An Agent-based Model for Analyzing Interaction of Two Stable Social Networks

Masatora Daito, and Hisashi Kojima

Abstract—In this research, the authors analyze network stability using agent-based simulation. Firstly, the authors focus on analyzing large networks (eight agents) by connecting different two stable small social networks (A small stable network is consisted on four agents.). Secondly, the authors analyze the network (eight agents) shape which is added one agent to a stable network (seven agents). Thirdly, the authors analyze interpersonal comparison of utility. The "star-network " was not found on the result of interaction among stable two small networks. On the other hand, "decentralized network" was formed from several combination. In case of added one agent to a stable network (seven agents), if the value of "c"(maintenance cost of per a link) was larger, the number of patterns of stable network was also larger. In this case, the authors identified the characteristics of a large stable network. The authors discovered the cases of decreasing personal utility under condition increasing total utility.

Keywords—Social Network, Symmetric Situation, Network Stability, Agent-Based Modeling.

I. INTRODUCTION

SOCIAL network is formed by friends, business, aerial route and so on. Many studies of social network has been done by social scientists. For example a study reported that more than 50 percent of people in Massachusetts got jobs using social ties [1]. Studies of relationships between buyers and sellers on business transaction[2, 3], a study of business ties and joint development [4] and a study of air transportation networks [5] are also used social network analysis.

In this research, the authors analyze stable network shapes which are the results of interaction among agents. In our model, agents make a decision about relationship with other agents. This model is applied by modified connection model [6].

In previous research [7], the authors used our modified connection model and carried out simulation of some cases which were interactions among four, five, six, seven and eight agents. In this research, the authors apply this stable networks [7] to analyzing interaction among two stable social networks. Firstly, the authors focus on analyzing large networks (eight agents) by connecting different two stable small social networks (4-4 agents cases). A small stable network is consisted on four agents. Secondly, the authors analyze the network (eight agents) shape which is added one agent to a stable network (seven agents)(7-1 agents cases). Thirdly, the authors analyze interpersonal comparison of utility on "7-1 agents cases".

M. Daito is with the Department of Business Administration, Okayama Shoka University, 2-10-1, Kyomachi Tsushima, Kitaku, Okayama City, Okayama 7008601 Japan e-mail: m-daito@po.osu.ac.jp.
H. Kojima is with the Department of Economics, Okayama Shoka University.

II. AGENT-BASED MODELING

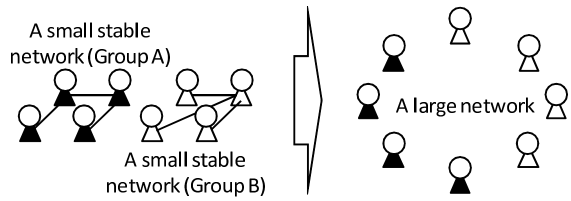

Fig. 1. An idea of 4-4 agents cases

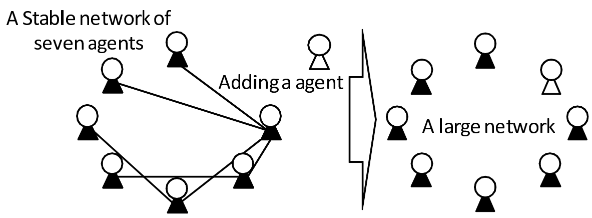

Fig. 2. An idea of 7-1 agents cases

TABLE I
THE DEGREE SEQUENCES OF STABLE NETWORK AND 4-4 AGENTS CASES

The Degree Sequences of Stable Networks			The Cost of Maintaining Links (Value of *C*)					
	Group 1	Group 2	0.2	0.3	0.4	0.5	0.6	0.7
(1)	1, 1, 1, 3	1, 1, 1, 3	✓	✓	✓	✓	✓	✓
(2)	2, 2, 2, 2	1, 1, 1, 3	✓					
(3)	2, 2, 2, 2	2, 2, 2, 2	✓					
(4)	1, 1, 2, 2	1, 1, 1, 3		✓	✓	✓	✓	✓
(5)	1, 1, 2, 2	1, 1, 2, 2		✓	✓	✓	✓	✓

The authors built agent-based model for analyzing interaction among two small stable networks. The authors defined this cases as "4-4 agents cases"(Shown in Fig.1). The authors built also agent-based model for analyzing interaction stable network of seven agents and one agent. The authors defined this cases as "7-1 agents cases"(Shown in Fig.2).

