

# Spanning Tree Transformation of Connected Graphs into Single-Row Networks

S.L. Loh, S. Salleh, and N.H. Sarmin

**Abstract**—A spanning tree of a connected graph is a tree which consists the set of vertices and some or perhaps all of the edges from the connected graph. In this paper, a model for spanning tree transformation of connected graphs into single-row networks, namely Spanning Tree of Connected Graph Modeling (STCGM) will be introduced. Path-Growing Tree-Forming algorithm applied with Vertex-Prioritized is contained in the model to produce the spanning tree from the connected graph. Paths are produced by Path-Growing and they are combined into a spanning tree by Tree-Forming. The spanning tree that is produced from the connected graph is then transformed into single-row network using Tree Sequence Modeling (TSM). Finally, the single-row routing problem is solved using a method called Enhanced Simulated Annealing for Single-Row Routing (ESSR).

**Keywords**—Graph theory, simulated annealing, single-row routing and spanning tree.

## I. INTRODUCTION

MANY engineering and science problems can be represented as a problem in graph theory. The scenario of the real-life applications is represented by the graph where the nodes in the graph can be treated as nodes in a network, and the communication links between the nodes are represented by the edges. To solve such a problem, the graph which represents the problem is transformed into single-row network where it is somehow the complexity of solution computations is reduced and solved as a single-row routing problem.

Single-row routing problem is an NP-complete problem. The necessary and sufficient condition for optimum single-row routing had been developed by Ting et al. [1] and Kuh et al. [2]. Many researches had been done to solve single-row routing problem such as the use of some heuristic algorithms [3] and applied graph theoretical representation [4]. The main objective for all these researches is to minimize the congestion. A model which minimizes both the congestion and

number of doglegs was developed, namely *Enhanced Simulated Annealing for Single-Row Routing* (ESSR) [5]. Congestion and doglegs in the single-row network are represented by the total energy, or  $E$ , which is the fitness value of the model.

The researches on transformation from connected graphs into single-row networks had been done for the first time in [6] and [7]. The optimality in nodes labeling was tackled in [8] based on simulated annealing [9]. The effort was extended from [8] to the single-row transformation of complete binary tree and the tree [10]. After the single-row transformation, the single-row routing problem is solved using ESSR.

In this work, the main concern is the spanning tree transformation of connected graph which can lead to the optimization of the single-row transformation. The connected graph is assumed to have one unit of weight for each edge. The relationship between connected graph with its spanning tree and the spanning tree with its single-row representation were studied in order to develop an efficient modeling which can optimize the spanning tree transformation of connected graph into single-row network.

The spanning tree of a connected graph is a tree that consists of all vertices and some (or perhaps all) of the edges from the connected graph. It can also be defined as the minimal set of edges that connects all the vertices. We found that the spanning tree transformation of a connected graph into the single-row network is optimal when that spanning tree has a set of nodes with minimum highest degree. In other words, the spanning tree has the fewest leaves. A model is developed based on this finding, namely *Spanning Tree of Connected Graph Modeling* (STCGM). The spanning tree which produced from the connected graph will be transformed into single-row network by Tree Sequence Modeling (TSM) [11] before ESSR is applied to solve the single-row routing problem.

This paper is organized into seven sections. Introduction is presented in Section I while Problem background is stated in Section II. The relationship between connected graph, spanning tree and its single-row network is shown in Section III. The fundamental concepts and findings from the study on relationship between a spanning tree from a connected graph and its single-row representation are included in Section IV. The developed model for spanning tree transformation of connected graphs into single-row networks, namely Spanning Tree of Connected Graph Modeling (STCGM) is shown in Section V. The experimental results are presented in Section

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VI. Summary and conclusion are consisted in Section VII while Section VIII is the acknowledgements.

## II. PROBLEM BACKGROUND

The problem can be stated as follows: *Given a connected graph, a spanning tree is produced from the connected graph and it needs to be transformed into single-row representation.*

The problem is illustrated in Fig. 1. A connected graph  $G_7$ , the spanning tree  $SP_7$  from the  $G_7$  and the single-row representation  $S_7$  of the spanning tree are shown in Fig. 1(a), (b) and (c) respectively.

