Sensitizing Rules for Fuzzy Control Charts

N. Pekin Alakoç and A. Apaydın

Abstract—Quality control charts indicate out of control conditions if any nonrandom pattern of the points is observed or any point is plotted beyond the control limits. Nonrandom patterns of Shewhart control charts are tested with sensitizing rules. When the processes are defined with fuzzy set theory, traditional sensitizing rules are insufficient for defining all out of control conditions. This is due to the fact that fuzzy numbers increase the number of out of control conditions. The purpose of the study is to develop a set of fuzzy sensitizing rules, which increase the flexibility and sensitivity of fuzzy control charts. Fuzzy sensitizing rules simplify the identification of out of control situations that results in a decrease in the calculation time and number of evaluations in fuzzy control chart approach.

Keywords—Fuzzy set theory, Quality control charts, Run Rules, Unnatural patterns.

I. INTRODUCTION

THE most popular technique used in statistical process control is the control charts. Control charts provide information in process shifts and are effective in examining the production process, which reduce the variability, improve the productivity and lead to take corrective actions if necessary.

It is important to detect malfunction in the production process before a failure occurs. Then, an investigation and corrective actions can be applied to take control of the causes of the variation and produce at the desired quality level.

Standpoint of quality has been changing tremendously since rules of competition have been altered in the last decade. Due to the fact that classical methods do not account for most of the real life problems, fuzzy logic was employed in real life applications such as quality control to a great extent. Fuzzy control charts that are constructed with fuzzy set theory reflect the uncertainty better than Shewhart control charts. Shewhart control charts give accurate results when the data are known with precisely. However, this is not possible if there are unavoidable errors or human subjectivity is concerned.

The purpose of this study is to present fuzzy sensitizing rules for fuzzy control charts. The fuzzy rules are proposed to apply with fuzzy control charts, so that efficiency and performance of the fuzzy control charts increase. The procedure is based on the most common sentizing rules of Shewhart control charts and fuzzy theory.

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A nonrandom pattern may be resulted from a fuzzy number or one of its characteristic that is different from other fuzzy numbers. For example, symmetry, variance and situation of the fuzzy number on the control chart. In addition to traditional sensitizing rules these characteristics are suggested to be used in fuzzy sensitizing rules and new rules are proposed.

The rest of this paper is organized as follows: The Shewhart control charts and the most popular sensitizing rules are reviewed in Section II. An overview of the studies in literature on fuzzy quality control charts and fuzzy nonrandom patterns are presented in Section III. In Section IV, the proposed fuzzy sensitizing rules are presented. Finally, the suggested fuzzy rules are discussed with the advantages of the rules and concluding remarks are presented in Section V.

II. CONTROL CHARTS

Control charts were first introduced by Walter A. Shewhart in 1920s [1]. This technique has spread to the world as a simple and effective way of monitoring the processes. Shewhart control charts display the quality characteristics that has been measured or calculated from the sample over time. All Shewhart control charts consists of three main parts: a center line (CL) and two horizontal lines, upper control limit (UCL) and lower control limit (LCL) which are chosen by the average and variance of the data. The points on the charts are the sample measurements or the averages of the measurements of the quality characteristic. Shewhart control charts have two important assumptions; the quality characteristic is normally distributed and independent from each other. These assumptions are utilized in the calculation of control limits.

In order to observe process shifts and reduce the variability of the process the control limits are compared by the sample data. If all of the sample points fall between the limits with a random pattern then the process is assumed to be in control. In other words, when the quality characteristic is only subject to random variation then the process is said to be "in control". Otherwise, the process is "out of control" and there are assignable causes of the variation. The existence of assignable causes of variation is observed with sensitizing rules which test the nonrandom patterns of control charts. Besides, the sensitivity of the control chart to small shifts of the process mean is increased with the supplementary rules.

Control charts indicate that "the process is out of control" if there exists any point beyond the control limits. However, a process may be out of control even if all the points are in control limits. The nonrandom patterns of a control chart are tested with sensitizing rules. In literature, nonrandom patterns of control charts have been investigated in several studies.