This model is applied by modified connection model[6]. Our modified connection model is as follows:

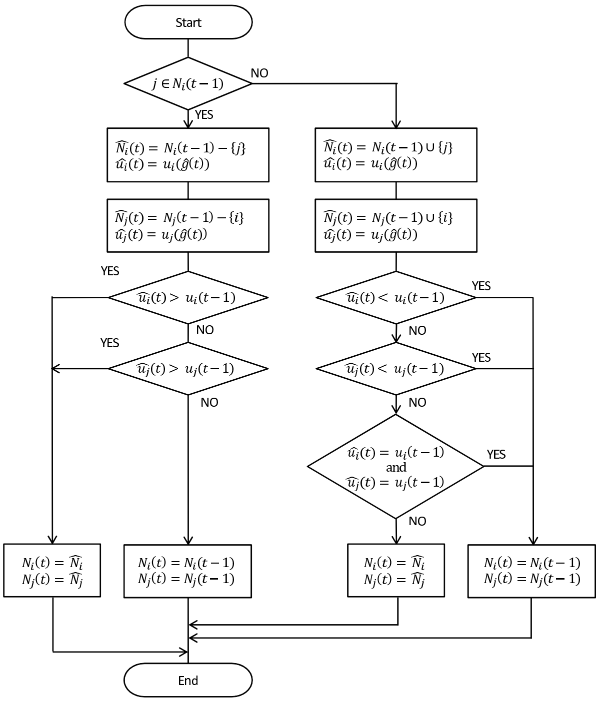

Fig. 3. Behavior rules of agent i and agent j

$$u_i(g) = \sum_{j \neq i} \delta^{s(ij)} - |N_i(g)|c$$

where left-hand side represents utility of agent i, first term of right-hand side represents that agent i obtains positive effect (e.g., valuable information, benefit) from directly or indirectly linked agents. δ represents that the more indirect links, the more effect is decreased. That is, $0 < \delta < 1$ is given.

Second term of right-hand side represents costs of maintaining links. Where, $N_i(g)$ is set of neighborhood (direct links) of agent i, $|N_i(g)|$ is cardinalities of neighborhood. That is, $N_i(g) := \{j \in N : ij \in g\}$ is given. c is positive real number and represents maintenance cost of per a link.

Rules of their behavior are represented in fig3. In previous research [7], the authors used this rules and got several stable network shapes. A network shape at period (t) is defined by $g(t), t = 0, 1, \ldots$. If every agent has not any links, that network shapes are expressed as $g(0)$. A neighborhood (links) set of agent i at period (t) is expressed as $N_i(t)$ Utilities of agent i at period (t) is expressed as $u_i(t)$. A agent i and agent j are chosen randomly at period (t). A network shape is dependent on past $(g(t-1))$ decision at that time. A network shape of this period$(g(t))$ is depend on decision of agent i and agent j.

If agent i and agent j are already linked, they decides to keep or to cut their links. In case of cutting the link, neighborhood (direct links) set of agent i is expressed as $\hat{g}(t) := g(t-1) \setminus \{ij\}$, $\hat{N_i}(t) := N_i(t-1) \setminus \{j\}$. Utilities of agent i is expressed as $\hat{u}_i(t) := u_i(\hat{g}(t))$. Agent j is also expressed same as that of agent i.

TABLE II
THE DEGREE SEQUENCES OF STABLE NETWORK AND 7-1 AGENTS CASES

The Degree Sequences of Stable Networks by Seven Players	The Cost of Maintaining Links (Value of C)					
	0.2	0.3	0.4	0.5	0.6	0.7
(1) 1, 1, 1, 1, 1, 2, 5		✓	✓	✓		
(2) 1, 1, 1, 1, 1, 3, 4		✓	✓	✓		✓
(3) 1, 1, 1, 1, 2, 2, 4 (a)					✓	✓
(4) 1, 1, 1, 1, 2, 2, 4 (b)					✓	✓
(5) 1, 1, 1, 1, 2, 3, 3 (c)					✓	
(6) 1, 1, 1, 1, 2, 3, 3 (d)						✓
(7) 1, 1, 1, 2, 2, 2, 3					✓	✓
(8) 1, 1, 2, 2, 2, 2, 4		✓	✓	✓		
(9) 1, 1, 2, 2, 2, 3, 3		✓	✓	✓		
(10) 1, 2, 2, 2, 2, 2, 3					✓	✓
(11) 2, 2, 2, 2, 2, 2, 2					✓	
(12) 2, 2, 2, 2, 2, 3, 3		✓	✓	✓		
(13) 2, 2, 2, 2, 2, 4, 4	✓					
(14) 2, 2, 2, 2, 2, 5, 5	✓					
(15) 2, 2, 2, 2, 3, 3, 4	✓					
(16) 2, 3, 3, 3, 3, 3, 3	✓					
(17) 3, 3, 3, 3, 4, 4, 4	✓					

If agent i and agent j are not linked, they decides to keep or to connect their links. In case of connecting the link, neighborhood (links) set of agent i is expressed as $\hat{g}(t), \hat{N_i}(t)$. Utilities of agent i is expressed as $\hat{u}_i(t)$. Agent j is also expressed same as that of agent i.