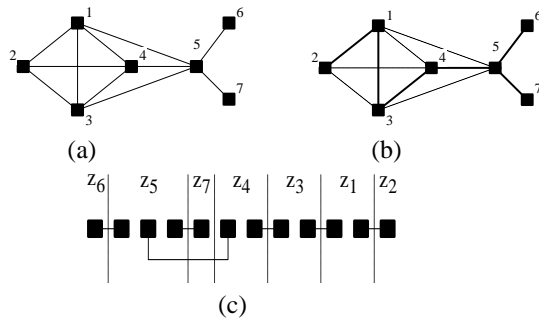


Fig. 1 (a) A connected graph  $G_7$ , (b) spanning tree from the  $G_7$  and (c) its single-row representation  $S_7$

## III. RELATIONSHIP BETWEEN CONNECTED GRAPH, SPANNING TREE AND ITS SINGLE-ROW NETWORK

In the spanning tree transformation of connected graphs into single-row networks, a spanning tree  $SP_n$  is produced from the given connected graph  $G_n$  before it is transformed into single-row network. The spanning tree is a tree which composed all of the vertices and some or maybe all of the edges from the connected graph. A spanning tree  $SP_n$  contains  $(n-1)$  edges. There are many forms of spanning trees that can be produced from the connected graph by selecting different sets of edges from  $G_n$ . In order to optimize the transformation from (a) to (c) as shown in Fig. 1, the necessary criterion in producing the optimal spanning tree is the course of this research.

In a connected graph,  $G_n$ , there are  $n$  nodes where each node  $v_i$  has  $d_i$  degree. After the spanning tree is produced from the connected graph, it will be transformed into single-row representation as shown in Fig. 1(c). Every node  $v_i$  in  $SP_n$  is transformed into a zone  $z_i$  for  $i=1,2,\dots,n$ , in the single-row network  $S_n$ . Each  $v_i$  has  $D_i$  degree in  $SP_n$  and thus each  $z_i$  has  $D_i$  number of terminals which evenly spaced aligned on node axis in  $S_n$ . The communication links

in  $SP_n$  are preserved where the edges are transformed into nets in  $S_n$ . The nets are made up of horizontal and vertical line segments which are drawn from left to right without crossing or backward movement. Congestion, or  $Q$ , is the maximum total number of horizontal tracks between upper and lower street congestion. A vertical crossing in node axis is called a dogleg, or  $D$ . The sequence of zones and the nets are arranged in such a way the congestion and dogleg are being minimized.

## IV. FUNDAMENTAL CONCEPTS

### A. G-SP

In this research, the connected graph  $G_n$  is assumed to have weight of one unit for each of the edges. Hence, there are various forms of spanning tree  $SP_n$  that can be formed from the connected graph. Now the question is: how is the form of  $SP_n$  that can optimize the single-row transformation? To obtain the answer, backward searching is needed where it starts with the study of relationship between  $SP_n$  and  $S_n$  before  $G_n$  and  $SP_n$ . In short, the form of  $SP_n$  which can lead to the optimal single-row transformation from  $SP_n$  to  $S_n$  is needed to be determined before the  $G_n$  is transformed into  $SP_n$ .

### B. G-SP

A spanning tree  $SP_n$  can be treated as a tree. A research had been carried out in studying the relationship between the tree and its single-row representation. A tree with  $n$  vertices gives the maximum value of  $E$ ,  $Q$  and  $D$  results in single-row network when the tree is in the form of 2-Level  $T_n$ . The optimum result for 2-Level  $T_n$  is  $E = Q(n-1-2Q)$  where  $Q = \lceil (n-3)/4 \rceil$  and  $D = 0$ . Proposition 1 is developed from 2-Level  $T_n$ 's results to find out the necessary criteria of spanning tree which optimizes the spanning tree transformation of connected graph into single-row network.

### C. Proposition

By having the same total number of nodes, a spanning tree with smaller value of  $D_{\max} = \max_{1 \leq i \leq n} \{D_i\}$  gives the lower congestion for single-row transformation of spanning tree.

