Modal Propagation Properties of Elliptical Core Optical Fibers Considering Stress-Optic Effects

M. Shah Alam and Sarkar Rahat M. Anwar

Abstract**—**The effect of thermally induced stress on the modal properties of highly elliptical core optical fibers is studied in this work using a finite element method. The stress analysis is carried out and anisotropic refractive index change is calculated using both the conventional plane strain approximation and the generalized plane strain approach. After considering the stress optical effect, the modal analysis of the fiber is performed to obtain the solutions of fundamental and higher order modes. The modal effective index, modal birefringence, group effective index, group birefringence, and dispersion of different modes of the fiber are presented. For propagation properties, it can be seen that the results depend much on the approach of stress analysis.

Keywords**—**Birefringence, dispersion, elliptical core fiber, optical mode analysis, stress-optic effect, stress analysis.

I. Introduction

POLARIZATION maintaining fibers (PMF) have been widely used for polarization control in fiber optic sensors, precision optical instruments, and optical communication systems [1]-[5]. Different structured fibers like bow-tie fibers, PANDA fibers, side-pit fibers, side-tunnel fibers have been analyzed to show different propagation properties previously [3]-[9]. These types of fibers are designed to introduce a differential stress in the core so that the state of polarization of the guided light wave can be maintained by means of stress-optic effect [3]-[9]. Another fiber is designed to have an asymmetric core shape, *i.e*., elliptical core to introduce birefringence in order to maintain the polarization [1]-[11].

In the elliptical core fibers, the birefringence primarily originated from the core ellipticity. But, like the other fibers, there is temperature induced birefringence also. Both the effects should be considered very carefully to find the accurate modal characteristics. In many cases, they have been considered, but only for fundamental modes [1]-[5], [7], [10]-[14]. In [10]-[11], modal analysis is carried out for fundamental and higher order modes, but stress effects have not been considered. As for analysis methods, many different approaches are used for analyzing PMFs [6]-[16]. Among them the finite element method (FEM) is suitable for producing accurate modal solutions of different types of fibers [10]-[11], [13], [17], though there are different approaches with this method. Very recently, the authors have reported modal solutions of fundamental and higher order modes of highly elliptical core fibers considering plane strain approach to incorporate stress-optic effects in the analysis [18], where only the dependence of birefringence properties with core ellipticity is discussed and field distributions of different modes are shown.

In this work, the structure with highly elliptical core optical fiber with a circular cladding is analyzed by using the FEM [17] to show the effects of the approach of stress analysis on modal characteristics. The plane strain approximation and also the generalized plane strain approach have been considered here to include the stress-optic effect and the results of propagation characteristics are compared to show the influences of stress analysis approach on optical analysis. After considering the stress-optic effect, an optical mode solver with the electric field as the working variable is employed to obtain the modal solutions. Thus, the effects of thermally induced stress on the fundamental and higher order modes of highly elliptical core optical fibers are studied. It has been seen that higher ellipticity of the core results much higher thermal stress in the fiber core, which in turn produces higher birefringence to produce non-degenerate modes in the fiber. Also, for propagation properties, the results depend much on the approach of stress analysis.

II. Theory

A high value of birefringence is usually desired for single mode operation in an optical fiber. During the manufacturing process, due to different thermal expansion coefficients of core and cladding material, the fiber becomes stressed. This stress causes the indices to become anisotropic and increases the birefringence of orthogonal polarized wave in the fiber. Along with this thermally induced birefringence, there can be geometrical birefringence, which is usually achieved by making the core a non-circular one [19]. For predicting light propagation performance of a fiber, both the geometrical and thermal induced birefringence effects have to be considered in the analysis [20].

To understand the overall birefringence of a fiber with elliptical core, it is necessary to carry out the stress analysis first to find the stress distribution and hence the change in refractive index due to stress-optic effect. Next, the optical analysis is to be performed with the new refractive index to obtain the modal solutions, birefringence, and propagation

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properties. In this work, the stress analysis has been carried out using two approaches and the difference in numerical results of the propagation properties have been studied.