World Academy of Science, Engineering and Technology
International Journal of Industrial and Manufacturing Engineering
Vol:7, No:5, 2013

Over the years, many rules have been proposed for pattern recognition of the charts. In 1956 Western Electric Handbook [2] suggested a set of rules for identifying the nonrandom patterns of control charts. Western Electric rules are based on three sigma, two sigma, and one sigma control limits which are called A, B, C zones respectively. For this reason, Western Electric rules are sometimes called zone rules.

The Western Electric suggests that if any one of the rule is satisfied then the process is out of control. The Western Electric rules are given below [1]:
1) One point plots outside the three sigma control limits,
2) Two out of three consecutive points plot beyond the two-sigma warning limits,
3) Four out of five consecutive points plot at a distance of one-sigma or beyond from the center line,
4) Eight consecutive points plot on one side of the center line.

Sensitizing rules which are the most well known in practice are suggested by Nelson [3], [4]. The rules that are most commonly used in applications are as follows [1]:
1) One or more points outside of the control limits,
2) Two of three consecutive points outside the two-sigma warning limits but still inside the control limits,
3) Four of five consecutive points beyond the one-sigma control limits,
4) A run of eight consecutive points on one side of the center line,
5) Six points in a row steadily increasing or decreasing,
6) Fifteen points in a row in zone C,
7) Fourteen points in a row altering up and down,
8) Eight points in a row on both sides of the center line with none in zone C,
9) An unusual or nonrandom pattern in the data,
10) One or more points near a warning or control limit.

The situations that satisfy these rules show that the process is actually out of control although the fuzzy numbers are between fuzzy control limits.

III. Fuzzy Control Charts

Fuzzy set theory was introduced by Lotfi. A. Zadeh in 1965 [5]. Fuzzy theory then used to explain and model real world systems containing uncertainty. Uncertainty exists in almost all real systems including the statistical process control problems. Therefore, fuzzy control charts are proposed to deal with this uncertainty in defining the processes. Fuzzy control charts have some important advantages compared to Shewhart control charts in handling uncertainty.

Although quality control charts have been studied since 1920's, the first fuzzy control chart was proposed after 1990s. Raz and Wang [6], [7] proposed two approaches, namely probabilistic and membership approaches, for the construction of fuzzy control charts for attributes. Kanagawa, Tamaki and Ohta [8] addressed linguistic terms as fuzzy numbers and defined probability density functions for the terms. Taleb and Limam [9] proposed to compare different fuzzy control charts generation procedures for linguistic data based on the fuzzy set and probability theories. A new approach for fuzzy control

chart based on the Shewhart's attribute control charts is described by Gülbay, Kahraman and Ruan [10]. Cheng [11] developed fuzzy control charts for attributes, plotted with expert opinion and fuzzy regression analysis with neural network. Gülbay and Kahraman [12] introduced direct fuzzy approach to monitor the performance of a process defined by triangular fuzzy numbers. The studies on statistical quality control are reviewed with theory of probability and fuzzy set theory by Hryniewicz [13]. Amirzadeh, Mashinchi and Parchami [14] proposed a fuzzy control chart based on degree of average nonconforming products. The first fuzzy control chart for quantitative data is proposed by Faraz and Moghadam [15]. In another study, Senturk and Erginel [16] discussed a procedure of constructing fuzzy $\bar{x}$, R and s control charts by using α - cut midrange transformation technique. Faraz, Kazemzadeh, Moghadam and Bazdar [17] introduced a fuzzy control chart by adjusting the control limits of Shewhart charts. Faraz and Shapiro [18] proposed an approach for fuzzy $\bar{x}$ and s chart by considering the assumptions of control charts. Shu and Wu [19] introduced fuzzy dominance approach for fuzzy $\bar{x}$ and R control charts.