III. CASES OF AGENT-BASED MODEL

Simulations of interaction among four agents were carried out and got several network formations (Shown in TABLE I) [7] . TABLE I represents combinations of the degree sequences of stable network and shows cases of value of c. The degree sequences 1113 of stable network were formed by value of c = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7. The degree sequences 2222 of stable network were formed by value of c = 0.2. The degree sequences 1122 of stable network were formed by value of c = 0.3, 0.4, 0.5, 0.6, 0.7. The check marks in the TABLE I represent results of [7] and show simulation cases in this research (4-4 agents cases).

Firstly, the authors carry out combination cases of (1), (2), (3), (4), (5) in TABLE I. These cases are carried out 1,000 times. There are 18 patterns. Therefore the authors are going to get 18,000 data.

Simulations of interaction among seven agents were carried out and got several network formations (Shown in TABLE II) [7] . TABLE II represents the degree sequences of stable network and shows cases of value of c. The degree sequences

TABLE III
THE RESULT OF 4-4 AGENTS CASES C = 0.2

C = 0.2	(1) 1113−1113		(2) 2222−1113		(3) 2222−2222	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
23333444	34.80	21.9%	34.80	34.6%	34.80	18.1%
22222345	34.96	14.9%	34.96	10.0%	34.96	0.1%
22333344	34.88	13.5%	34.88	20.6%	34.88	6.6%
33333344	34.80	13.4%	34.80	12.1%	34.80	37.3%
22223344	34.96	8.5%	34.96	2.3%	34.96	4.0%
33333333	34.88	7.3%	34.88	12.7%	34.88	32.5%
	Four other forms, 20.5%.		Three other forms, 7.7%.		Two other forms, 1.4%.	

TABLE IV
THE RESULT OF 1113 AND 1113 NETWORKS ON 4-4 AGENTS CASES

(1) 1113−1113	C = 0.3		C = 0.4		C = 0.5	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
12222234	32.04	69.6%	30.24	67.8%	28.44	74.9%
11111144	31.58	10.9%	30.18	11.9%	28.78	10.8%
12222333	31.78	7.7%	29.98	4.3%	28.18	0.3%
11122234	31.55	4.8%	29.95	6.8%	28.35	6.7%
	Three other forms, 7.0%.		Three other forms, 9.2%.		Two other forms, 7.3%.	

	C = 0.6		C = 0.7	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
22222233	26.13	44.2%	24.33	42.9%
12222223	25.57	15.6%		
11122333	26.09	13.6%		
11111144	27.38	8.4%	25.98	11.0%
11111234	26.45	4.5%	24.24	38.2%
	Four other forms, 13.7%.		One other form, 7.9%.	

1111125, 1111134, 1122224, 1122233, and 2222233 of stable network were formed by value of c = 0.3, 0.4, 0.5. The degree sequences 1111224 (a)[1], 1111224 (b), 1122224, 1122233, and 2222233 of stable network were formed by value of c = 0.6, 0.7. The degree sequences 1111233 and 2222222 of stable network were formed by value of c = 0.6. The degree sequences 1111134 and 1111233 (d) of stable network were formed by value of c = 0.7. The degree sequences 2222244, 2222255, 2222334, 2333333, and 3333444 of stable network were formed by value of c = 0.2. The check marks in the TABLE II represent results of [7] and show simulation cases in this research (7-1 agents cases). One agent is added to each case.

Secondly, the authors carry out combination cases of (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), (17) in TABLE II. These cases are carried out 1,000 times. There are 32 patterns. Therefore the authors are going to get 32,000 data.

Thirdly, the authors analyze interpersonal comparison of utility on each case of TABLE II.

IV. RESULTS OF THE SIMULATION

A. 4-4 agents cases

The result of 4-4 agents cases of the simulation, network shape was categorized by value of "c". That is, network shapes of c = 0.2, network shapes of c = 0.3, 0.4, 0.5, and network shapes of c = 0.6, 0.7. In case of c = 0.2, network shape of (1)1113-1113, (2)2222-1113, (3)2222-2222

[1]In the case of 111124 (a),(b) are same degree sequences, but network shapes are different each other. In the case of 1111233 (c),(d) are also same degree sequences, but network shapes are different each other.

