World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:6, No:8, 2012

A robust approach to the load frequency control problem with speed regulation uncertainty

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Abstract—The load frequency control problem of power systems has attracted a lot of attention from engineers and researchers over the years. Increasing and quickly changing load demand, coupled with the inclusion of more generators with high variability (solar and wind power generators) on the network are making power systems more difficult to regulate. Frequency changes are unavoidable but regulatory authorities require that these changes remain within a certain bound. Engineers are required to perform the tricky task of adjusting the control system to maintain the frequency within tolerated bounds. It is well known that to minimize frequency variations, a large proportional feedback gain (speed regulation constant) is desirable. However, this improvement in performance using proportional feedback comes about at the expense of a reduced stability margin and also allows some steady-state error. A conventional PI controller is then included as a secondary control loop to drive the steady-state error to zero. In this paper, we propose a robust controller to replace the conventional PI controller which guarantees performance and stability of the power system over the range of variation of the speed regulation constant. Simulation results are shown to validate the superiority of the proposed approach on a simple single-area power system model.

Keywords—Robust control, Power system, Integral action, Minimax LQG control

I. INTRODUCTION

FREQUENCY regulation is one of the important control problems in electric power system design and operation. As demand for energy keeps soaring, alternative and cleaner sources of energy (such as wind and solar power) have been gradually making their way to represent a non-negligible percentage of the total power delivered to customers. Moreover, numerous schemes are put in place by governments and regulatory authorities to encourage customers to sell any excess generated power back to the utilities. While these measures are positive and aim to minimize the release of green-house gases and their environmental impact, the dynamics of power systems are now affected by a much wider set of parameters and variables than before. At the same time, given the variety of energy sources on power lines, the requirement of the regulatory bodies to maintain a certain quality standard have become more stringent which usually amounts to operating at optimal conditions under all situations. More than ever, the frequency regulation problem is now a complex problem and designing controllers for power systems presents bigger challenges.

The basic requirement of a power system is to meet the load demands and satisfy the power flow equations; see [1]. In addition to that, both the bus voltage and the operating frequency need to be within prescribed limits. The household or mains voltage used vary from region to region and can range from 100 to 240 V. In Europe for example, with voltage harmonization, the mains nominal voltage is now $230 \pm 6\%$ V while the North American region (United States and Canada) operate at $120 \pm 5\%$ V. Similarly, many different frequencies have been used ranging quite widely from 25 Hz to 140 Hz during the development of commercial electric power systems in the late 19th and early 20th centuries. However, as of the 21st century, frequencies of 50 and 60 Hz predominate. In particular, places that use 50 Hz tend to operate at 230 V and those that use 60 Hz operate at 120 V, giving rise to the “modern standard combinations” of 230 V/50 Hz and 120 V/60 Hz. Still, there are exceptions like Japan, where the western part operate at 100 V/60 Hz and the east at 100 V/50 Hz.

Frequency regulation is necessary to control the flow of alternating current power from multiple generators through the network. The change in system frequency provides a measure of mismatch between demand and generation, and thus is a necessary parameter for load control. Frequency changes are an unavoidable consequence of changing demand/generation. Rapidly changing mains frequency is often a sign that a distribution network is operating close to its capacity limits, dramatic examples of which can be observed just before major power outages. During an overload caused by the failure of generators or transmission lines, the power system frequency will decline, due to an imbalance of load versus generation. On the other hand, the sudden loss of an interconnection, while exporting power will cause the system frequency to rise. Automatic generation control (AGC) is used to regulate frequency and interchange power flows; see [2]. Control systems in power plants detect changes in the network-wide frequency and adjust mechanical power input to generators back to their target frequency. This is usually done by adjusting the position of a steam control valve which in turn modifies the turbine torque to match changes in the load torque.

Power systems are complex nonlinear systems which are usually linearized for the purpose of controller design. This naturally implies that any controller can be suitable only at or in a small region about the linearized point. In practice, power systems are subjected to a wide range of disturbances and unless the controllers are re-tuned, they can experience excursions from the equilibrium point which can be regarded as larger than “small signals”. In such cases, the system can show sub-optimal or poor performance with larger frequency deviations than tolerated by the regulatory authorities. Re-tuning the controller by manipulating the speed regulation