A. Stress Analysis

The effect of thermally induced stress on the refractive index in different domains in the cross section of the fiber can be determined using generalized plane strain approach or the plane strain approximation [17]. The generalized plane strain approach differs from the plane strain approximation in that the longitudinal strain is no longer zero in the generalized plane strain approach. Also, the strain is linearly varying over the cross section of the fiber in generalized approach. The system formulation is carried out for unknown displacements due to strain over the cross section of the fiber. After solving the system for unknown displacements, one can easily obtain the stresses σ_x, σ_y, and σ_z and can find the anisotropic change in refractive indices due to stress-optic effect. The new refractive indices are given by

$$\begin{aligned} n_x &= n_0 - C_2\sigma_x - C_1(\sigma_y + \sigma_z) \\ n_y &= n_0 - C_2\sigma_y - C_1(\sigma_z + \sigma_x) \cdot \\ n_z &= n_0 - C_2\sigma_z - C_1(\sigma_x + \sigma_y) \end{aligned} \quad (1)$$

Here C_1 and C_2 are the stress-optic coefficients and n_0 is the isotropic refractive index of the unstressed fiber material. The n_x, n_y, and n_z are the principal diagonal components of anisotropic refractive index tensor when the stress-optic effect is incorporated. The refractive index of unstressed fiber material at different wavelengths may be obtained using Sellmeier equation [6].

B. Optical Analysis

In the optical analysis, the resulting anisotropic refractive indices as calculated by (1) are used. The modal analysis is carried out assuming that the wave propagates along the z-direction and the electric field of the wave has the form [17]

$$\mathbf{E}(x, y, z, t) = \mathbf{E}(x, y)\exp[j(\omega t - \beta z)], \quad (2)$$

where ω is the angular frequency and β is the propagation constant. An eigenvalue equation in terms of the electric field can be obtained from the Helmholtz equation

$$\nabla \times (n^{-2}\nabla \times \mathbf{E}) - k_0^2\mathbf{E} = \mathbf{0}, \quad (3)$$

and is solved for modal effective index, $n_{\text{eff}} = \beta/k_0$, as the eigenvalue. The boundary condition for electric field at the outside of the cladding boundary was set to zero. In [17], however, a module based on the perpendicular hybrid mode wave using transversal fields is used for finding the modal solutions.

III. Results and Discussion

The cross section of the optical fiber with elliptical core is shown in Fig. 1, where the core dimensions in x-direction (major axis) and y-direction (minor axis) are $2a$ and $2b$, respectively. The cladding is circular with radius, r=25μm. The refractive indices of core and cladding are n_1 and n_2, respectively. The cross section of the fiber is meshed with quadratic triangular elements in both stress analysis and optical analysis. For stress analysis, the boundary conditions are taken in such a way that along x-axis through the center of the core, no displacement is allowed in y-direction, and along y-axis through the center of the core, no displacement is allowed in x-direction, and in other regions, no restrictions are applied.

The values taken for stress-optic coefficients are C_1=4.184×10^{-12}m^2/N and C_2=7.5714×10^{-13}m^2/N. The thermal expansion coefficients of core and cladding are, α_{core}=2×10^{-6}K^{-1} and α_{clad}=1×10^{-6}K^{-1}. The Young's modulus of the fiber core and cladding material, E=72.324×10^{9}Pa and the Poisson's ratio, υ=0.186. The initial temperature of the fiber is 1020°C and the final temperature when the fiber is cooled down is 20°C. So, the temperature difference is 1000°C, which causes the thermal strain over the fiber cross section. As the core is not circular and have different thermal expansion coefficient than the cladding, the stress distribution will not be equal in all the directions.