TABLE V
THE RESULT OF 1122 AND 1113 NETWORKS ON 4-4 AGENTS CASES

(4) 1122−1113	C = 0.3		C = 0.4		C = 0.5	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
22223333	32.53	36.8%	30.53	38.3%	28.53	31.4%
11122234	31.55	32.7%	29.95	30.4%	28.35	22.2%
12222234	32.04	19.9%	30.24	19.9%	28.44	22.9%
12222333	31.78	7.5%	29.98	8.5%	28.18	6.2%
	Two other forms, 3.1%.		Two other forms, 2.9%.		Three other forms, 17.3%.	

	C = 0.6		C = 0.7	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
12222223	25.57	29.3%	24.03	49.1%
22222233	26.13	19.6%	24.33	0.8%
11122333	26.09	17.1%		
11111234	26.25	15.5%	24.85	18.7%
	Four other forms, 18.5%.		Seven other forms, 31.4%.	

TABLE VI
THE RESULT OF 1122 AND 1122 NETWORKS ON 4-4 AGENTS CASES

(5) 1122−1122	C = 0.3		C = 0.4		C = 0.5	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
22223333	32.53	48.9%	30.53	49.0%	28.53	40.5%
12222333	31.78	30.9%	29.98	31.3%	28.18	26.8%
12222234	32.04	9.3%	30.24	9.6%	28.44	10.4%
11122234	31.55	8.2%	29.95	8.4%	28.35	3.9%
	One other form, 2.7%.		Two other forms, 1.7%.		Two other forms, 18.4%.	

	C = 0.6		C = 0.7	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
11222233	25.83	55.8%	24.23	42.7%
22222233	26.13	19.7%	24.33	30.8%
12222223	25.57	19.1%	23.97	15.8%
	Two other forms, 5.4%.		Six other forms, 10.7%.	

in TABLEIII was analyzed for tendency to form network shape. The authors define concentrated links to one agent (e.g., the degree sequences 1113) as"star network". The authors define also dispersed links (e.g., the degree sequences 2222) as "decentralized network". In cases of (1)1113-1113 and (2)2222-1113 were formed the degree sequences 23333444, 22222345, 22333344 frequently. In case of (3)2222-2222 was formed the degree sequences 33333344 and 33333333 frequently. The star network wasn't reformed by (1)1113-1113 and (2)2222-1113 cases. The range of degrees was from two to five. On the other hand, "decentralized network" was reformed by (3)2222-2222 case. The range of degrees was from 3 to 4. Initial shapes affect large stable network shapes.

The results of (1)1113-1113 and c = 0.3, 0.4, 0.5, 0.6, 0.7 were shown in TABLE IV. In cases of c = 0.3, 0.4, 0.5 were formed the degree sequences 12222234 frequently. In cases of c = 0.6, 0.7 were formed the degree sequences 22222233 frequently. c = 0.7 was also formed the degree sequences 11111234 frequently.

The results of (4)1122-1113 and c = 0.3, 0.4, 0.5, 0.6, 0.7 were shown in TABLE V. In cases of c = 0.3, 0.4, 0.5 were formed the degree sequences 22223333 and 11122234 frequently. In cases of c = 0.6, 0.7 were formed the degree sequences 12222223 frequently.

TABLE VII
THE RESULT OF CASES OF ADDING ONE AGENT TO 1111125 NETWORKS

(1) 1111125	C = 0.3		C = 0.4		C = 0.5	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
11122225	32.06	68.9%	30.46	71.2%	28.86	73.0%
11111135	31.83	17.3%	30.43	13.1%	29.03	14.1%
11111126	32.60	13.8%	31.20	15.7%	29.80	12.9%

TABLE VIII
THE RESULT OF CASES OF ADDING ONE AGENT TO 1111134 NETWORKS

(2) 1111134	C = 0.3		C = 0.4		C = 0.5		C = 0.6	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
11111234	31.55	70.8%	29.95	71.1%	28.35	71.5%	25.05	44.5%
11111234							24.85	27.5%
11111144	31.58	14.8%	30.18	14.3%	28.78	14.2%	25.98	12.5%
11111135	31.83	14.4%	30.43	14.6%	29.03	14.3%	26.23	15.5%

TABLE IX
THE RESULT OF CASES OF ADDING ONE AGENT TO 1111224 (a) NETWORKS

(3) 1111224	C = 0.6		C = 0.7	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
11112224	25.99	31.2%	24.59	28.2%
11222224			24.49	27.3%
11111234	26.45	29.2%	25.05	27.2%
11111225	27.17	13.4%	25.77	17.3%
22222233	26.13	11.6%		
Four other forms, 14.6%.				