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constant usually results in improved performance. It is well known and will be shown in the sequel that a large feedback gain results in tighter control on the system's frequency with reduced frequency droop. However, this comes about at the expense of the stability margin. In this paper, we propose a robust controller design approach which guarantees stability and a certain level of performance for a range of feedback gains. We model the range of values of the speed regulation constant that the power system is expected to operate within as a bounded uncertainty and design a minimax LQG controller to replace the traditional PI controller. Moreover, we show a simple approach to include integral action in the designed controller. This requirement will be explained in detail in Section IV-A and is a direct consequence of the fact that the minimax LQG control methodology operates on the principle of minimization of a cost function which includes control action as a parameter. The main aim of this paper is to show the suitability and power of the proposed control approach to a common application problem. The versatility of the approach however makes it amenable to many more complex problems. The paper can also be considered as an appropriate reference/tutorial paper for the practicing control engineer in robust control which deliberately attempts to limit the mathematical details and rigor usually expected of audiences in the area.

The paper is organized as follows. The power system frequency regulation problem is presented in Section II and the need for two levels of control as is the current practice is explained. The sub-components of a power system are analyzed in Section III and the effect of changing the speed regulation constant on the stability of the system is investigated. In Sections IV and V, the uncertain power system is suitably modeled and the equations of the controller for the proposed minimax LQG approach are presented. To illustrate the effectiveness of the controller and to compare it with conventional control designs, simulation results are given in Section VII and we finally conclude with Section VIII.

II. Power System Frequency Control

Generators supply both *real* (P) and *reactive* (Q) powers. The real and reactive power components can be viewed as separate control inputs acting on the system and the obvious way of keeping a perfect power balance is to continuously keep the generated powers P_M and Q_M in balance with the loads P_L and Q_L. Thus, each generator is equipped with two separate feedback loops to regulate the real and the reactive powers respectively. The voltage in a power system is regulated by controlling the reactive power output and this is achieved by manipulating the field current supplied to the generator. On the other hand, frequency regulation is achieved by controlling the real power output. This is done by adjusting the position of a steam control valve which in turn modifies the turbine torque to match changes in the load torque. Both control loops are designed to operate around an equilibrium point with small excursions tolerated about that point. As long as the system operates about a chosen operating point, it may be modelled with linear differential equations and represented using linear transfer functions. In turn, linear control techniques can be used to design controllers for such a system.

Our aim in this paper is to consider the frequency regulation problem and we shall restrict our attention to this problem from this point onwards. Traditionally, frequency control is achieved using two different control actions known as the primary and secondary speed control. The primary speed control takes care of the main bulk of the frequency regulation and shares load variations among the generators in the control area in proportion to their capacities. The response time of this loop is usually quite fast. On its own however, the primary control loop will only be able to stabilize the frequency of the system at some value, which can be considerably different (up to 5%) from its nominal value. In other words, different loading conditions will introduce varying steady-state errors in the frequency. For example, Fig. 1 shows the response of a typical power system when a load change of 10% is introduced at a time of 1 second, with only primary loop feedback being used. The frequency stabilizes at a value other than the nominal one.

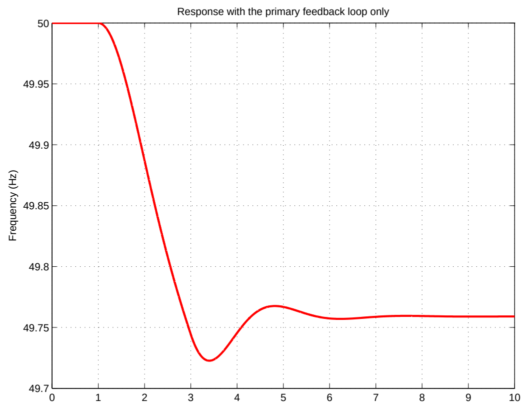

Fig. 1. Dynamic response with primary feedback loop only.

The role of the secondary control loop is to ensure the fine adjustment of the frequency by resetting the frequency error to zero. This is invariably achieved through integral action. The secondary loop has a relatively larger time constant and can be considered to come into play after the primary loop has acted.