### Proof:

A spanning tree  $SP_n$  has a total degree  $D$  where  $D = \sum_{i=1}^n D_i = 2(n-1)$ . The spanning tree  $SP_n$  can be treated as a tree  $T_n$  and the highest degree among the nodes in the

$SP_n$  denoted by  $D_{\max} = \max_{1 \leq i \leq n} \{D_i\}$ . Different sets of  $D_i$  for  $i = 1, 2, \dots, n$ , present the different structures of  $SP_n$ . For every  $D_i$ , the corresponding  $z_i$  has  $D_i$  number of nets to be formed. As a minimum, the  $z_i$  will contribute to the total congestion  $Q = \lceil (D_i - 2)/4 \rceil$  in  $S_n$  by replacing  $D_i = n - 1$  in the optimum result from 2-Level  $T_n$ . Obviously, the larger value of  $D_i$  will lead to the larger value of total congestion. Thus,  $D_{\max}$  among the  $D_i$  affects the result significantly where the set of  $D_i$  which gives the smallest value of  $D_{\max}$  will contribute to the lower congestion in single-row transformation. The optimum result for the optimum set of  $D_i$  is  $E = Q = D = 0$  given by  $D_{\max} = 2$  where it is exactly a Hamiltonian path.

#### D. G-S

In the spanning tree transformation of  $G_n$  into single-row network, the result is optimum ( $E = Q = D = 0$ ) when the spanning tree  $SP_n$  produced from  $G_n$  is a Hamiltonian path. A graph that contains a Hamiltonian path is called the traceable graph. For the connected graph  $G_n$  other than the traceable graph, the  $SP_n$  that produced from  $G_n$  need to be the  $SP_n$  with the minimum value of  $D_{\max}$ . By having the same total number of nodes, the connected graph with larger number of edges tends to give a Hamiltonian path or a spanning tree with fewer leaves compared with the connected graph with lower number of edges. Spanning tree  $SP_n$  with minimum value of  $D_{\max}$  minimizes the total energy value and congestion as well as doglegs in single-row network.

### V. OUR STCGM MODEL

From a connected graph, a spanning tree is produced and being transformed into single-row representation. For optimal spanning tree transformation of connected graph into single-row network, the necessary requirement is the spanning tree produced from the connected graph has the minimum value of  $D_{\max}$ . To obtain the  $SP_n$  with minimum value of  $D_{\max}$ , the *Path-Growing Tree-Forming* algorithm is developed to produce the  $SP_n$  from the  $G_n$ .

#### A. Path-Growing Tree-Forming Algorithm

Obviously, Path-Growing Tree-Forming algorithm is made up of two algorithms. *Path-Growing* algorithm produces the paths while *Tree-Forming* algorithm connects all the paths that are formed by Path-Growing algorithm into a spanning tree with the minimum value of  $D_{\max}$ . *Vertex-Prioritized* is applied in Path-Growing Tree-Forming algorithm to minimize the  $D_{\max}$  in the  $SP_n$ .

Initially, notation  $C_{1k}$  is defined as the set of nodes of the  $SP_n$  and notation  $C_{2k}$  is defined as the set of nodes of the path while notation  $\overline{C}_k$  denotes the set of unconnected nodes at  $k^{th}$  iteration. In the Path-Growing Tree-Forming algorithm, Vertex-Prioritized is applied where the node with the least degree is given the highest priority to be chosen. Notation  $c\_d$  represents the degree of nodes in set  $C_{1k}$  for  $SP_n$  while  $\overline{c\_d}$  represents the degree of nodes in set  $\overline{C}_k$  where it is involving only the edges connected by the nodes in the set itself.

First of all, a path is formed by Path-Growing algorithm. The node with the least degree in set  $\overline{C}_k$  is chosen to be the initial node  $i^*$  for the path to be formed. This step enables the possible of obtaining the path with maximum length. Once the node is included in the path construction, it will be moved from set  $\overline{C}_k$  to set  $C_k$ . In Path-Growing, when the path is impossible to be connected to the tree by any edge in length of one unit, *Backward Tracing* will be applied in Path-Growing and a walk will be produced instead of a path.

The path is expanded by choosing node  $j^*$  from  $\overline{C}_k$  under Vertex-Prioritized in which it is connected with the node  $i^*$ . The node  $i^*$  is updated as node  $j^*$  whenever a node  $j^*$  is found. The first formed path acts as the spanning tree. When there is no such a node  $j^*$  is found in  $\overline{C}_k$ , the next path need to be produced from set  $\overline{C}_k$  if set  $\overline{C}_k$  is not yet a null set. The process is repeated for Path-Growing algorithm applied with Vertex-Prioritized to form another path. Tree-Forming algorithm is developed to join the new formed path to the spanning tree by joining the new formed path to the node with the least  $c\_d$  from the spanning tree.

