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Robust UKF Insensitive to Measurement Faults for Pico Satellite Attitude Estimation

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***Abstract*—**In the normal operation conditions of a pico satellite, conventional Unscented Kalman Filter (UKF) gives sufficiently good estimation results. However, if the measurements are not reliable because of any kind of malfunction in the estimation system, UKF gives inaccurate results and diverges by time. This study, introduces Robust Unscented Kalman Filter (RUKF) algorithms with the filter gain correction for the case of measurement malfunctions. By the use of defined variables named as measurement noise scale factor, the faulty measurements are taken into the consideration with a small weight and the estimations are corrected without affecting the characteristic of the accurate ones. Two different RUKF algorithms, one with single scale factor and one with multiple scale factors, are proposed and applied for the attitude estimation process of a pico satellite. The results of these algorithms are compared for different types of measurement faults in different estimation scenarios and recommendations about their applications are given.

***Keywords*—**attitude algorithms, Kalman filters, robust estimation.

I. Introduction

SINCE it was proposed, Kalman filter has been widely used as an attitude determination technique and different Kalman filter types have been developed with that purpose. As a known fact; attitude estimation problem of a pico satellite cannot be solved by linear Kalman filters because of the inherent nonlinear dynamics and kinematics. In such case Extended Kalman Filter (EKF) may be used instead. By using EKF, it is possible to estimate attitude parameters of a satellite which has three onboard magnetometers as the only measurement sensors [1]. However, mandatory linearization phase of EKF procedure may cause filter to diverge and usually, Jacobian calculations required for this phase are cumbersome and time-consuming [2].

Unscented Kalman Filter (UKF) is a relatively new Kalman filtering technique that generalizes Kalman filter for both linear and nonlinear systems and in case of nonlinear dynamics, UKF may afford considerably more accurate estimation results than the former observer design methodologies such as Extended Kalman Filter. UKF is based on the fact that; approximation of a nonlinear distribution is easier than the approximation of a nonlinear function or transformation [2]. UKF introduces sigma points to catch higher order statistic of the system and by securing higher order information of the system, it satisfies both, better estimation accuracy and convergence characteristic [3]. Besides, as the estimation characteristic of the UKF is not affected by the level of nonlinearity, it can be preferred for the systems, which has highly nonlinear dynamics and measurements models such as the spacecrafts [4].

On the other hand, UKF has no capability to adapt itself to the changing conditions of the measurement system. Malfunctions such as abnormal measurements, increase in the background noise etc. affects instantaneous filter outputs and process may result with the failure of the filter. In order to avoid from such condition, the filter must be operated robustly.

UKF can be made adaptive and hence insensitive to the priori measurements or system uncertainties by using various different techniques. Multiple Model Based Adaptive Estimation (MMAE), Innovation Based Adaptive Estimation (IAE) and Residual Based Adaptive Estimation (RAE) are three of basic approaches to the adaptive Kalman filtering. In the first approach, more than one filters run parallel under different models for satisfying filter's true statistical information. However that can be only achieved if the sensor/actuator faults are known. Also, this approach requires several parallel Kalman filters to run and the processing time may increase in such condition [5]. In IAE or RAE methods, adaptation is applied directly to the covariance matrices of the measurement and/or system noises in accordance with the differentiation of the residual or innovation sequence. To realize these methods, the innovation or residual vectors must be known for *m epoch* and that causes an increment in the storage burden, as well as the requirement to know the width of the *moving window* [6]. Besides, in order to estimate covariance matrix of the measurement noise based on the innovation or residual vector; number, type and distribution of measurements must be consistent for all epochs within a window.

Another concept is to scale the noise covariance matrix by multiplying it with a time dependent variable. One of the methods for constructing such algorithm is to use a single scale factor as a multiplier to the process noise covariance matrices [5, 7]. This algorithm, which may be named as

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Adaptive Fading Kalman Filter (AFKF), can be both used when the information about the dynamic process or the priori measurements is absent [8]. However, when the point at issue is the recent measurements, another technique to scale measurement noise covariance matrix and make filter robust (insensitive to recent measurement faults) should be proposed. Therefore, if there is a malfunction in the measurement system, Robust Kalman Filter (RKF) algorithm can be utilized and via correction applied to the filter gain, good estimation behaviour of the filter can be secured without being affected from faulty current measurements [6].