III. Power System Model

The frequency regulation loop can be analysed by modelling its building blocks. In general, it comprises of a hydraulic amplifier, a turbine and a generating unit. A reference power setting is fed to the hydraulic amplifier which controls the opening position of a steam valve. The reference power is chosen such that the frequency of generated voltage is 50 Hz. The amplifier can be modelled as a first-order transfer function with a time constant T_H as follows:

$$G_H = \frac{1}{1 + sT_H}. \tag{1}$$

The flow rate of steam regulates the angular speed of rotation of the turbine. The dynamics of turbines vary widely depending on the type used. Here, we will consider a "Non-Reheat Steam Turbine" where steam enters through a steam-chest, before going through the turbine and back to the condenser. The steam-chest introduces a delay T_T in the system. The turbine can be modeled as a first order system with transfer function,

$$G_T = \frac{1}{1+sT_T}. \quad (2)$$

Finally, the generator-load dynamic relationship between the incremental power mismatch $(\Delta P_M - \Delta P_L)$ and the frequency deviation Δf can be expressed as:

$$\Delta P_M - \Delta P_L = 2H\frac{d\Delta f}{dt} + D\Delta f, \quad (3)$$

where ΔP_M is the mechanical power change, ΔP_L is the load change, Δf is the frequency deviation, H is the inertia constant and D is the load damping coefficient. Eqn. (3) can be represented by the following transfer function,

$$G_P = \frac{1}{2Hs + D}. \quad (4)$$

A. Stability Analysis

Conventional control strategies for the load frequency control problem utilize the frequency error and the integral of the frequency error as the control signal. Fig. 2 shows the interconnection of the differential blocks that make up a single-area power system as well as the conventional feedback scheme used. ΔP_L which represents the local load change can be considered to be the input to the system.

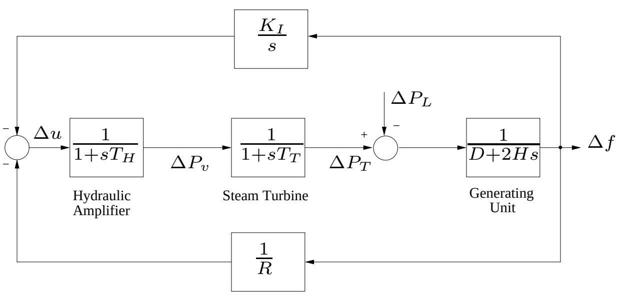

Fig. 2. Conventional integral feedback scheme.

In designing for the parameters K_I and R, we make use of classical control methodologies such as root locus and Bode plots. As mentioned before, the integral action is responsible for correcting any offset from the nominal frequency and is usually a process with a relatively long time-constant. On the other hand, the speed regulation constant is chosen to ensure quick sharing of load among connected generators during a change in the loading condition. The value of R is a measure of the static drop in frequency as caused by increased power output and is typically set at 5%. This means that a change in power from zero to full load will result in a frequency drop of 5% of the nominal frequency, or 2.5 Hz. Hence, R is usually chosen to have a nominal value of 2.5 Hz/per-unit. If tighter frequency control is required, the value of speed regulation R can be adjusted to say 1% or 0.5 Hz by the operator. An increase in the frequency feedback gain $(1/R)$ however reduces the stability margin of the system. We can analyze the stability of the system in Fig. 2 by treating the regulation constant R as a variable parameter. The plant $G(s)$ will have as input ΔP_L and output Δf and is formed by including the feedback loop K_I/s. The resulting transfer function for $G(s)$ can be easily worked out as:

$$G(s) = \frac{s(1+sT_H)(1+sT_T)}{s(1+sT_H)(1+sT_T)(2Hs+D)+K_I}. \quad (5)$$

And the corresponding characteristic equation is given by

$$1 + \frac{1}{R}\frac{s(1+sT_H)(1+sT_T)}{s(1+sT_H)(1+sT_T)(2Hs+D)+K_I} = 0. \quad (6)$$

To analyze the stability of the system, we select suitable values for the parameters of a typical power system. These parameters are given in Table I.

TABLE I
PARAMETERS OF SINGLE-AREA POWER SYSTEM

Governor time constant T_H	0.080
Turbine time constant T_T	0.400
Damping coefficient D	0.015
Inertia constant H	0.083

The root-loci of the system is plotted for various values of R as shown in Fig. 3. We assume a value of $K_I = 0.01$. It is clear from Fig. 3 that reducing R from 2.5 to 0.3 (equivalent to reducing frequency droop) moves the dominant poles of the system closer to the right half plane, making the system very lightly damped to the point of being unstable $(R = 0.3)$.