Sensitizing rules are widely used with quality control charts in all types of processes. As a result of the uncertainty in process control, fuzzy sensitizing rules are suggested in literature. Gwee, Lim and Soong [20] studied fuzzy logic in recognizing the unnatural statistical patterns of control charts. Kahraman, Tolga and Ulukan [21] used triangular fuzzy numbers in the tests of control charts for unnatural patterns. Instead of using traditional zones of control charts, corresponding membership degrees of each zones are proposed. Wang and Rowlands [22] developed a fuzzy statistical process control evaluation and control method. The traditional $\bar{x}$ control chart is used and fuzzy logic is applied in zone rules. Hsu and Chen [23] suggested a new on line diagnostic system based fuzzy reasoning and genetic algorithms to monitor the process and justify the possible causes of unnatural patterns. An application of fuzzy control charting method for inviduals is described by Tannock [24]. Two fuzzy sets namely, centered fuzzy set and random fuzzy set are used to examine three typical unnatural patterns: Shift, trend and cyclical patterns. In an other study, Gülbay and Kahraman [25] discussed direct fuzzy approach by defining the unnatural patterns of a fuzzy control chart. Fazel Zarandi, Alaeddini and Turkşen [26] presented a hybrid method which combines adaptive sampling and run rules with fuzzy theory. Recently, a subjective fuzzy rule based fuzzy inference system is developed by Demirli and Vijayakumar [27].

IV. Fuzzy Sensitizing Rules

In a fuzzy control chart plotting the quality characteristic as fuzzy numbers on the control chart increases the flexibility of the result. In this study, the fuzzy sensitizing rules are developed in considering fuzzy control charts which is monitored with fuzzy quality characteristic and fuzzy control limits as fuzzy numbers. It is assumed that the fuzzy control limits are obtained from fuzzy data so that they have the same

World Academy of Science, Engineering and Technology
International Journal of Industrial and Manufacturing Engineering
Vol:7, No:5, 2013

characteristics. The type of the fuzzy numbers are not restricted and the proposed rules are developed in such a way that the rules can be applied with different fuzzy control chart approaches to a wide variety of processes.

The fuzzy sensitizing rules are given in the following list. If any one of the rules is satisfied, then the pattern of the fuzzy control chart is nonrandom and the process is out of control. It is suggested to use fuzzy sensitizing rules along with the fuzzy control charts.

Rule 1: If any fuzzy number is completely out of the fuzzy control limits, then the process is out of control.

Rule 2: If fuzzy number includes both of the fuzzy control limits, then the process is out of control. This rule is satisfied when the variability of the number is greater than the remaining data. The rule should be redefined with α - cut fuzzy number and control limits. Fig. 1 shows triangular fuzzy control limits, $U\widetilde{C}L$ and $L\widetilde{C}L$, and a triangular fuzzy number, $\widetilde{A}$, which satisfies the rule.
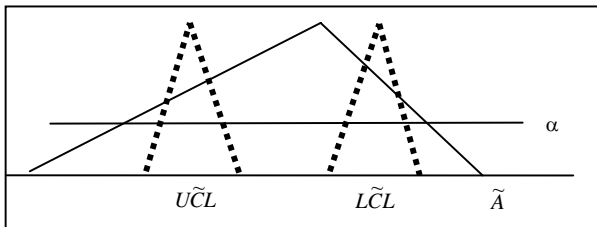

Fig. 1 Rule 2: Triangular control limits and a fuzzy number

In Fig. 1 the fuzzy control limits are given by dashed lines and the triangular fuzzy number, $\widetilde{A} = (a,b,c)$, has a large variability such that $a^{\alpha} < LCL_{a}^{\alpha}$ and $c^{\alpha} > UCL_{c}^{\alpha}$ are satisfied.

Rule 3: If symmetry of consecutive fuzzy numbers is altering steadily, then this is the result of a fact that affects the process. Fig. 2 shows six triangular fuzzy numbers, which satisfies the rule.
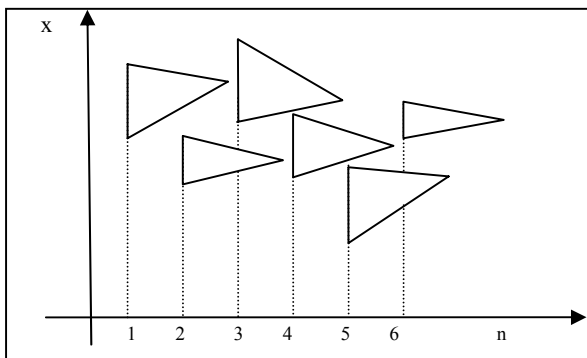