However, estimation performance of the Kalman filter differs for each variable when it is utilized for complex systems with multivariable and it may be not sufficient to use single measurement noise scale factor (MNSF) as a multiplier for the measurement noise covariance matrices [9]. Single MNSF may not reflect corrective effects for the faulty measurement to the estimation process, accurately. Technique, which can be implemented to overcome this problem, is to use multiple MNSFs to fix relevant component of the gain matrix, individually. Hence a robust unscented Kalman filter (RUKF) algorithm with multiple MNSFs may be considered instead of RUKF with single MNSF.

In literature, it is possible to meet with a limited number of RUKF applications, which also considers measurement noise scaling as well as process noise and uses unscented Kalman filter. In [10] a two-stage adaptive UKF is proposed in the base of the process noise and measurement noise covariances matrices adaptation. Basically, it applies the methodology presented in [8] to the nonlinear systems by the use of UKF. However, as a disadvantage, it secures the adaptation using only single fading factor and as it is aforementioned, that may be a problem for implementations on complex systems like spacecrafts.

In this paper, Robust Unscented Kalman Filter (RUKF) algorithms with single and multiple measurement noise scale factors, which make filter insensitive to current measurement faults, are introduced and applied for the attitude parameter estimation process of a pico satellite. Results of these algorithms are compared for different types of measurement malfunctions in different estimation scenarios and the recommendations about their utilization are given.

II. Pico Satellite Mathematical Model

If the kinematics of the pico satellite is derived in the base of Euler angles, then the mathematical model can be expressed with a 6 dimensional system vector which is made of attitude Euler angles (φ is the roll angle about x axis; θ is the pitch angle about y axis; ψ is the yaw angle about z axis) vector and the body angular rate vector with respect to the inertial axis frame,

$$\bar{x} = \begin{bmatrix} \varphi & \theta & \psi & \omega_x & \omega_y & \omega_z \end{bmatrix}^T, \tag{1}$$

$$\bar{\omega}_{BI} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T, \tag{2}$$

where $\bar{\omega}_{BI}$ is the angular velocity vector of body frame with respect to the inertial frame. Besides, dynamic equations of the satellite can be derived by the use of the angular momentum conservation law;

$$J_x \frac{d\omega_x}{dt} = N_x + \left(J_y - J_z\right)\omega_y\omega_z, \tag{3}$$

$$J_y \frac{d\omega_y}{dt} = N_y + \left(J_z - J_x\right)\omega_z\omega_x, \tag{4}$$

$$J_z \frac{d\omega_z}{dt} = N_z + \left(J_x - J_y\right)\omega_x\omega_y, \tag{5}$$

where J_x, J_y and J_z are the principal moments of inertia and N_x, N_y and N_z are the terms of the external moment affecting the satellite. If the gravity gradient torque is taken into the consideration for the Low Earth Orbit (LEO) satellite, these terms can be written as

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = -3\frac{\mu}{r_0^3}\begin{bmatrix} \left(J_y - J_z\right) A_{23}A_{33} \\ \left(J_z - J_x\right) A_{13}A_{33} \\ \left(J_x - J_y\right) A_{13}A_{23} \end{bmatrix}. \tag{6}$$

Here μ is the gravitational constant, r_0 is the distance between the centre of mass of the satellite and the Earth and A_{ij} represents the corresponding element of the direction cosine matrix of [11];

$$A = \begin{bmatrix} c(\theta)c(\psi) & c(\theta)s(\psi) & -s(\theta) \\ -c(\varphi)s(\psi)+s(\varphi)s(\theta)c(\psi) & c(\varphi)c(\psi)+s(\varphi)s(\theta)s(\psi) & s(\varphi)c(\theta) \\ s(\varphi)s(\psi)+c(\varphi)s(\theta)c(\psi) & -s(\varphi)c(\psi)+c(\varphi)s(\theta)s(\psi) & c(\varphi)c(\theta) \end{bmatrix}. \tag{7}$$

In matrix A, $c(\cdot)$ and $s(\cdot)$ are the cosines and sinus functions successively. Kinematic equations of motion of the pico satellite with the Euler angles can be given as