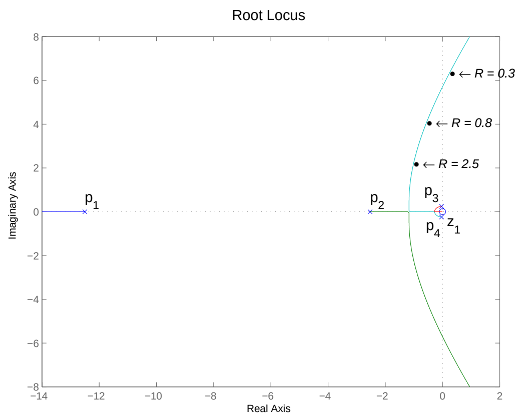

Fig. 3. Effect of changing droop constant on location of roots

IV. PROBLEM STATEMENT

While it is possible and usually straightforward to use classical control design techniques to synthesize a controller that ensures good performance, it is not always clear how the controlled system will behave in the face of uncertainty. The *robustness* of a control system is a measure of its ability to maintain a certain level of performance in the face of variations in plant dynamics. In our current problem, the uncertainty arises as a result of changes in the speed regulation constant. As we have seen before in Sec. III-A, manipulating R can result in instability. In particular, when the operator is manipulating the speed regulation constant during times of sudden and large changes in load demand in order to maintain the frequency within regulated bounds, the degradation in stability and performance needs to be quantified. With classical control scheme, this would require designing the proportional-integral (PI) controller for the smallest value of R which will then result in a conservative performance for most of the operating time of the power system. On the other hand, if the parameter R is treated as an uncertainty which is suitably modeled and it is made to form an integral part of the design process, a much better performance for the overall system can be expected. The plant is then represented by a nominal model together with the type of uncertainty present. Such a plant model is referred to as an uncertain system and when it is possible to separate the nominal system from the uncertainty in a feedback interconnection, it can be represented as shown in Fig. 4.

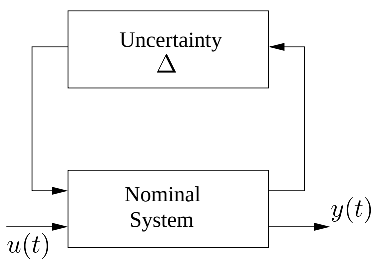

Fig. 4. Uncertain system block diagram

The uncertainty operator Δ in such an uncertain model description is typically a quantity which is unknown but bounded in magnitude. The form of the uncertainty can vary but is usually represented as a time-varying uncertain norm bounded matrix $\Delta(t)$ or in the form of a stable uncertain norm bounded transfer function matrix $\Delta(s)$. We will discuss, in detail, the requirements and constraints on the uncertainty block in Sec. V. In this work, we design a robust controller which takes into account the full range of variation of the speed regulation constant. The system is appropriately modeled to take into account the uncertainty present in Sec. IV-A and we make use of the minimax LQG control theory to design an output feedback controller which will minimize a given worst-case cost functional.

A. Modelling for controller design

The problem we are considering involves finding a controller which will be *robust* in the face of variation in the parameter R; see Fig. 2. The idea is to replace the slower secondary control loop as represented usually by simple integral action with a more appropriate controller. With this in mind, we re-frame the uncertain parameter varying plant as shown in Fig. 5. As can be seen in Fig. 5, we have included an integrator at the output of the plant. The reason for this will be explained next.

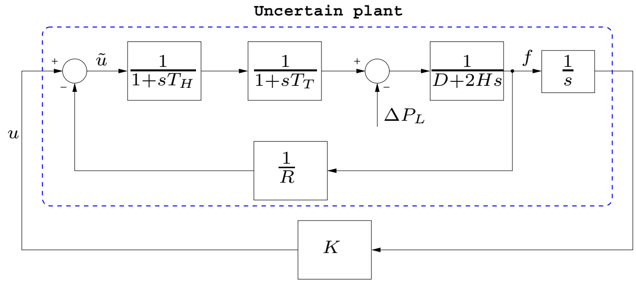

Fig. 5. Modeling for controller design

The standard LQG control design approach works by minimizing a given cost functional which is an integral of a function of the states of the system as well as the control signals over an infinite time horizon. The noises or disturbances present are assumed to be stochastic in nature or Gaussian white noise. In power systems, load frequency regulation requires the rejection of loading conditions which can be modeled as d.c. or slowly varying signals. In such cases, a suitable controller will need to provide a constant or slowly varying control signal u to offset the disturbances and drive the frequency back to its nominal value. However, such a control signal will make the cost functional as described above infinite. Thus, the standard LQG technique turns out to be inappropriate for these kinds of problems. One approach to circumvent this issue and to ensure zero steady-state error is to introduce an integrator at the output of the plant and augment the plant states accordingly. A controller is then designed using the augmented plant and after the design stage, the integrator is included as part of the designed controller. This controller will then be able to provide the integral action necessary to correct for offsets from the nominal frequency and at the same time ensure a stable closed-loop system. The same reasoning applies to the minimax LQG control design approach which is based on the LQG approach. The only difference being that we will then minimize the cost functional for the worst-case uncertainty.