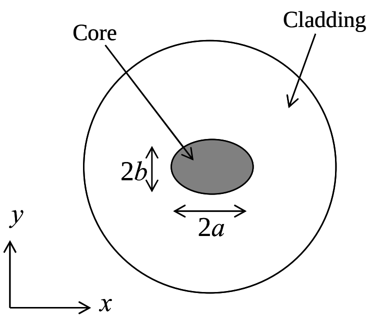


Fig. 1: Cross section of the optical fiber with elliptical core.

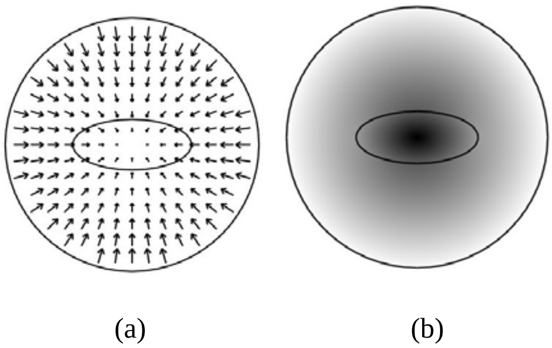
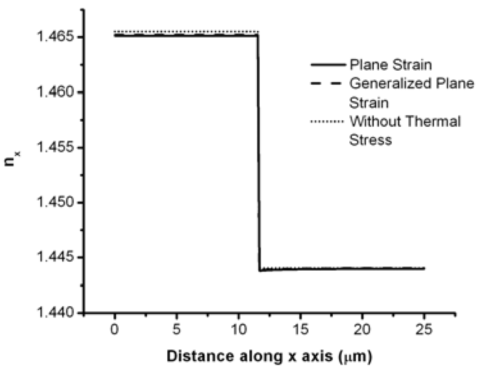


Fig. 2: (a) Vector displacement, and (b) Stress distribution, over the cross section of the fiber.

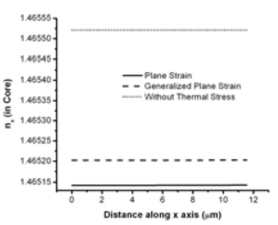
Fig. 2 shows the vector displacement due to deformation caused by strain in the fiber and stress distribution over the cross section of the fiber that occur because of different thermal expansion coefficients. The length of the arrow in Fig. 2(a) is proportional to the amount of displacement and the direction of the arrow is the direction of the displacement. It can be seen easily that the displacement is towards the center of the fiber and lateral contraction occurs. From Fig. 2(b), it can be seen that the stress is more at the core than at the cladding region. This is expected as the displacement is towards the center, more stress develops at the center of the fiber. Here a=11.67μm, b=5μm, and ellipticity, ξ=0.4 are taken throughout the calculation and the ellipticity is defined by

$$\xi = \frac{(a-b)}{(a+b)}.$$

Due to the stress-optic effect, the refractive index of both core and cladding changes. This is shown for n_x in Fig. 3. With the development of stress, it can be seen that in the core and cladding regions, the refractive index is decreased and at the interface of core and cladding, it decreases abruptly. However, the changed value depends on the associated stress analysis technique. With the plane strain approach, the new value of the index is much lowered than that given by generalized plane strain approach. It can be seen that in the core region, the refractive index, n_x, is decreased from 1.46552 to 1.4652 for generalized plane strain and to 1.46514 for plane strain approximation, while in the cladding region, for generalized plane strain, the refractive index shows a minimum value of 1.44384 at the interface of core and cladding, it increases towards the edge of the fiber to 1.44404, which is greater than the original value of 1.44402. But when the plane strain approximation is used, n_x increases from 1.44378, at the interface to 1.44398, at the edge of the fiber to 1.44402, which



(a)