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s(\varphi)t(\theta) & c(\varphi)t(\theta) \\ 0 & c(\varphi) & -s(\varphi) \\ 0 & s(\varphi)/c(\theta) & c(\varphi)/c(\theta) \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix}. \tag{8}$$

Here $t(\cdot)$ stands for tangent function and p, q and r are the components of $\bar{\omega}_{BR}$ vector which indicates the angular velocity of the body frame with respect to the reference frame. $\bar{\omega}_{BI}$ and $\bar{\omega}_{BR}$ can be related via,

$$\bar{\omega}_{BR} = \bar{\omega}_{BI} + A\begin{bmatrix} 0 \\ -\omega_0 \\ 0 \end{bmatrix}. \tag{9}$$

where ω_0 denotes the angular velocity of the orbit with respect to the inertial frame, found as $\omega_0 = \sqrt{\left(\mu / r_0^3\right)}$.

III. The Measurement Sensor Models

A. The Magnetometer Model

As the satellite navigates along its orbit, magnetic field vector differs in a relevant way with the orbital parameters. If those parameters are known, then, magnetic field tensor vector that affects satellite can be shown as a function of time analytically [3, 11]. Note that, these terms are obtained in the orbit reference frame.

$$H_1(t)=\frac{M_e}{r_0^3}\left\{\cos(\omega_0 t)\left[\cos(\varepsilon)\sin(i)-\sin(\varepsilon)\cos(i)\cos(\omega_e t)\right]\right.$$
$$\left.-\sin(\omega_0 t)\sin(\varepsilon)\sin(\omega_e t)\right\} \quad (10)$$

$$H_2(t)=-\frac{M_e}{r_0^3}\left[\cos(\varepsilon)\cos(i)+\sin(\varepsilon)\sin(i)\cos(\omega_e t)\right], \quad (11)$$

$$H_3(t)=\frac{2M_e}{r_0^3}\left\{\sin(\omega_0 t)\left[\cos(\varepsilon)\sin(i)-\sin(\varepsilon)\cos(i)\cos(\omega_e t)\right]\right.$$
$$\left.-2\sin(\omega_0 t)\sin(\varepsilon)\sin(\omega_e t)\right\} \quad (12)$$

Here

- $M_e = 7.943\ x\ 10^{15}\ Wb.m$; the magnetic dipole moment of the Earth,
- $\mu = 3.98601\ x\ 10^{14}\ m^3/s^2$; the Earth Gravitational constant,
- $i = 97^\circ$; the orbit inclination,
- $\omega_e = 7.29\ x\ 10^{-5}\ rad/s$; the spin rate of the Earth,
- $\varepsilon = 11.7^\circ$; the magnetic dipole tilt,
- $r_0 = 6,928,140\ m$; the distance between the centre of mass of the satellite and the Earth.

Three onboard magnetometers of pico satellite measures the components of the magnetic field vector in the body frame. Therefore for measurement model, which characterizes the measurements in the body frame, gained magnetic field terms must be transformed by the use of direction cosine matrix, A. Overall measurement model may be given as;

$$\begin{bmatrix} H_x(\varphi,\theta,\psi,t) \\ H_y(\varphi,\theta,\psi,t) \\ H_z(\varphi,\theta,\psi,t) \end{bmatrix} = A \begin{bmatrix} H_1(t) \\ H_2(t) \\ H_3(t) \end{bmatrix} + \eta_1, \quad (13)$$

where, $H_1(t)$, $H_2(t)$ and $H_3(t)$ represent the Earth magnetic field vector components in the orbit frame as a function of time and $H_x(\varphi,\theta,\psi,t)$, $H_y(\varphi,\theta,\psi,t)$ and $H_z(\varphi,\theta,\psi,t)$ show the Earth magnetic field vector components in body frame as a function of time and varying Euler angles. Furthermore η_1 is the zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{1k}\eta_{1j}^T\right] = I_{3x3}\sigma_m^2\delta_{kj}\,. \quad (14)$$

Here, I_{3x3} is the identity matrix with the dimension of 3×3, σ_m is the standard deviation of each magnetometer error and δ_{kj} is the Kronecker symbol.