We represent the speed regulation constant $R = \bar{R} + \delta R$, where $\bar{R}$ represents the nominal value of R and ΔR is the parameter uncertainty. Since R appears in the denominator in the state-space model to be derived, we instead define a new variable $\mu = 1/R = \bar{\mu} + \delta\mu$ to model the uncertainty. After some manipulation, the state equations of the system shown in Fig. 5 can be written as:

$$\begin{aligned}
\dot{x}_1 &= -\frac{1}{T_H}x_1 - \frac{\bar{\mu}+\delta\mu}{T_H}x_3 + \frac{1}{T_H}u;\\
\dot{x}_2 &= -\frac{1}{T_T}x_2 + \frac{1}{T_T}x_1;\\
\dot{x}_3 &= -\frac{D}{2H}x_3 + \frac{1}{2H}x_2;\\
\dot{x}_4 &= x_3;
\end{aligned} \tag{7}$$

and the output is given by $y = x_4$. Next, we show how we account for the uncertainty in the system and design a controller which is guaranteed to be stable and provide satisfactory performance for a range of values of the speed regulation constant R.

V. Minimax LQG control

Our aim in this section is to present the minimax LQG control approach and to show its usefulness in solving the power system problem considered in this paper. We will set the problem appropriately such that the proposed technique can be effectively applied. However, we will not provide proof of the theorems that we use and instead refer the interested reader to more complete and mathematically rigorous description of minimax LQG control technique which can be found in [3]–[6].

We apply the minimax LQG method to a stochastic uncertain system of the form shown in Fig. 6 where the nominal system is described by the following stochastic state equations:

$$\begin{aligned} \dot{x} &= Ax + B_1 u + B_2 \xi + B_2 w; \\ z &= C_1 x + D_1 u; \\ y &= C_2 x + D_2 \xi + D_2 w. \end{aligned} \tag{8}$$

Here, $x(t) \in \mathbb{R}^n$ represents the state, $u(t)$ is the control input, $w(t)$ is a unity covariance Gaussian white noise process, $z(t)$ is the uncertainty output, $\xi(t)$ is the uncertainty input and $y(t)$ is the measured output. It is assumed that the uncertainty input $\xi(t)$ is generated from the uncertainty output $z(t)$ as shown in Fig. 6. Note that although in general B_1 and B_2 are different, in this particular configuration $B_1 = B_2$.

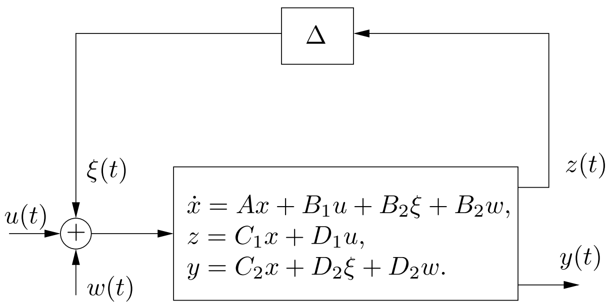

Fig. 6. Stochastic uncertain system representation

The constraint on the uncertainty follows from a natural stochastic generalization of the integral quadratic constraint (IQC) uncertainty description, as used in deterministic robust control theory; see [7]. In this case, we bound the noise acting on the system as well as the uncertainty in system dynamics as follows:

$$\lim_{T\to\infty} \frac{1}{T}\mathbf{E}\left[\int_0^T \|\xi(t)\|^2 dt - \int_0^T \|z(t)\|^2 dt\right] \le d. \tag{9}$$

The constant $d > 0$ determines the size of the uncertainty in the probability distribution of the total disturbance signal acting on the system. The minimax LQG control problem generates a controller which minimizes the maximum of a given cost functional where the maximum is taken over all admissible uncertainties satisfying the uncertainty constraint (9). Since the minimax LQG problem can be considered as a robust version of the standard LQG problem, it makes sense to consider a cost functional which is of similar form as that of the LQG problem. We assume therefore that the performance index is given by:

$$J = \lim_{T\to\infty} \frac{1}{2T}\int_0^T \left[x(t)^T Q x(t) + u(t)^T G u(t)\right] dt, \tag{10}$$

where $Q \ge 0$ and $G > 0$. Our aim in the frequency regulation control problem is to reduce frequency deviation from a nominal value. However, the rapid changes in the power line frequency is already taken care of by the speed regulation constant. Here, we are concerned with minimizing the offset and setting it to zero. Hence, we choose the matrix Q in the cost functional (10) as $Q = C_2^T C_2$. That is, the term $x(t)^T Q x(t)$ in the cost functional (10) corresponds to the norm squared value of integral of the frequency deviation.