Here is the algorithm for Path-Growing Tree-Forming.

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 $C_{1k}$  = set of nodes of spanning tree at iteration  $k$ .
 $C_{2k}$  = set of nodes of path at iteration  $k$ .
 $\overline{C}_k$  = set of unconnected nodes.
 $c\_d_k$  = current degree of every node in  $C_k$ .
 $\overline{c\_d}_k$  = current degree of every node in  $\overline{C}_k$ .
 $E_{ij}$  = set of edges of the spanning tree
Set  $C_{10} = 0$ ,  $C_{20} = 0$ ,  $\overline{C}_0 = N$  and  $a = 0$ ;
for  $i = 1$  to  $n$ 
     $c\_d_i = 0$ ;
     $\overline{c\_d}_i = d_i$ ;
Select the node with least  $\overline{c\_d}_i$ , set  $C_{21} = \{i\}$ , set
 $i^* = i$  and  $\overline{C}_1 = N - \{i\}$ ;
for  $k = 1$  to  $n - 1$ 

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Identify the unconnected node  $j^*$  in the  $\bar{C}_k$ 
that has the least  $\bar{c}_d$  which connected with
node  $i^*$  in  $C_{2k}$ ;
if such  $j^*$  is found
    Connect these two nodes:
     $E[i^*][j^*] = 1$ ;
    Set  $i^* = j^*$ ;
    /*Update degree of nodes in  $C_k$  and
     $\bar{C}_k$ */
     $c_d[j^*]++$ ;
     $\bar{c}_d--$  for nodes in  $\bar{C}_k$  which
    connected (in  $G_n$ ) with node  $i^*$  in
     $C_{2k}$ ;
    //Check connection of path to tree
    Identify the connected node  $j^{**}$  in
    the  $C_{1k}$  that has the least  $c_d$  that
    connected with node  $i^*$  in  $C_{2k}$ 
        Set  $i^{**} = i^*$ ;
else
     $a++$ ;
    if  $a = 1$  or ( $i^{**}$  and  $j^{**}$  are found)
        Set  $C_{1k} = C_{2k}$ ;
    if  $a > 1$  //Connect the path formed
    with the spanning tree
        if  $i^{**}$  and  $j^{**}$  are found
            Connect these two
            nodes:  $E[i^{**}][j^{**}] = 1$ ;
            Update degree of nodes
            in  $C_k$  and  $\bar{C}_k$ 
             $k++$ ;
        else
            Backward Tracing to get the
             $i^*$ ;
             $k--$ ;