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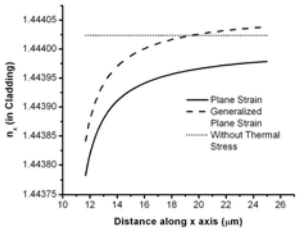
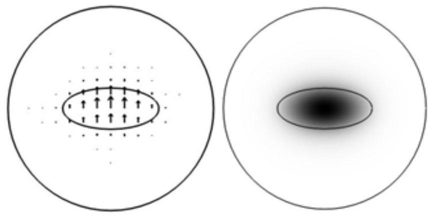
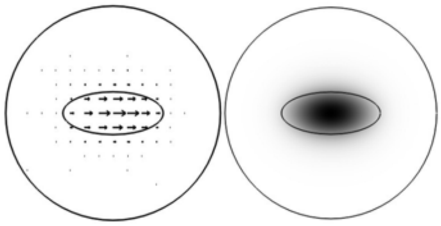


(c)

Fig. 3: Variation of refractive index along x-axis, (a) in the fiber from center of the core to the cladding boundary, (b) in the core region, (c) in the cladding region.



(a)



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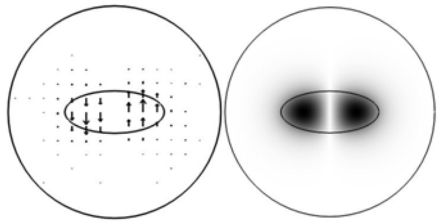

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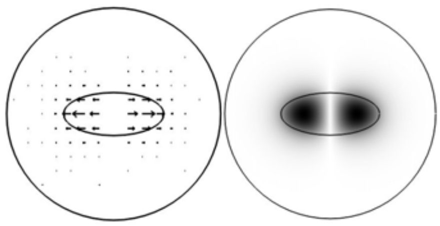

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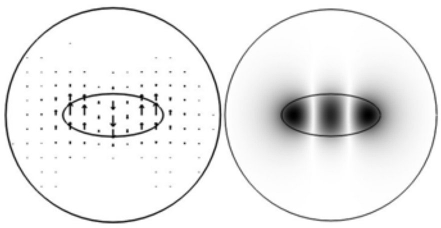

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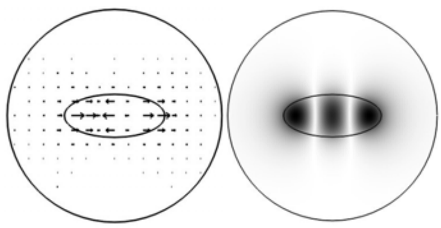

(f)

Fig. 4: Electric field intensity distribution of different modes of the fiber, (a) HE^y_{11}, (b) HE^x_{11}, (c) TE_{01}, (d) TM_{01}, (e) HE^y_{12}, (f) HE^x_{12}.

is less than the original value. May be this is the reason why the optical characteristics of the fiber varies with the associated stress analysis technique.

The anisotropic change in refractive index due to stress-optic effect will cause a higher value of birefringence in the fiber and the birefringence will cause the fundamental and higher order modes to split, which are usually two-fold degenerate modes in unstressed fiber. The electric field intensity distribution of different modes over the cross section of the fiber are shown in Fig. 4. In Fig. 4, fundamental and higher order modal solutions are shown, where part (a) and (b) show *y*-polarized and *x*-polarized fundamental HE_{11} modes, respectively, part (c) and (d) show *y*-polarized TE_{01} mode and *x*-polarized TM_{01} modes, respectively, part (e) and (f) show *y*-polarized HE_{12} mode and *x*-polarized HE_{12} modes, respectively.

Next, in Fig. 5, the dominant electric field intensity of the fibers for thermally stressed condition and unstressed condition are shown for different modes. The electric field value is taken over the radial line along the *x*-axis from the center of the fiber. From these curves, it can be seen that for all the modes considered here, the field intensity distribution remain similar in all the three cases of analysis.