The minimax LQG control theory developed in [5], [8], [9] and [6] leads to a computationally tractable method of synthesizing the minimax LQG controller. This involves solving the following two algebraic Riccati equations dependent on a single scaling parameter τ as follows:

$$\begin{aligned} &[A - B_2 D_2^T (D_2 D_2^T)^{-1} C_2] Y_\infty \\ &\quad + Y_\infty [A - B_2 D_2^T (D_2 D_2^T)^{-1} C_2]^T \\ &\quad - Y_\infty [C_2^T (D_2 D_2^T)^{-1} C_2 - \frac{1}{\tau} R_\tau] Y_\infty \\ &\quad + B_2 [I - D_2^T (D_2 D_2^T)^{-1} D_2] B_2^T = 0; \end{aligned} \tag{11}$$

and

$$\begin{aligned} &X_\infty (A - B_1 G_\tau^{-1} \Upsilon_\tau^T) + (A - B_1 G_\tau^{-1} \Upsilon_\tau^T)^T X_\infty \\ &\quad + (R_\tau - \Upsilon_\tau G_\tau^{-1} \Upsilon_\tau^T) \\ &\quad - X_\infty [B_1 G_\tau^{-1} B_1^T - \frac{1}{\tau} B_2 B_2^T] X_\infty = 0; \end{aligned} \tag{12}$$

where the solutions are required to satisfy the conditions $X_\infty > 0, Y_\infty > 0$ and $(I - \frac{1}{\tau} X_\infty Y_\infty) > 0$. Here, $R_\tau \doteq R + \tau C_1^T C_1, G_\tau \doteq G + \tau D_1^T D_1$, and $\Upsilon_\tau \doteq \tau C_1^T D_1$. Then, the minimax LQG controller is defined by the equations:

$$\begin{aligned} \dot{x}_c &= \left[A - B_1 G_\tau^{-1} \Upsilon_\tau^T - (B_1 G_\tau^{-1} B_1^T - \frac{1}{\tau} B_2 B_2^T) X_\infty\right] x_c \\ &+ \left[I - \frac{1}{\tau} Y_\infty X_\infty\right]^{-1} (Y_\infty C_2^T + B_2 D_2^T)(D_2 D_2^T)^{-1} \\ &\times \left[y - (C_2 + \frac{1}{\tau} D_2 B_2^T X_\infty) x_c\right]; \\ u_\tau &= -G_\tau^{-1} (B_1^T X_\infty + \Upsilon_\tau^T) x_c. \end{aligned} \tag{13}$$

As stated before, the two Riccati equations (11) and (12) together with the corresponding controller (13) are dependent upon the parameter τ. For each value of the parameter $\tau > 0$, we determine the corresponding upper bound on the cost functional as:

$$W_\tau = \frac{1}{2}\mathrm{tr}\left[\begin{array}{c} Y_\infty R_\tau + (Y_\infty C_2^T + B_2 D_2^T)(D_2 D_2^T)^{-1} \\ \times (C_2 Y_\infty + D_2 B_2^T) X_\infty (I - \frac{1}{\tau} Y_\infty X_\infty)^{-1} \end{array}\right] + \tau d. \tag{14}$$

VI. Controller Design

In our problem, the control signal u, the noise signal w and the uncertainty signal ξ all feed through the same channel as shown in Fig. 6. This implies that $B_1 = B_2$. It is clear from

(11) and (13) that solutions to the Riccati equation and to the minimax controller respectively require that $D_2 D_2^T > 0$. This is achieved by adding a small amount of measurement noise to the system in addition to process noise. We chose $D_2 = 0.0001$. Also, the parameter d determines the level of uncertainty in the probability distribution of the disturbance signal. In our case, the main source of uncertainty arises in the system dynamics rather than in the probability distribution of the disturbance signal. The parameter was selected as $d = 0.001$ to reflect its negligible influence. Finally, the term $u^T G u$ is included in the cost functional (10) to ensure that the gain of the resulting controller is not too large. After some tuning, a value of $G = 0.009$ was selected.

To solve the minimax LQG control problem, the parameter τ is chosen such that the smallest upper bound on (14) is obtained. For the uncertain system model as described by (7), the cost functional as defined by (10) and the set of design parameters chosen as described above, a plot of W_τ versus τ is generated as shown in Fig. 7. The optimal value of τ for our problem turns out to be $\tau = 0.0651$.