TABLE X
THE RESULT OF CASES OF ADDING ONE AGENT TO 1111224 (b) NETWORKS

(4) 1111224	C = 0.6		C = 0.7	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
22222233	26.13	29.9%	24.33	1.1%
11111234	26.25	14.9%	24.24	13.9%
11111234			24.85	13.8%
12222223	25.57	14.2%		
11111225	26.56	13.1%	25.16	15.5%
11222233	25.83	12.6%	24.03	1.7%
11222224			24.49	53.4%
	Three other forms, 15.3%.		Two other forms, 0.6%.	

TABLE XI
THE RESULT OF CASES OF ADDING ONE AGENT TO 1111233 (c) AND (d) NETWORKS

(5) 1111233	C = 0.6	
The Degree Sequences	Total Utility	Appearance Ratio
11222233	25.83	50.8%
11111234	26.25	16.2%
11112233	25.33	12.7%
11111234	26.45	12.6%
11122333	26.09	7.7%

(6) 1111233	C = 0.7	
The Degree Sequences	Total Utility	Appearance Ratio
11222233	24.03	56.7%
11111234	24.24	29.9%
11111333	24.39	13.4%

TABLE XII
THE RESULT OF CASES OF ADDING ONE AGENT TO 1112223 NETWORKS

(7) 1112223	C = 0.6		C = 0.7	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
11112233	25.33	45.9%	23.93	43.3%
22222233	26.13	25.3%	24.33	44.2%
11112224	25.99	12.8%	24.59	12.5%
Three other forms, 16.0%.				

TABLE XIII
THE RESULT OF CASES OF ADDING ONE AGENT TO 1122224 NETWORKS

(8) 1122224	C = 0.3		C = 0.4		C = 0.5	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
12222234	32.04	47.5%	30.24	42.6%	28.44	57.2%
11122234	31.55	33.8%	29.95	36.8%	28.35	29.4%
11122225	32.06	12.5%	30.46	13.4%	28.86	13.4%
12222333	31.78	6.2%	29.98	7.2%		

TABLE XIV
THE RESULT OF CASES OF ADDING ONE AGENT TO 1122233 NETWORKS

(9) 1122233	C = 0.3		C = 0.4		C = 0.5	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
12222333	31.78	38.9%	29.98	36.8%	28.18	38.3%
22223333	32.53	33.5%	30.53	33.3%	28.53	34.7%
11122234	31.55	27.6%	29.95	29.9%	28.35	27.0%

TABLE XV
THE RESULT OF CASES OF ADDING ONE AGENT TO 1222223 NETWORKS

(10) 1222223	C = 0.6		C = 0.7	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
11222233	25.83	39.0%	24.03	29.9%
11222233			24.23	28.9%
22222233	26.13	37.2%	24.33	28.7%
12222223	25.57	18.3%		
11222224			24.49	12.5%
Three other forms, 5.5%.				

TABLE XVI
THE RESULT OF CASES OF ADDING ONE AGENT TO 2222222 AND 2222233 NETWORKS”

(11) 2222222	C = 0.6	
The Degree Sequences	Total Utility	Appearance Ratio
12222223	25.57	100.0%

(12) 2222233	C = 0.3		C = 0.4		C = 0.5	
The Degree Sequences	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio	Total Utility	Appearance Ratio
22223333	32.53	55.8%	30.53	58.8%	28.53	58.0%
12222234	32.04	29.8%	30.24	26.7%	28.44	26.9%
12222333	31.78	14.4%	29.98	14.5%	28.18	15.1%

TABLE XVII
THE RESULT OF 7-1 AGENTS CASES $C = 0.2$

(13) 2222244	C = 0.2	
The Degree Sequences	Total Utility	Appearance Ratio
23333444	34.80	62.5%
22222345	34.96	21.8%
22222255	34.96	15.7%

(15) 2222334	C = 0.2	
The Degree Sequences	Total Utility	Appearance Ratio
22333344	34.88	40.2%
33333344	34.80	37.6%
22222345	34.96	22.2%

(16) 2333333	C = 0.2	
The Degree Sequences	Total Utility	Appearance Ratio
33333344	34.80	45.5%
23333444	34.80	44.8%
22333344	34.88	9.7%

(14) 2222255	C = 0.2	
The Degree Sequences	Total Utility	Appearance Ratio
33333555	34.64	73.5%
22222266	34.88	26.5%

(17) 3333444	C = 0.2	
The Degree Sequences	Total Utility	Appearance Ratio
44444444	34.56	58.6%
33333555	34.64	41.4%

TABLE XVIII
THE CASE DECREASING PERSONAL UTILITY (1111224 (a))

