World Academy of Science, Engineering and Technology
International Journal of Physical and Mathematical Sciences
Vol:5, No:11, 2011

Non-reflection Boundary Conditions for Numerical Simulation of Supersonic Flow

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Abstract**—**This article presents the boundary conditions for the problem of turbulent supersonic gas flow in a plane channel with a perpendicular injection jets. The non-reflection boundary conditions for direct modeling of compressible viscous gases are studied. A formulation using the NSCBC (Navier- Stocks characteristic boundary conditions) through boundaries is derived for the subsonic inflow and subsonic non-reflection outflow situations. Verification of the constructed algorithm of boundary conditions is carried out by solving a test problem of perpendicular sound of jets injection into a supersonic gas flow in a plane channel.

Keywords— WENO scheme; non-reflection boundary conditions; NSCBC; Supersonic flow.

I. Introduction

FOR the problem of turbulent supersonic gas flow in a plane channel with a perpendicular injection jets there are two types of boundary conditions, the first is physical which is specifies the known physical behavior of one or more of the dependent variables at the boundaries, and the second - is numerical boundary conditions which are necessary when the number of physical boundary conditions are less than the number of dependent variables and also to exclude all false reflections of propagating incoming waves from the inside of the domain to the outside (subsonic non-reflecting outflow) as in [1]and false reflections of propagating incoming waves from the outside of the domain to the inside (subsonic inflow). These waves require a specific treatment as quoted here from [2-3] for the Navier-Stokes equations. In this article based on the method developed for solving the Navier-Stokes equations for multi-component gas mixture flow is numerically simulated planar turbulent supersonic air flow with a transverse hydrogen injection from the channel walls. When studying for the convenience of computation we consider the injecting of the jet with the bottom wall. Flow scheme is shown in Fig. 1. The computational code is developed On the basis of the fourth order weighted essentially non-oscillatory (WENO) schemes. the NSCBC for supersonic flows of compressible viscous multi-component gas at the boundaries is introduced.

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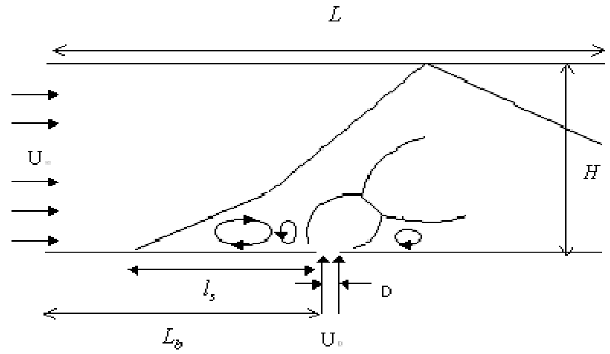

Fig. 1 Flow diagram

II. The Model Equations

Now, In Cartesian coordinates two-dimensional Reynolds-averaged Navier-Stokes equations for multi- components flow in conservation forms are:

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial\left(\vec{E}-\vec{E}_v\right)}{\partial x} + \frac{\partial\left(\vec{F}-\vec{F}_v\right)}{\partial z} \tag{1}$$

Where $\vec{U} = \left(\rho, \rho u, \rho w, E_t, \rho Y_i\right)^T$,

The invisid and viscous flux vectors are defined by

$$\vec{E} = \left(\rho u, \rho u^2 + P, \rho u w, (E_t + P)u, \rho u Y_i\right)^T,$$

$$\vec{F} = \left(\rho w, \rho u w, \rho w^2 + P, (E_t + P) w, \rho w Y_i\right)^T$$

$$\vec{E}_v = \left(0, \tau_{xx}, \tau_{xz}, u\tau_{xx} + u\tau_{xz} - q_x, J_{ix}\right)^T$$

$$\vec{F}_v = \left(0, \tau_{xz}, \tau_{zz}, w\tau_{xz} + w\tau_{zz} - q_z, J_{iz}\right)^T$$

$$P = \frac{\rho T}{\gamma_\infty M_\infty^2}\left(\sum_{i=1}^{N}\frac{Y_i}{W_i}\right), \ \sum_{i=1}^{N} Y_i = 1, \tag{2}$$