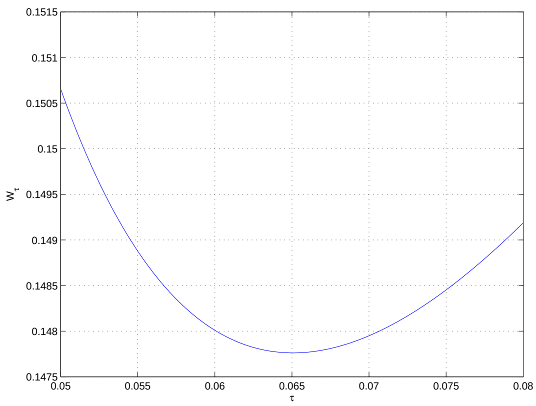

Fig. 7. W_τ versus τ

Using this value of τ, the controller is constructed using (13) by first solving the Riccati equations (11) and (12) for Y_∞ and X_∞ respectively. The controller obtained is a stable one, with the eigenvalues lying at $-15.95 \pm 37.60j$ and $-44.05 \pm 25.97j$. The final controller is obtained by augmenting the controller with the integrator introduced at the output of the plant in the design stage. The Bode magnitude and phase plots of the final controller is shown in Fig. 8.

The stability and performance of the closed-loop system is next analysed with the minimax LQG controller in place. In Fig. 9, we show a series of Bode plots of the loop gain $L(s) = P(s)K(s)$ for the range of values of the speed regulation constant R that the controller was designed for. In the worst case (smallest value of R), the controlled system has a gain margin of 7.45 dB at 23.7 rad/s and a phase margin of 30.7° at 11.3 rad/s.

VII. Simulation Results

To show the performance of the designed controller, it is usually much more informative to show the time-domain simulations. Here, we simulate an increase of 10% in the loading conditions of the power system which happens over a period of 2 seconds, as shown in Fig. 10. The resulting frequency deviation as well as the control signal generated by the controller are observed.

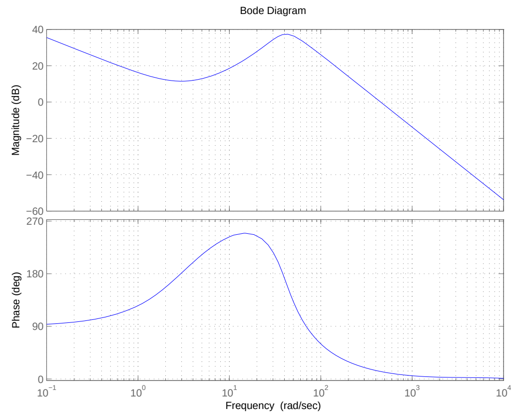

Fig. 8. Magnitude and phase Bode plot of the controller

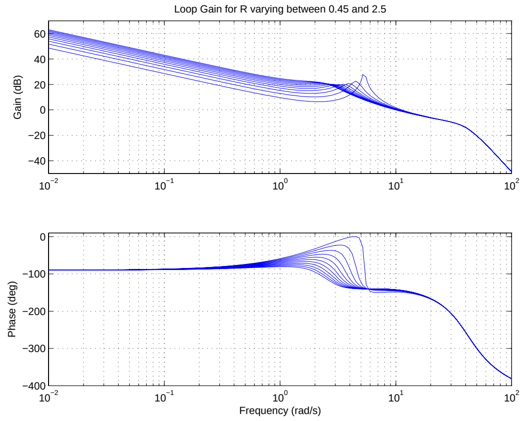

Fig. 9. Loop gain for multiple values of R in the range 0.45 to 2.5

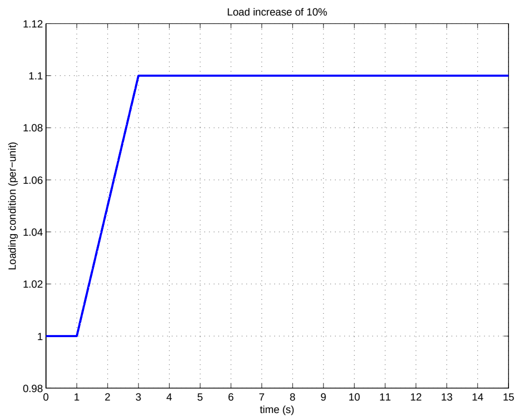

Fig. 10. Change in loading condition

Moreover, in an effort to provide a certain measure of comparison, we also design a proportional-integral (PI) controller for the system. The PI controller is optimally designed for one value of the speed regulation constant R, given the parameters of the plant as shown in Table I. We choose $R = 2.5$, that is the nominal value. It turns out that the optimized controller has an integral component only and the corresponding transfer function is given as