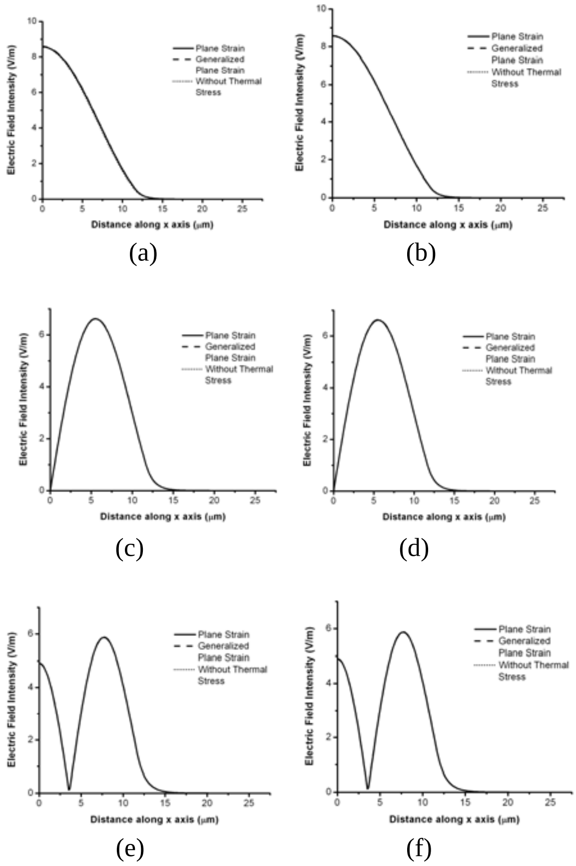

Fig. 5: Electric field intensity over the radial distance along *x*-axis, (a) HE^x_{11}, (b) HE^y_{11} , (c) TE_{01}, (d) TM_{01}, (e) HE^x_{12}, and (f) HE^y_{12} mode.

The birefringence of optical fiber for fundamental and higher order modes are shown as a function of wavelength in Fig. 6. Part (a), (b), and (c) of this figure show the birefringence of fundamental HE modes, TE-TM modes, and higher order HE modes, designated by B_{11}, B_{01}, and B_{12}, respectively. The solid lines and dashed lines show the birefringence when the modal optical analysis incorporates indices of plane strain approach and generalized plane strain approach, respectively. The dotted line shows the birefringence when no thermal stress is considered. Birefringence differs much when thermal stress in the fiber is