B. The Rate Gyro Model

Inertial Measurement Unit (IMU) consists of three rate gyros aligned through three axes, orthogonally to each other. Rate gyros supply directly the angular rates of the body frame with respect to the inertial frame. Hence the model for rate gyros can be given as;

$$\bar{\omega}_{BI,meas} = \bar{\omega}_{BI} + \eta_2\,. \quad (15)$$

where, $\bar{\omega}_{BI,meas}$ is the measured angular rates of the satellite and η_2 is the zero mean Gaussian white noise with the characteristic of

$$E\left[\eta_{2k}\eta_{2j}^T\right] = I_{3x3}\sigma_g^2\delta_{kj}\,, \quad (16)$$

Here, σ_g is the standard deviation of each rate gyro random error.

IV. Robust Unscented Kalman Filters for Attitude Estimation

A. Robust Unscented Kalman Filter with Single Measurement Noise Scale Factor

In case of normal operation of the measurement system, filter works correctly. However when there is a malfunction in the estimation system such as abnormal measurements, step-like changes or sudden shifts in the measurement channel etc. filter fails and estimation outputs become faulty [6].

Therefore, a robust algorithm must be introduced such that filter makes itself insensitive to faults in case of malfunctions and corrects estimation process without affecting good estimation behaviour.

Robust algorithm affects characteristic of filter only when the condition of the measurement system does not correspond to the model used in the synthesis of the filter. Otherwise filter works with regular UKF algorithm [2] in an optimal way. Adaptation occurs as a change in the covariance matrix of the innovation sequence,

$$P_{vv}(k+1|k) = P_{yy}(k+1|k) + S(k)R(k+1), \quad (17)$$

where $S(k)$ is the scale factor calculated in the base of innovation sequence, $e(k+1)$, analyses. In robust case filter gain becomes

$$K(k+1) = P_{xy}(k+1|k)\left[P_{yy}(k+1|k) + S(k)R(k+1)\right]^{-1} \quad (18)$$

The gain matrix is changed when the condition of

$$tr\left[e(k+1)e^T(k+1)\right] \geq tr\left[P_{yy}(k+1|k) + R(k+1)\right] \quad (19)$$

is the point at issue. Here $tr(\cdot)$ is the trace of the related matrix. Left hand side of (19) represents the real filtration error while the right hand side is the accuracy of the innovation sequence known as a result of priori information. When the predicted observation vector $\hat{y}(k+1|k)$ is reasonably different from the current measurement vector , $y(k+1)$, real filtration error exceeds the theoretical one. Hence, gain matrix must be fixed hereafter by the use of robust algorithm and so scale factor $S(k)$. In order to calculate MNSF equality of

$$tr\left[e(k+1)e^T(k+1)\right] = tr\left[P_{yy}(k+1|k) + S(k)R(k+1)\right] \quad (20)$$

is used. Equation (20) can be rewritten as

$$tr\left[e(k+1)e^T(k+1)\right]=tr\left[P_{yy}(k+1|k)\right]+S(k)tr\left[R(k+1)\right] \quad (21)$$

If the knowledge of

$$tr\left[e(k+1)e^T(k+1)\right]=e^T(k+1)e(k+1) \quad (22)$$

is taken into consideration, (21) becomes

$$e^T(k+1)e(k+1)=tr\left[P_{yy}(k+1|k)\right]+S(k)tr\left[R(k+1)\right]. \quad (23)$$

As a result, MNSF can be obtained as

$$S(k)=\frac{e^T(k+1)e(k+1)-tr\left[P_{yy}(k+1|k)\right]}{tr\left[R(k+1)\right]}. \quad (24)$$

MNSF increases in case of malfunctions. That makes up an increment in covariance matrix of innovation sequence and a decrement in Kalman gain as it can be seen from (17) and (18). Consequently, current faulty measurements are regarded with a small weight in the estimation process and filter outputs are not affected.

B. Robust Unscented Kalman Filter with Multiple Measurement Noise Scale Factor

As it is discussed, robustness of the filter may be secured by using single MNSF as a corrective term on the filter gain. However that is not a healthy procedure as long as the filter performance differs for each state for the complex systems with multivariable [9]. The preferred method is to use a matrix built of multiple scale factors to fix the relevant term of the Kalman gain matrix, individually.

Robust algorithm affects characteristic of filter only when the condition of the measurement system does not correspond to the model used in the synthesis of the filter. Otherwise filter works with regular UKF algorithm in an optimal way. In case, where the system operates normally, the real and the theoretical innovation covariance matrix values match as in (25).

$$\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}e(k+1)e^T(k+1)\geq P_{yy}(k+1|k)+R(k+1), \quad (25)$$

here, μ is the width of the moving window.