$$K(s) = \frac{0.25}{s}. \tag{15}$$

Fig. 11 shows the response of the plant controlled by the minimax LQG controller and the PI controller respectively when subjected to the disturbance shown in Fig. 10. Here, we used the same value of R for which the PI controller was designed, i.e., $R = 2.5$. From Fig. 11, it is clear that the frequency deviation when the system is controlled by the minimax LQG controller is much less than when the PI controller is in use. Moreover, the minimax LQG controller also forces the system's frequency to quickly resettle back to its nominal value of 50 Hz.

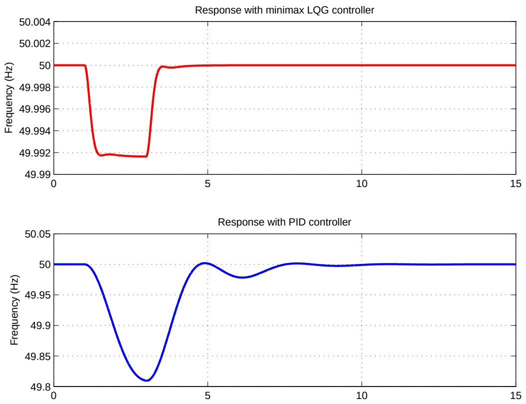

Fig. 11. Frequency deviation of controlled system with $R = 2.5$

The exercise is then repeated with the system being simulated with a smaller value of $R = 0.45$. In this case, we expect the power plant to be driven harder since the feedback component through $1/R$ will be much larger. Fig. 12 shows that the frequency deviation of the plant controlled by the minimax LQG controller is again well within tolerated bounds and that although the system takes a bit longer to settle back to its original frequency, the performance is still very good. This result is as expected since the minimax controller was designed for speed regulation constant lying within the range $R = 0.45 \rightarrow 2.5$. On the other hand, the PI controlled plant starts to show oscillatory behaviour and is on the verge of instability. It is also interesting to note that the frequency deviation is less than when $R = 2.5$. It can also be shown that a further slight drop in R results in instability with the PI controller while the minimax LQG controller not only maintain stability but still provides good performance. This is a remarkable result since the minimax LQG controller is then operating beyond the uncertainty bound for which it was designed. This result also confirms the gain and phase margins we found in Fig. 9 for the worst-case scenario.

VIII. Conclusion

The frequency regulation problem of a single-area power system has been considered in this paper. Using the root-locus technique, we analysed a power system model and showed how the poles of the system move towards the right-hand plane when the speed regulation constant R is decreased. It has also been shown through time-domain simulations that a simple PI controller as is conventionally used in practice to regulate frequency does not only result in poor performance but also runs the risk of making the system oscillatory. The minimax LQG control approach is proposed as a robust alternative to cater for the uncertain parameter(s) in the system. In this case, the speed regulation constant is the parameter that is manipulated by power systems operators to maintain frequency droop at a minimum and it is modeled as the main source of uncertainty in the system. We have shown that the minimax LQG controller is robust for the whole range of variation of the speed regulation constant and also provides better performance with smaller frequency deviations than the PI controller. Another interesting feature of our approach is that integral action was easily included into our design to ensure zero steady-state error. It needs to be pointed out that while the approach presented here was used on a single-area power system, the same idea can be easily extended to multi-area systems. In fact, the minimax LQG approach is particularly suited for multivariable systems.

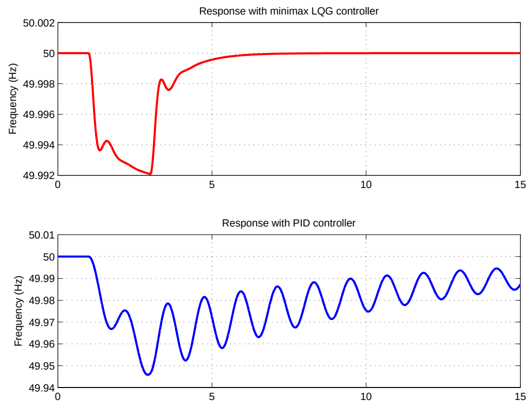

Fig. 12. Frequency deviation of controlled system with $R = 0.45$

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