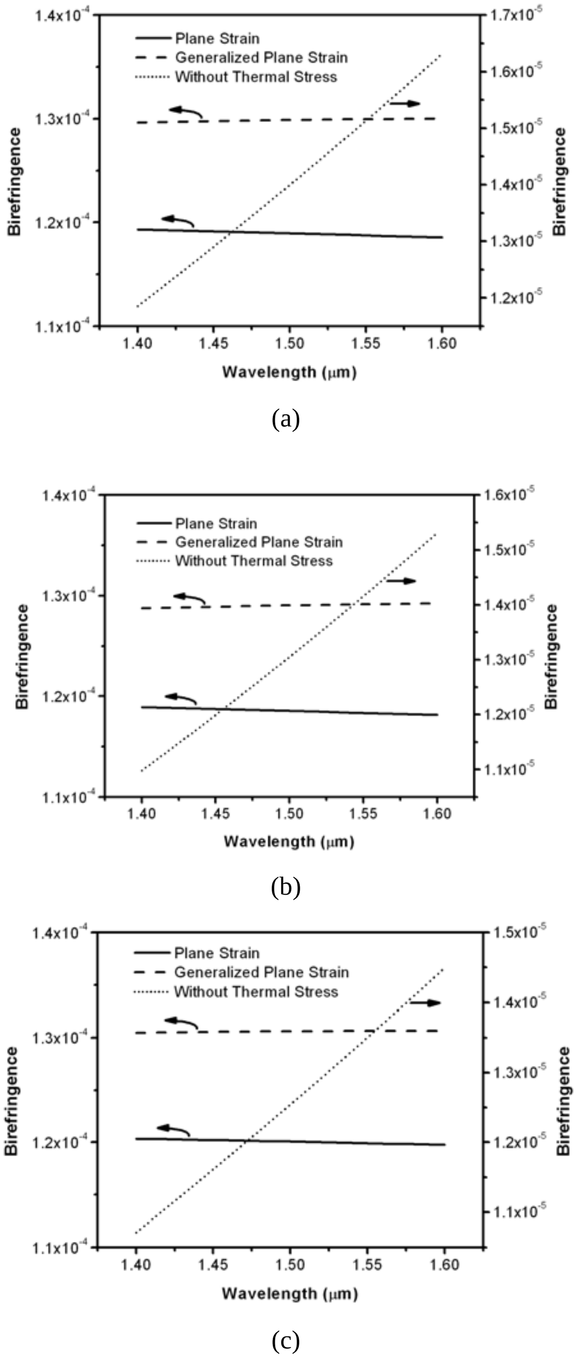

Fig. 6: Variation of Birefringence with wavelength, (a) B_{11}, (b) B_{01}, and (c) B_{12}.

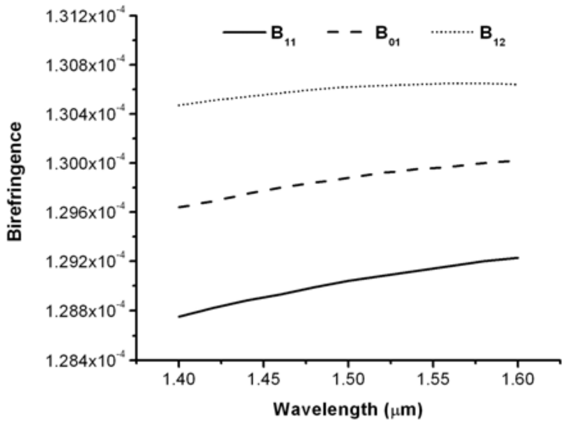

Fig. 7: Variation of Birefringence with wavelength (extracted results from Fig. 6).

considered. Also, the approach of stress analysis to incorporate the stress-optic effect plays a vital role to obtain the predicted value of the birefringence. The difference in the results of plane strain based analysis and generalized plane strain based analysis is about 8-10% for all birefringences over the spectrum considered here. The generalized plane strain based analysis, however, gives higher values.

Most importantly, the numerical results show that the value of birefringence is improved by at least one decimal place when thermal stress induced changes in material index is incorporated. As in reality, the thermal stress induced effect is always there in the fiber material, any sort of modeling must incorporate the stress-optic effect. Without considering the effect may lead to erroneous prediction of performance. Now, it can also be seen that the birefringence increases slightly with the increase in wavelength and this is shown in Fig. 7. These results are the extracted values from Fig. 6 when generalized plane strain is considered for incorporating the stress-optic effect. In this figure, the dotted, dashed, and solid lines are showing the birefringence values for fundamental HE modes, TE-TM modes, and higher order HE modes, respectively.

Next, considering the stress-optic effect using generalized plane strain, the wavelength dependence of other propagation properties, e.g., beat length, chromatic dispersion, group birefringence, and group effective index of different modes are studied. Fig. 8 shows the variation of beat length with wavelength. The solid line, dashed line, and dotted line show beat lengths for fundamental HE modes, transverse modes, higher order HE modes, respectively. For all the three cases, beat length increases linearly with the increase in wavelength. Here the beat lengths are obtained from the phase birefringence of the respective modes.

In Fig. 9, the dependence of group effective indices of different modes with wavelength is shown. It is interesting to see that the group effective indices increase with the increase in the wavelength. The rate of increase or the slope of the curve is larger for higher order modes than the immediate lower order modes. A clear difference can be seen in the group index of orthogonal pairs of modes over the wavelength range considered here.