Fig. 2 Rule 3: Triangular fuzzy numbers with regularly changing symmetry

Rule 4: The consecutive fuzzy numbers with steadily increasing or decreasing variance show a nonrandom pattern. An example of the rule is given in Fig. 3. The

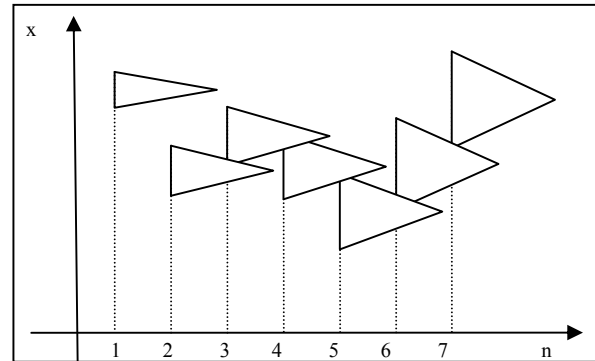variabilities of the triangular fuzzy numbers are increasing.

Fig. 3 Rule 4: Triangular fuzzy numbers with steadily increasing variability

Rule 5: If variation of a fuzzy number or subgroups is not in specified limits, then the process is out of control. It is assumed that the limits are determined by the experiences of decision maker.

In some cases, the rule involves Rule 2. If the range of α - cut fuzzy number is greater than an acceptable limit then also the variability of the number is also large enough to satisfy the Rule 5.

Rule 6: An unusual or nonrandom pattern of variances indicates an out of control situation of the process.

Rule 7: If α – cut of four of the five consecutive fuzzy numbers are beyond the α – cut fuzzy one – sigma control limits and at the same side of the control chart, then the process is said to be out of control.

Rule 8: If α – cut of eight consecutive fuzzy numbers are out of the α – cut fuzzy center line, then the process is out of control.

Rule 9: If six consecutive fuzzy numbers are steadily increasing or decreasing, then the process is out of control. Fig. 4 illustrates five triangular fuzzy numbers with a downward trend.
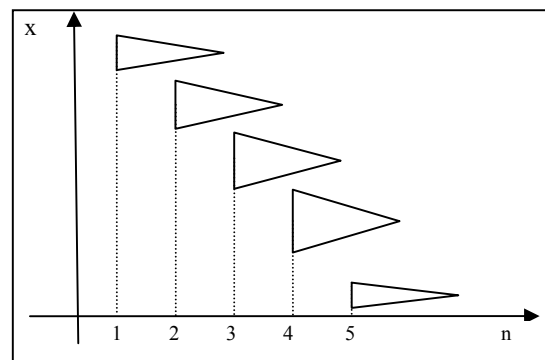

Fig. 4 Rule 9: Triangular fuzzy numbers decreasing regularly

The fuzzy numbers given in Fig. 4 are definitely decreasing as the sample number increases. A method for ranking fuzzy numbers should be used to test the given fuzzy rule. Ranking

World Academy of Science, Engineering and Technology
International Journal of Industrial and Manufacturing Engineering
Vol:7, No:5, 2013

the numbers necessiates to evaluate and compare a fuzzy number with all other fuzzy numbers. In order to compare fuzzy numbers several methods have been suggested in literature. An early review of these methods is presented by Bortolan and Degani [28]. Wang and Kerre [29], [30] reviewed and classified reasonable properties for the ordering of fuzzy quantities.

Rule 10: If fourteen consecutive fuzzy numbers are alternating up and down, then the process is out of control. Fig. 5 illustrates the seven triangular fuzzy numbers which are higher (or lower) than the previous fuzzy number with an upward trend.
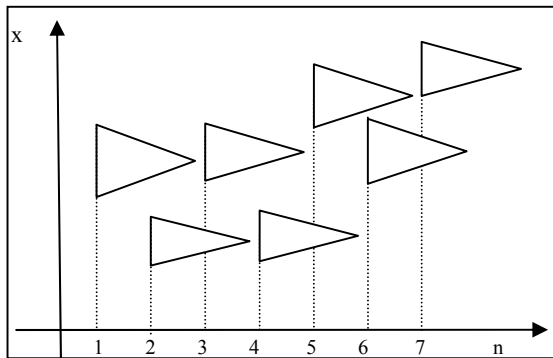

Fig. 5 Rule 10: Triangular fuzzy numbers regularly decreasing and increasing

Similar to rule 3; in order to test the rule the fuzzy numbers need to be ranked by a fuzzy ranking method.