    if  $i^{**}$  and  $j^{**}$  are found
        //Select a node to form a path
        Select the node with least
         $\bar{c}_d$ , set  $i^* = i$ ;
        Check connection of path to
        tree;
        if  $\bar{C} \neq 0$ 
             $k--$ ;

```

### B. Tree Alteration for Single-Row Transformation

For illustration, after the spanning tree  $SP_n$  is produced from connected graph  $G_n$  which is shown in Fig. 2, the  $SP_n$  will be transformed into a rooted tree as shown in Fig. 3(b) and the rooted tree is solved using Tree Sequence Modeling (TSM).

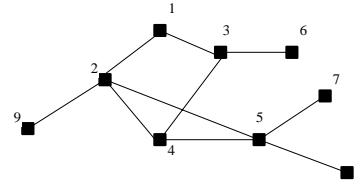
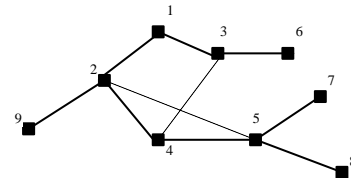
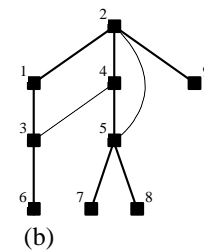


Fig. 2 A connected graph,  $G_9$



(a)



(b)

Fig. 3 The two depictions of the output ( $SP_9$ ) from Path-Growing Tree-Forming

From Fig. 3(b), node  $v_2$  is chosen to be the root node as it has the highest degree  $D_{\max}$  among the nodes in the  $SP_9$ . Actually, the results are slightly different based on the chosen node for the root node. However, this problem can be overcome by TSM where it is a specifically developed model to deal with the single-row transformation of tree with all kinds of structures. Hence, any node in the  $SP_n$  can act as the root node in the tree. To standardize the algorithm, the node with the highest degree is chosen as the root node in the transformation.

### C. TSM Model

The Tree Sequence Modeling (TSM) transforms the trees into single-row networks. The transformation involves the formation of zones, terminals and intervals. Formation of partitions and insertion mechanisms are consisted in the formation of zones. A tree is divided into a few partitions and groups. There are two groups of nodes where each group has a different arrangement for zones in insertion mechanisms. Formation of zones optimizes the sequence of zones which contributes to the optimal transformation result. The arrangement of zones leads to the formation of shortest intervals set with maximum number of zero energy nets in order to obtain the optimal result.

After the formation of zones, the pins and the intervals are formed using Formation of Terminals and Formation of Intervals [8]. The single-row routing problem is then solved

using the basic of simulated annealing [5] from earlier work called ESSR to obtain the result.

## VI. EXPERIMENTAL RESULTS

STCGM has been applied on some connected graphs with different number of nodes. The results are simulated by a developed program for STCGM using Microsoft Visual C++ 6.0. The results for energy values ( $E$ ), congestion ( $Q$ ) and number of doglegs ( $D$ ) are shown in Table I.

TABLE I  
COMPUTATIONAL RESULTS OF STCGM

# nodes	$d_{\max}$	# $d_{\max}$	$E$	$Q$	$D$
10	4	1	2	1	0
20	5	1	3	1	0
30	3	1	1	1	0
40	4	2	6	1	0
50	3	3	3	1	0
60	3	7	5	1	0

By having the same total number of nodes, the connected graphs with more communication links will give the spanning tree with lower value of  $D_{\max}$ . The figures from Table I shows the results are not primarily affected by total number of nodes in  $G_n$ . In fact, the core factor is the maximum degree of node ( $D_{\max}$ ) in the spanning tree  $SP_n$  together with the total number of nodes with degree of  $D_{\max}$  that exist in the  $SP_n$ . The  $SP_n$  with the same  $D_{\max}$  may have the different results by having the different total number of nodes with degree of  $D_{\max}$ . From the experimental results, the  $G_{30}$  and the  $G_{50}$  both have  $D_{\max}$  of three. Since there is only exist one node that gives the  $D_{\max}$  in  $G_{30}$  compared with three nodes in  $G_{50}$ , total energy for the  $G_{30}$  is then lower than the  $G_{50}$ . This shows the higher of  $D_{\max}$  and the higher number of nodes with  $D_{\max}$  in the spanning tree will lead to the higher value of total energy.

## VII. CONCLUSION AND SUMMARY

In this research, a model which transforms the connected graph ( $G_n$ ) into spanning tree ( $SP_n$ ) before the single-row transformation into single-row network ( $S_n$ ) is introduced, namely Spanning Tree of Connected Graph Modeling (STCGM). To obtain the optimal single-row transformation, the relationships between  $G_n$  to  $SP_n$  and  $SP_n$  to  $S_n$  are studied in order to find out the relationship from  $G_n$  to  $S_n$ . The connected graph  $G_n$  in this research has weight of one unit for all edges. From the study, the spanning tree with lower value of  $D_{\max}$  which is the highest degree of node in

$SP_n$  was found to give the lower value of total energy for the complete transformation. Hence, STCGM is developed based on the finding.

STCGM consists Path-Growing Tree-Forming algorithm which is developed by applying Vertex-Prioritized to produce the  $SP_n$  with minimum value of  $D_{\max}$  from the  $G_n$ . There are two parts in Path-Growing Tree-Forming algorithm. The Path-Growing produces the paths from the  $G_n$  and the Tree-Forming connects all these paths into a  $SP_n$ . Sometimes, Backward Tracing is needed in Path-Growing to form a walk in order to join the walk to the tree in Tree-Forming. After the  $SP_n$  is produced from the  $G_n$ , the  $SP_n$  will be transformed into  $S_n$  using Tree Sequence Modeling (TSM). The single-row routing problem is then solved by ESSR [5] to obtain the result. The experimental results show STCGM can efficiently optimize the spanning tree transformation of connected graphs into single-row networks.

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