0$, and $\tilde{x}_j \underset{\Re}{=} 0$, for all $j \neq B_i \cup l$. Since $y_{il} \leq 0, i = 1,...,m$, hence

$$\tilde{y}_{i0} - y_{il}\tilde{x}_l \underset{\Re}{\geq} 0 \qquad (20)$$

Therefore, the current fuzzy basic solution will remain feasible. Now, the value of $\hat{z}$ for the above fuzzy feasible solution as following:

$$\hat{z} \underset{\Re}{=} c_B \tilde{x}_B + c_N \tilde{x}_N \underset{\Re}{=} \sum_{i=1}^m c_{B_i} (\tilde{y}_{i0} - y_{il}\tilde{x}_l) + c_l \tilde{x}_l$$

$$\underset{\Re}{=} \sum_{i=1}^m c_{B_i}\tilde{y}_{i0} - (\sum_{i=1}^m c_{B_i} y_{il} - c_l)\tilde{x}_l$$

$$\underset{\Re}{=} c_B \tilde{y}_0 - (c_B y_l - c_l)\tilde{x}_l \underset{\Re}{=} \tilde{z} - (z_l - c_l)\tilde{x}_l$$

So,

$$\hat{z} \underset{\Re}{=} \tilde{z} - (z_l - c_l)\tilde{x}_l. \qquad (21)$$

Hence, we can enter $\tilde{x}_l$ into the basis with arbitrarily large fuzzy value. Then, from (21) we have unbounded solution.

VII. A FUZZY SIMPLEX METHOD

Suppose that we are given a basic feasible solution with basis B. Then:

1. The basic feasible solution is given by $\tilde{x}_B \underset{\Re}{=} B^{-1}\tilde{b} \underset{\Re}{=} \tilde{y}_0$ and $\tilde{x}_N \underset{\Re}{=} 0$. The fuzzy objective $\tilde{z} \underset{\Re}{=} c_B B^{-1}\tilde{b} \underset{\Re}{=} c_B \tilde{y}_0$.

2. Calculate $w = c_B B^{-1}$, and $y_0 = \Re(\tilde{y}_0)$. For each nonbasic variable, calculate $\gamma_j = z_j - c_j = c_B B^{-1}a_j - c_j = wa_j - c_j$. Let $\gamma_l = \min_j \{\gamma_j\}$. If $\gamma_l \geq 0$, then stop; the current solution is optimal. Otherwise go to step 3.

3. Calculate $y_l = B^{-1}a_l$. If $y_l \leq 0$, then stop; the optimal solution is unbounded. Otherwise determine the index of the variable $\tilde{x}_{B_r}$ leaving the basis as follows:

$$\frac{y_{r0}}{y_{rl}} = \min_{1 \leq i \leq m}\{\frac{y_{i0}}{y_{il}} \mid y_{il} > 0\}$$

Update $\tilde{y}_{i0}$ by replacing $\tilde{y}_{i0} - \frac{\tilde{y}_{r0}}{y_{rl}} y_{il}$ for $i \neq r$ and $\tilde{y}_{r0}$ by

replacing $\frac{\tilde{y}_{r0}}{y_{rl}}$. Also, update $\tilde{z}$ by replacing

$\tilde{z} - \frac{\tilde{y}_{r0}}{y_{rl}}(z_l - c_l)$. Then, update B by replacing a_{B_r} with a_l and go to step 2.

VIII. A NUMERICAL EXAMPLE

For an illustration of the above method we solve a FVLP problem by use of fuzzy simplex method.

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Example 8.1.

$$\max \quad \tilde{z} \underset{\Re}{=} 3\tilde{x}_1 + 4\tilde{x}_2$$

$$\text{s.t.} \quad 3\tilde{x}_1 + \tilde{x}_2 \underset{\Re}{\leq} (2,4,1,3)$$

$$2\tilde{x}_1 - 3\tilde{x}_2 \underset{\Re}{\leq} (3,5,2,1)$$

$$\tilde{x}_1, \tilde{x}_2 \underset{\Re}{\geq} 0$$

Now, we may rewrite the above problem in form (10):

$$3\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 \underset{\Re}{=} (2,4,1,3)$$

$$2\tilde{x}_1 - 3\tilde{x}_2 + \tilde{x}_4 \underset{\Re}{=} (3,5,2,1)$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \underset{\Re}{\geq} 0$$

Therefore, using fuzzy simplex tableau (Table I), we obtain first tableau as follow:

basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	R.H.S.	$\Re(R.H.S.)$
$\tilde{z}$	-3	-4	0	0	$\tilde{0}$	0
$\tilde{x}_3$	3	1	1	0	(2,4,1,3)	7
$\tilde{x}_4$	2	-3	0	1	(3,5,2,1)	7.5

From the above tableau, we obtain $\gamma_1 = z_1 - c_1 = -3 < 0$ and $\gamma_2 = z_2 - c_2 = -4 < 0$. Then, $\gamma_2 < \gamma_1$. Hence, related fuzzy nonbasic variable to γ_2, that is $\tilde{x}_2$ is an entering variable. Therefore, according to the minimum ration test is given in the step 3 of the fuzzy simplex algorithm, $\tilde{x}_3$ is a leaving variable. Now, after pivoting (as given in the part B of section 6) the new tableau is:

basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	R.H.S.	$\Re(R.H.S.)$
$\tilde{z}$	9	0	4	0	(8,16,4,12)	28
$\tilde{x}_2$	3	1	1	0	(2,4,1,3)	7
$\tilde{x}_4$	11	0	3	1	(9,17,5,10)	28.5

According to above tableau, for fuzzy nonbasic variables $\tilde{x}_1, \tilde{x}_3$ we have $\gamma_1 = 9 > 0, \gamma_3 = 4 > 0$. Hence, using the optimality condition for the FVLP problems is given in Theorem 5.1, the optimal fuzzy solution is obtained $\tilde{x}_1^* \underset{\Re}{=} (0,0,0,0)$, $\tilde{x}_2^* \underset{\Re}{=} (2,4,1,3)$, $\tilde{x}_3^* \underset{\Re}{=} (0,0,0,0)$, $\tilde{x}_4^* \underset{\Re}{=} (9,17,5,10)$ and $\tilde{z} \underset{\Re}{=} (8,16,4,12)$ with $\Re(\tilde{z}) = 28$.

IX. CONCLUSION

We considered fuzzy variable linear programming problems and introduced the fuzzy basic feasible solution for these problems. Finally, we proposed a new algorithm for solving these problems directly, by use of linear ranking function.

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