Rule 11: If α - cut of consecutive fuzzy numbers are intersecting with α - cut of fuzzy control limits then the process is out of control. In other words, the fuzzy numbers are tend to fall on control limits but not outside the limits, far away from the center line or with few numbers near or on the fuzzy center line. This rule is an extension of mixture pattern of Shewhart control charts. Fig. 4 shows 10 α - cut fuzzy numbers on a fuzzy control chart.
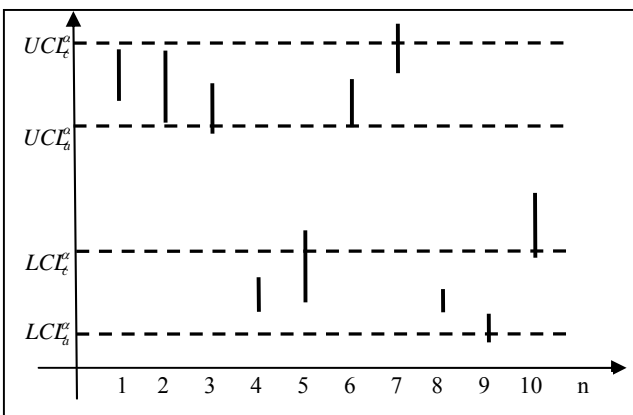

Fig. 6 Rule 11: α - cut triangular fuzzy numbers and α - cut fuzzy control limits

Rule 12: If α - cut of fifteen fuzzy numbers in a row is in α -cut of zone C, then the process is out of control.

The rule is arranged for triangular fuzzy numbers: Let $\widetilde{C}^+ = \left(C_a^+, C_b^+, C_c^+\right)$ and $\widetilde{C}^- = \left(C_a^-, C_b^-, C_c^-\right)$ are one sigma upper and lower control limits for the fuzzy chart then, zone C is the region between the limits. α - cut triangular fuzzy number is in α - cut of zone C, if $\widetilde{A}^\alpha = \left(a^\alpha, c^\alpha\right) \subset \widetilde{C}^\alpha = \left(C_a^-, C_c^+\right)$.

Rule 13: If α - cut of eight fuzzy numbers in a row is out of the α - cut of zone C, then the process is out of control.

A triangular fuzzy number is out of α - cut of zone C, if $a^\alpha > C_c^+$ or $c^\alpha < C_a^-$ is satisfied.

Rule 8 and 9 can be extended such that α - cut fuzzy numbers that intersect with the α - cut of zone C may be included by the rule to give an out of control signal. Then, a defuzzification is required to define limits of the rule certainly.

Rule 14: An unusual or nonrandom pattern in the fuzzy control chart indicates an out of control situation of the process.

V. CONCLUSIONS

In this study, out of control situations of fuzzy quality control charts are defined and a set of sensitizing rules is proposed for these situations. Some of the sensitizing rules are obtained by combining traditional rules and fuzzy theory, such as Rules 7, 8, and 9. On the other hand, Rules 2 - 6 are only applicable for fuzzy control charts. These rules are related to the patterns of symmetry and variance characteristics of fuzzy numbers.

The fuzzy sensitizing rules are not limited to any type of fuzzy control chart. Also, the type of the fuzzy numbers and fuzzy control limits are not specified. Fuzzy control charts, which are monitored with the fuzzy sensitizing rules, are more sensitive and flexible than fuzzy control charts and Shewhart control charts. Another important advantage of the rules is that the use of the fuzzy rules affects positively the performance of the fuzzy control charts. Most of the fuzzy control chart approaches entails lots of calculations and evaluations in defining the processes. If the process is tested with the rules, the result of the chart is derived more easily and loss of time caused by calculations of the approach will be reduced especially when the process is out of control. The sensitizing rules are flexible so that the fuzzy rules could be modified or similar rules could be developed for different types of control charts or fuzzy numbers. Similar to Shewhart control charts, a combination of the proposed fuzzy rules should be used in the fuzzy charts.

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Vol:7, No:5, 2013

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