(3) 1111224 (a) C = 0.6

The Degree Sequences	ID	A	B	C	D	E	F	G	H
1111224	Degree	1	1	1	1	2	2	4	
Total Utility	Utility	3.144	3.144	2.7856	2.7856	2.832	2.832	2.08	
19.6032	Linked ID	G	G	E	F	C, G	D, G	A, B, E, F	
Degree Seq.	ID	A	B	C	D	E	F	G	H
12222223	Degree	3	2	2	1	2	2	2	2
Total Utility	Utility	**2.904**	**3.114**	3.344	3.3232	3.114	3.216	3.344	3.216
25.5744	Linked ID	C, D, G	E, H	B, H	B	B, F	E, G	A, F	B, C
Degree Seq.	ID	A	B	C	D	E	F	G	H
11112233	Degree	3	2	1	1	2	1	3	1
Total Utility	Utility	**2.904**	3.216	3.3232	3.0928	3.216	3.323	3.16	3.0928
25.328	Linked ID	C, F, G	G, H	A	E	D, G	A	A, B, E	B
Degree Seq.	ID	A	B	C	D	E	F	G	H
11112233	Degree	3	1	3	1	1	2	2	1
Total Utility	Utility	3.16	**3.093**	2.904	3.0928	3.323	3.216	3.216	3.3232
25.328	Linked ID	C, F, G	G	A, E, H	F	C	A, D	A, B	C
Degree Seq.	ID	A	B	C	D	E	F	G	H
11222233	Degree	2	3	3	2	1	2	1	2
Total Utility	Utility	**3.114**	**3.032**	3.032	3.1136	3.426	3.344	3.4256	3.344
25.8304	Linked ID	D, H	C, F, G	B, E, H	A, F	C	B, D	B	A, C
Degree Seq.	ID	A	B	C	D	E	F	G	H
22222233	Degree	2	3	2	2	2	2	2	3
Total Utility	Utility	3.344	**3.032**	3.344	3.344	3.344	3.344	3.344	3.032
26.128	Linked ID	G, H	E, F, G	E, H	F, H	B, C	B, D	A, B	A, C, D

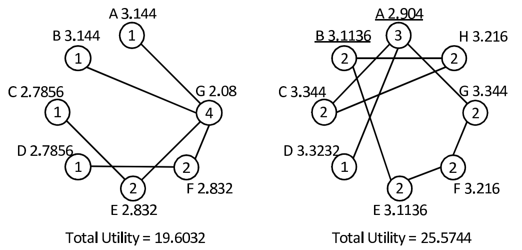

Fig. 4. Stable network shape of 1111224 and 12222223

The results of (1)1122-1122 and c = 0.3, 0.4, 0.5, 0.6, 0.7 were shown in TABLE VI. In cases of c = 0.3, 0.4, 0.5 were formed the degree sequences 22223333 and 12222333 frequently. In cases of c = 0.6, 0.7 were formed the degree sequences 11222233 and 22222233 frequently.

B. 7-1 agents cases

The result of 7-1 agents cases of the simulation, network shape was categorized by value of "c". That is, network shapes of c = 0.2, network shapes of c = 0.3, 0.4, 0.5, and network shapes of c = 0.6, 0.7.

The results of cases of adding one agent to 1111125 networks and c = 0.3, 0.4, 0.5 were shown in TABLE VII. In cases of c = 0.3, 0.4, 0.5 were formed the degree sequences 11122225 frequently. Tendency of occurrence frequency 11122225 was similar.

The result of cases of adding one agent to 1111134 networks and c = 0.3, 0.4, 0.5, 0.6 were shown in TABLE VIII. In cases of c = 0.3, 0.4, 0.5 were formed the degree sequences 11111234 frequently. In cases of c = 0.6 were also formed the degree sequences 11111234 frequently. Notice that in the case of 11111234 were same degree sequences, but network shapes were different each other.

The result of cases of adding one agent to 1111224 (a) networks were shown in TABLE IX. Notice that in the case of

TABLE XIX
THE CASE DECREASING PERSONAL UTILITY (1111224 (b), 1111233, 1222223)

(4) 1111224 (b) C = 0.6

The Degree Sequences	ID	A	B	C	D	E	F	G
1111224	Degree	1	1	1	1	2	2	4
Total Utility	Utility	3.042	3.042	3.0416	2.5808	2.576	2.96	1.952
19.1936	Linked ID	G	G	G	E	D, F	E, G	A, B, C, F