However, when there is a measurement malfunction in the estimation system, the real error will exceed the theoretical one. Hence, if a scale matrix, $S(k)$, is added into the algorithm as,

$$\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}e(k+1)e^T(k+1)=P_{yy}(k+1|k)+S(k)R(k+1), \quad (26)$$

then, it can be determined by the formula of,

$$S(k)=\left\{\frac{1}{\mu}\sum_{j=k-\mu+1}^{k}e(k+1)e^T(k+1)-P_{yy}(k+1|k)\right\}R^{-1}(k+1). \quad (27)$$

In case of normal operation, the scale matrix will be a unit matrix as $S(k)=I$. Here I represents the unit matrix.

Nonetheless, as μ is a limited number because of the number of the measurements and the computations performed with computer implies errors such as the approximation errors and the round off errors; $S(k)$ matrix, found by the use of (27) may not be diagonal and may have diagonal elements which are "negative" or lesser than "one" (actually, that is physically impossible).

Therefore, in order to avoid such situation, composing scale matrix by the following rule is suggested:

$$S^*=diag\left(s_1^*,s_2^*,\ldots,s_n^*\right) \quad (28)$$

where,

$$s_i^*=\max\left\{1,S_{ii}\right\} \qquad i=1,n. \quad (29)$$

Here, S_{ii} represents the i[th] diagonal element of the matrix $S(k)$. Apart from that point, if the measurements are faulty, $S^*(k)$ will change and so affect the Kalman gain matrix;

$$K(k+1)=P_{xy}(k+1|k)\left[P_{yy}(k+1|k)+S^*(k)R(k+1)\right]^{-1}. \quad (30)$$

In case of any kinds of malfunctions, the related element of the scale matrix, which corresponds to the faulty component of the measurement vector, increases and that brings out a smaller Kalman gain, which reduces the effect of the faulty innovation term on the state update process. As a result, accurate estimation results can be obtained even in case of measurement malfunctions.

On the other hand, robust algorithms are used only in case of faulty measurements and in all other cases procedure runs optimally with regular Unscented Kalman filter. Checkout is satisfied via a kind of statistical information. In order to achieve that, following two hypotheses may be introduced:

- γ_o ; the system is normally operating
- γ_1 ; there is a malfunction in the estimation system.

Failure detection is realized by the use of following statistical function,

$$\beta(k)=e^T(k+1)\left[P_{yy}(k+1|k)+R(k+1)\right]^{-1}e(k+1). \quad (31)$$

This statistical function has χ^2 distribution with M degree of freedom where M is the dimension of the innovation vector.

If the level of significance, α, is selected as,

$$P\left\{\chi^2>\chi^2_{\alpha,M}\right\}=\alpha; \qquad ;\ 0<\alpha<1, \quad (32)$$

the threshold value, $\chi^2_{\alpha,M}$ can be determined. Hence, when the hypothesis γ_1 is correct, the statistical value of $\beta(k)$ will be greater than the threshold value $\chi^2_{\alpha,M}$, i.e.:

$$\gamma_0:\beta(k)\leq\chi^2_{\alpha,M} \qquad \forall k$$
$$\gamma_1:\beta(k)>\chi^2_{\alpha,M} \qquad \exists k \quad (33)$$

V. Simulations

In order to understand the efficiency of the proposed robust unscented Kalman filter algorithms and examine the advantages of each algorithm, RUKF with single MNSF and RUKF with multiple MNSFs, various estimation scenarios are performed. In the first scenario, it is considered that the pico satellite has only three magnetometers onboard as the measurement sensors. Different kinds of measurement malfunctions are implemented to one of these magnetometers and robust Kalman filter algorithms are tested for these cases. On the other hand, in the second scenario, pico satellite has also three gyros for measuring attitude rates with respect to the inertial frame, so there are six measurement devices in total. Besides, the measurement error is implemented to one of the gyros in this case.