In Fig. 10, the change of group birefringence with wavelength is shown for fundamental and higher order modes. In this figure, the solid line, dashed line, and the dotted line are showing the group birefringence values for fundamental HE modes, TE-TM modes, and higher order HE modes, respectively. It can be seen that the value of group birefringence of all the modes considered here increases with the increase of wavelength and the values are in the order of 10^{-4}. Finally, Fig. 11 shows group velocity dispersion versus wavelength of different modes. The dispersion curves show that they are polarization independent and show increasing tendency with the increase in wavelength.

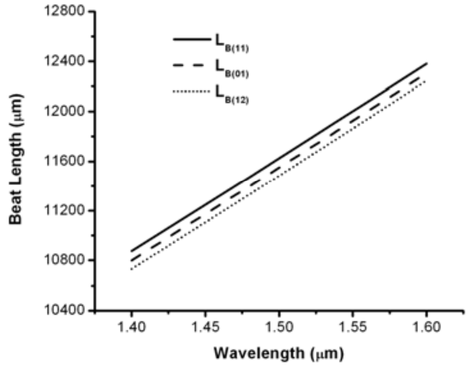

Fig. 8: Variation of Beat Length with wavelength.

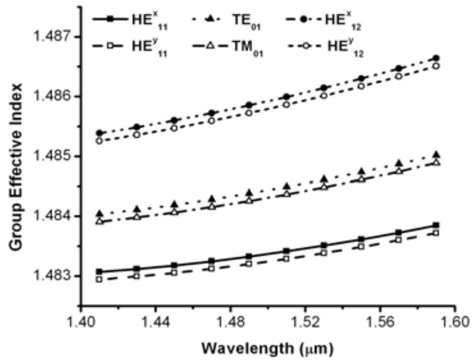

Fig. 9: Group effective index versus wavelength.

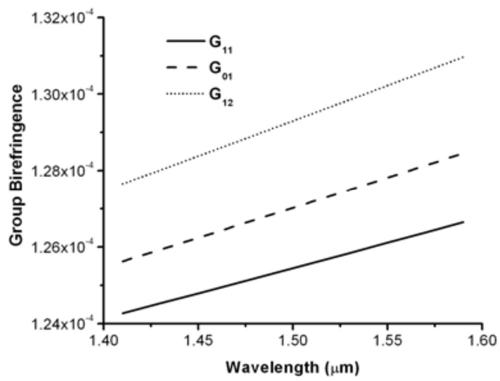

Fig. 10: Variation of Group Birefringence with wavelength.

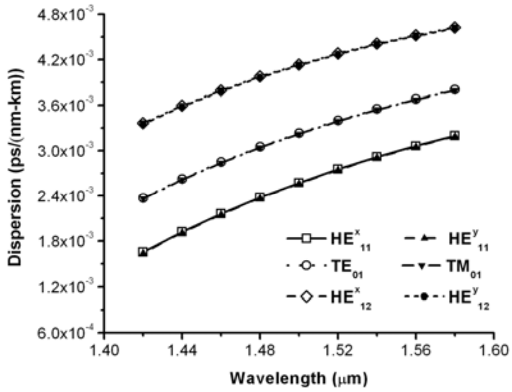

Fig. 11: Variation of group velocity dispersion with wavelength for different modes.

IV. Conclusion

The effect of thermally induced stress on fundamental and higher order modes of a highly elliptical core optical fiber is studied in this work. The birefringence of different modes show lower values when the modal analysis is carried out without considering thermal stress in the fiber. The stress developed in a fiber during manufacturing process, causes higher values of birefringence of fundamental and higher order modes of the fiber. However, the computation of modal birefringence and other propagation properties depend much on the approach of associated stress analysis. Also, it has been found that the birefringence and the dispersion of fundamental and higher order modes increase with an increase in the wavelength.

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