(4) 1111224 (b) C = 0.7

The Degree Sequences	ID	A	B	C	D	E	F	G
1111224	Degree	1	1	1	1	2	2	4
Total Utility	Utility	2.942	2.942	2.9416	2.4808	2.376	2.76	1.552
17.9936	Linked ID	G	G	G	E	D, F	E, G	A, B, C, F

(5) 1111233 (b) C = 0.7

The Degree Sequences	ID	A	B	C	D	E	F	G
1111233	Degree	1	1	1	1	2	3	3
Total Utility	Utility	2.814	2.814	2.916	2.5832	2.504	2.22	2.092
17.9424	Linked ID	G	G	F	E	D, F	C, E, G	A, B, F

(10) 1222223 C = 0.6

The Degree Sequences	ID	A	B	C	D	E	F	G
1222223	Degree	1	2	2	2	2	2	3
Total Utility	Utility	2.914	2.832	2.704	2.6016	2.704	2.832	2.392
18.9792	Linked ID	G	C, G	B, D	C, E	D, F	E, G	A, B, F

1111224 (a) and (b) were same degree sequences, but network shapes were different each other. In cases of c = 0.6, 0.7 were formed the degree sequences 11112224 and 11111234 frequently. c = 0.7 was also formed the degree sequences 11222224 frequently.

The result of cases of adding one agent to 1111224 (b) networks were shown in TABLE X. In cases of c = 0.6 was formed the degree sequences 22222233 frequently. On the other hand, c = 0.7 was also formed the degree sequences 11222224 frequently. In this cases, variation of the degree sequences were more than other cases.

The result of cases of adding one agent to 1111233 (c) and (d) networks were shown in TABLE XI. Notice that in the case of 1111233 (c) and (d) were same degree sequences, but network shapes were different each other. In cases of 1111233 (c) c = 0.6, 1111233 (c) c = 0.7 were formed the degree sequences 11222233 and 11111234 frequently.

The result of cases of adding one agent to 1112223 networks were shown in TABLE XII. In cases of c = 0.6, 0.7 were formed the degree sequences 11112233 and 22222233 frequently. Tendency of occurrence frequency 11112233 was similar. But variation of the degree sequences on c = 0.6 were more than cases of c = 0.7.

The result of cases of adding one agent to 1122224 networks were shown in TABLE XIII. In cases of c = 0.3, 0.4, 0.5 were formed the degree sequences 12222234 and 11122234 frequently.

The result of cases of adding one agent to 1122233 networks were shown in TABLE XIV. In cases of c = 0.3, 0.4, 0.5 were formed the degree sequences 12222333 and 22223333 frequently. In cases of c = 0.3, 0.4, 0.5, tendency of occurrence frequency was similar.

The result of cases of adding one agent to 1222223 networks were shown in TABLE XV. In cases of c = 0.6, 0.7 were formed the degree sequences 11222233 frequently. In cases of c = 0.7, notice that in the case of 11222233 were same degree sequences, but network shapes were two types.

The results of cases of adding one agent to 2222222 networks and c = 0.6 were shown in TABLE XVI. In cases of c = 0.6 were formed the degree sequences only 12222223.

The result of cases of adding one agent to 2222233 networks were also shown in TABLE XVI. In cases of c = 0.3, 0.4, 0.5 were formed the degree sequences 22223333, 12222234 frequently. In cases of c = 0.3, 0.4, 0.5, tendency of occurrence frequency was similar.

In case of c = 0.2, network shape of (13)2222244, (14)2222255, (15)2222334, (16)2333333, (17)3333444 in TABLE XVII was analyzed for tendency to form network shape. Variation of the degree sequences on (14)2222255 and (17)3333444 were only two types.

C. Analyzing interpersonal comparison of utility on "7-1 agents cases"

The authors analyzed the network shape which was added one agent to a stable network of seven agents (7-1 agents cases). In this section, the authors analyze interpersonal comparison of utility on "7-1 agents cases".

In our model, if amount of agents was added, total utility of the network was increasing. Therefore, in a lot of cases, each personal utility of agents was increased by adding one agent to stable network. However, in some cases, despite adding one agent to stable network, personal utility of some agents was decreased.

TABLE XVIII represents the cases of (3)1111224(a) c = 0.6 and shows that utility of some agent was decreased despite of added one agent. Stable network shape of seven agents (total utility was 19.6032) is shown in Fig.4. Stable network shape after interaction among eight agents (total utility was 25.5744) is also shown in Fig.4. Personal utilities of agent "A" and agent "B" of Fig.4 were decreased by added one agent.