Simulations are realized for 2000 seconds with a sampling time of $\Delta t = 0.1\sec$ As an experimental platform a cubesat model is used and the inertia matrix is taken as;

$$J = \begin{bmatrix} 2.1x10^{-3} & 0 & 0 \\ 0 & 2.0x10^{-3} & 0 \\ 0 & 0 & 1.9x10^{-3} \end{bmatrix}$$

Nonetheless the orbit of the satellite is a circular orbit with an altitude of $r = 550km$. Other orbit parameters are same as it is presented in the section for the Earth Magnetic Field Model (Section III-A).

Simulations are also done with regular UKF so as to compare results with both RUKF algorithms. For robust Kalman filters taken $\chi^2_{\alpha,M}$ value corresponds to the reliability level of %95.

Results are summarized by tables which represent the absolute estimation errors. Note that, for all presented tables, highlighted results are gained at seconds where the measurement malfunction is implemented.

A. Instantaneous Abnormal Measurements

For the first scenario, where there are only magnetometers on the pico satellite, instantaneous abnormal measurements are simulated by adding a constant term to the magnetic field tensor measurement of one magnetometer at the 500th second.

As it is seen from Table I, both RUKF algorithms (with single and multiple MNSFs) give more accurate estimation results than UKF in case of the instantaneous abnormal measurements. The results obtained by regular UKF are not reliable when the measurements are gained with an error. However, RUKFs with single and multiple MNSFs maintain their estimation characteristic for the whole process and afford precise estimation outputs in case of the abnormal measurements, as well as the normal operation condition.

Although RUKF with multiple MNSFs gives better estimation results, there is not any significant difference between the estimation outputs of two filters, RUKF with single MNSF and RUKF with multiple MNSFs, as the Table I presents. Nonetheless, as a result of the moving window data keeping and the matrix calculations, it is apparent that RUKF with multiple MNSFs demands a larger calculation effort and that is an extra load for the onboard computers of the pico satellite. Under this circumstances, because of the lesser computational burden, which is more essential for the limited processor of the pico satellite, RUKF with single MNSF may be preferred.

For the second scenario, in which there are also gyros onboard, measurement malfunction is implemented to one of the gyros at the 500th second. Results are summarized at Table II by comparing the absolute estimation errors of three filters; UKF, RUKF with single MNSF and RUKF with multiple MNSFs.

TABLE I
Comparison of Absolute Values of Error in Case of Instantaneous Abnormal Measurements for Scenario of Faulty Magnetometer Measurement.

Par.	Abs. Values of Err. for Regular UKF		Abs. Values of Err. for RUKF with Single MNSF		Abs. Values of Err. for RUKF with Multi. MNSFs	
	500 s.	1000 s.	500 s.	1000 s.	500 s.	1000 s.
φ(deg)	2.6941	0.9987	0.071	0.1709	0.0652	0.1286
θ(deg)	1.1066	0.8125	0.4872	0.0861	0.3746	0.0705
ψ(deg)	0.7437	5.1317	0.9257	1.2101	0.7132	0.9247
ω_x(deg/s)	0.028	0.002	0.0003	0.0005	0.0003	0.0003
ω_y(deg/s)	0.0181	0.0055	0.001	0.0009	0.0007	0.0007
ω_z(deg/s)	0.0007	0.0005	0.0016	0.0001	0.0013	0.0004

TABLE II
Comparison of Absolute Values of Error in Case of Instantaneous Abnormal Measurements for Scenario of Faulty Gyro Measurement.

Par.	Abs. Values of Err. for Regular UKF		Abs. Values of Err. for RUKF with Single MNSF		Abs. Values of Err. for RUKF with Multi. MNSFs	
	500 s.	1000 s.	500 s.	1000	500 s.	1000
φ(deg)	20.54	1.663	0.008	0.1626	0.0086	0.0192
θ(deg)	37.63	0.005	1.0302	0.0482	0.1126	0.0032
ψ(deg)	33.70	3.491	1.7212	1.0506	0.1956	0.1371
ω_x(deg/s)	0.063	0.002	0.0002	0.0003	0.0001	0.0001
ω_y(deg/s)	0.013	0.001	0.0005	0.0004	0.0001	0.0001
ω_z(deg/s)	0.085	0.003	0.0037	0.0005	0.0005	0.0001

As it is seen from the Table II, unlike the first scenario, this time estimation characteristic of the RUKF with multiple MNSFs is much better than RUKF with single MNSF. Hence, if the pico satellite has also gyros onboard for attitude rate measurements, utilizing RUKF with multiple MNSFs may be more sensible despite the increased computational demands.