In cases of (4)1111224(b) c = 0.6 (shown in TABLE XIX), utility of agent "A", agent "B" and agent "C" are same. That is, these three agents are replaceable. If results of the simulation were degree sequences 11112233 (total utility 25.328), 11122333 (26.0864), 11222233 (25.8304), 12222223 (25.5744), or 22222233 (26.128), then the degree on one of agent "A", "B" and "C" was three. In addition personal utility of the agent was decreased. If results of the simulation were degree sequences 12222223 (total utility 25.5744) or 11112233 (25.328), then the degree of agent "F" was three. In addition personal utility of the agent was decreased.

In case of (4)1111224(b) c = 0.7 is shown in TABLE XIX. If results of the simulation were degree sequences 11111234 (total utility 25.0544) or 11111144 (25.976), then the degree of agent "D" was four. In addition personal utility of the agent was decreased. If results of the simulation were degree sequences 11111234 (total utility 24.2352), then the degree of agent "D" was three in some case. In addition personal utility of the agent was decreased in some case.

In case of (5)1111233 (b) c = 0.6 is shown in TABLE XIX. If results of the simulation were degree sequences 11111234 (total utility 24.2352), then the degree of agent "F" or "G" was four in some case. In addition personal utility of the agent was decreased.

In case of (4) 1222223(b) c = 0.6 is shown in TABLE XIX. If results of the simulation were degree sequences 11112233 (total utility 25.328), 11122333 (26.0864) or 12222223 (25.5744), then the degree of agent "A" was four in some case. In addition personal utility of the agent was decreased.

V. Conclusion

In this research, the authors analyzed interaction among two stable networks. Firstly the authors carried out simulation of 4-4 agents cases. The result of the simulation, some patterns of network shapes was formed. Notice that "star-network" was not found on the result of interaction among eight agents. On the other hand, "decentralized network" was formed from several combination.

Secondly, the authors analyzed the network (eight agents) shape which was added one agent to a stable network of seven agents (7-1 agents cases). The result of the simulation, almost close to "star network" shapes (e.g. (1)1111125 and (2)1111134) of stable network by seven agents became one shape at a higher rate than that of other cases (e.g. (1)111115 became 11122225 at about 70 percent ratio). In case of c = 0.2, patterns of network shape were two or three. In addition tendency of degree sequences was "decentralized network". In case of c = 0.3, 0.4 and 0.5, patterns of network shape were three or four. In case of c = 0.6, patterns of network shape were from five to eight except a case of (11) 2222222. In case of c = 0.7, patterns of network shape were from three to eight. If the value of "c" was larger, the number of patterns of stable network was also larger.

Thirdly, the authors analyzed interpersonal comparison of utility on "7-1 agents cases". In our model, if amount of agents was added, total utility of the network was increasing. Therefore, in a lot of cases, each personal utility of agents was increased by adding one agent to stable network. However, in some cases, despite adding one agent to stable network, personal utility of some agents was decreased. The authors discovered patterns of decreasing personal utility on "c" = 6 or 7. For example, In case of (4) 1222223(b) c = 0.6, if degree sequences were 11112233 (total utility 25.328), 11122333 (26.0864) or 12222223 (25.5744), then the degree of agent "A" was four in some case. In addition personal utility of the agent was decreased.

In this research, the authors identified the characteristics of a large stable network which was the result of interaction among eight agents("7-1 agents cases"). Especially, the thing that the authors discovered the cases of decreasing personal utility under condition increasing total utility, has important meaning. For instance, what that means is that if one company enters into an industry by easing of regulations, then total utility of industry may increase, but utility of other companies may decrease.

Our model can apply to concrete issues for social environment.

Acknowledgment

The authors would like to thank Kozo Keikaku Engineering Inc. for providing the Multi Agent Simulator, named "artisoc".

REFERENCES

[1] Granovetter, M. (1973) "The Strength of Weak Ties," *American Journal of Sociology*, 78, 1360-1380.
[2] Corominas-Bosch, M. (2004) "Bargaining in a network of buyers and sellers," *Journal of Economic Theory*, 115 (1), 35-77.
[3] Kranton, R. and D. Minehart (2001) "A Theory of Buyer-Seller Networks," *The American Economic Review*, 91, 485-508.
[4] Bramoullé, Y. and R. Kranton (2007) "Risk-sharing networks," *Journal of Economic Behavior and Organization*, 64, 275-294.
[5] Hendricks, K., M. Piccone and G. Tan (1995) "The Economics of Hubs: The Case of Monopoly," *Review of Economic Studies*, 62, 83-100.
[6] Jackson, M.O. and A. Wolinsky (1996) "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory*, 71, 44-74.
[7] Daito, M. and H. Kojima (2011), "A Dynamic Analysis for Social Network Formation under Symmetric Situation" *Journal of Okayama Shoka University* 47(2), (forthcoming).