B. Continuous Bias at Measurements

For the first scenario, continuous bias term is formed by adding a constant term to the measurements of one of the magnetometers in between 500th and 530th seconds. As Table III shows, again optimal UKF fails about estimating states accurately. Moreover, implemented faulty measurement affects the estimation process even after 530th second and estimation results of UKF worsens by time as the results for 1000th second reflects. Per contra, RUKF algorithms with single and multiple MNSFs reduce the effect of the innovation sequence and eliminate the estimation error which is caused by the biased measurements of one magnetometer.

For the second scenario, in which there are also gyros onboard, bias term is implemented to one of the gyros in between 500th and 530th seconds. Results are summarized at Table IV by comparing the absolute estimation errors of three filters; UKF, RUKF with single MNSF and RUKF with multiple MNSFs. As it reflects, RUKF with multiple MNSFs is again preferable if the measurement input is also taken from the gyros.

TABLE III
COMPARISON OF ABSOLUTE VALUES OF ERROR IN CASE OF CONTINUOUS BIAS AT MEASUREMENTS FOR SCENARIO OF FAULTY MAGNETOMETER MEASUREMENT.

Par.	Abs. Values of Err. for Regular UKF		Abs. Values of Err. for RUKF with Single MNSF		Abs. Values of Err. for RUKF with Multi. MNSFs	
	500 s.	1000 s.	500 s.	1000 s.	500 s.	1000 s.
φ(deg)	5.0535	15.759	0.1064	0.2743	0.0928	0.2329
θ(deg)	2.7733	23.431	0.7773	0.1348	0.6617	0.1156
ψ(deg)	1.2492	192.76	1.4749	1.9333	1.256	1.6444
ω_x(deg/s)	0.0558	0.0798	0.0006	0.0007	0.0005	0.0006
ω_y(deg/s)	0.0352	0.2307	0.0015	0.0015	0.0013	0.0013
ω_z(deg/s)	0.0043	0.1422	0.0025	0.0001	0.0021	0.0001

TABLE IV
COMPARISON OF ABSOLUTE VALUES OF ERROR IN CASE OF CONTINUOUS BIAS AT MEASUREMENTS FOR SCENARIO OF FAULTY GYRO MEASUREMENT.

Par.	Abs. Values of Err. for Regular UKF		Abs. Values of Err. for RUKF with Single MNSF		Abs. Values of Err. for RUKF with Multi. MNSFs	
	500 s.	1000s.	500 s.	1000 s.	500 s.	1000 s.
φ(deg)	4.0597	954.43	0.0265	0.0519	0.0125	0.0269
θ(deg)	8.3862	405.73	0.2955	0.0135	0.1572	0.0051
ψ(deg)	7.567	760.22	0.5159	0.3726	0.2733	0.1927
ω_x(deg/s)	0.0141	0.9859	0.0001	0.00003	0.0001	0.00002
ω_y(deg/s)	0.0019	0.0724	0.00002	0.0002	0.00001	0.0001
ω_z(deg/s)	0.0198	0.8744	0.0012	0.0003	0.0006	0.0001

VI. CONCLUSIONS AND DISCUSSION

In this paper Robust Unscented Kalman Filter algorithms with single and multiple measurement noise scale factors for the case of measurement malfunctions are developed. By the use of defined variables named as scale factor, current faulty measurements are taken into consideration with small weight and the estimations are corrected without affecting the characteristic of the accurate ones. In the presented RUKFs, the filter gain correction is performed only in the case of malfunctions in the measurement system.

If the magnetometers are used as the only onboard measurement devices, and the computational resources are restricted, utilization of RUKF with single MNSF is recommended for pico satellite attitude estimation because of the lower computational demands even though it gives similar results with RUKF with multiple MNSFs. Besides, if any other measurement devices are also used, i.e. gyroscopes, in this case, RUKF with multiple MNSFs gives significantly better estimation results and its utilization is recommended.

The proposed approaches do not require a priori statistical characteristics of the faults and can be used for both linear and nonlinear systems. Furthermore the presented RUKF algorithms are simple for practical implementation. These characteristics make introduced RUKF algorithms extremely important in point of view of supplying reliable parameter estimation for the attitude control system of a pico satellite.

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