$$E_t = \frac{\rho}{\gamma_\infty M_\infty^2}\sum_{i=1}^{N} Y_i h_i - P + \frac{1}{2}\rho\left(u^2 + w^2\right) \tag{3}$$

$$h_i = h_i^0 + \int_{T_0}^{T} c_{pi}\, dT, \ c_{pi} = C_{pi}/W.$$

where the molar specific heat C_{pi} of the i-th components is given in terms of the fourth degree polynomial with respect to temperature which constants can be found in the JANAF Thermo-chemical Tables [4], Y_i is the mass fraction of the i-th components, τ_{xx}, τ_{xz}, τ_{zx}, τ_{zz} are the viscous stress tensors, q_x, q_z, J_{ix}, J_{iz} are the heat and diffusion flux (diffusion fluxes are defined from Fick's Law), $\mu = \mu_l + \mu_t$ is the sum of the coefficients of the laminar and turbulent viscosity. The Baldwin — Lomax model is used for determining μ_t.

The system (1)-(3) is written in a dimensionless form and conventional notation. The governing parameters are the entrance parameters, the pressure and total energy are normalized to $\rho_\infty u_\infty^2$, the enthalpy to $R^0 T_\infty / W_\infty$ the molar specific heat to R^0.

The boundary conditions have the following form:

At the entrance:

$W_i = W_{i\infty}$, $P = P_\infty$, $T = T_\infty$, $u = M_\infty \sqrt{\gamma_\infty R^0 T_\infty / W_\infty}$, $w = 0$, $Y_i = Y_{i\infty}$, $x = 0$, $0 \le z \le H$, the boundary layer is specified in the inlet section near the wall, and the velocity and temperature are approximated by the a power law;

On the slot: $W_k = W_{k0}$, $P_0 = nP_\infty$, $T = T_0$, $u = M_0 \sqrt{\gamma_0 R^0 T_0 / W_0}$, $u = 0$, $Y_i = Y_{i0}$, $z = 0$, $L_b \le x \le L_b + h$; ($n = P_0 / P_\infty$ is the jet pressure ratio, P_0 is the jet pressure, and P_∞ is the flow pressure); on the lower wall the no-slip condition and the adiabatic wall condition are imposed; on the upper boundary the condition of symmetry is assumed; on the outflow the nonreflecting boundary condition [2] is considered .

III. METHOD OF THE SOLUTION

The numerical solution of the system (1)-(3) is calculated by the two steps. The first is defined dynamic parameters and second mass species. The approximation of convection terms is performed on the basis of the fourth order weighted essentially non-oscillatory (WENO) scheme. The WENO scheme is constructed on the basis of ENO scheme [1]. However, in WENO scheme instead of choosing one interpolating polynomial, we use a convex combination of all corresponding polynomials. This is done by introducing weight coefficients to the convex combination. In approximation of derivatives in diffusion terms were used second-order central-difference operators.

IV. THE IMPLEMENTATION OF NSCBC METHOD

The NSCBC approach for the outflow was considered in the situation of a subsonic non-reflection outflow in [1]. Here we introduce only The NSCBC approach for the inlet in the situation of a subsonic inflow, first we consider a boundary located at x=L. Using the characteristic analysis [2] to modify the hyperbolic terms of equations (1)-(3) corresponding to waves propagating in x=L direction, we can write this system as:

$$\frac{\partial \rho}{\partial t} + d_1 + \frac{\partial(\rho w)}{\partial z} = 0 \tag{4}$$

$$\frac{\partial(\rho u)}{\partial t} + ud_1 + \rho d_2 + \frac{\partial(\rho w)}{\partial z} = \frac{\partial(\tau_{xx})}{\partial x} + \frac{\partial(\tau_{xz})}{\partial z} \tag{5}$$

$$\frac{\partial(\rho w)}{\partial t} + wd_1 + \rho d_3 + \frac{\partial(\rho w)^2}{\partial z} + \frac{\partial P}{\partial z} = \frac{\partial(\tau_{xz})}{\partial x} + \frac{\partial(\tau_{zz})}{\partial z} \tag{6}$$

$$\frac{\partial E_t}{\partial t} + \frac{1}{2}(u^2 + w^2)d_1 + \frac{d_4}{\gamma - 1} + \rho u d_2 + \rho w d_3 + \frac{\partial((E_t + P)w)}{\partial z} = = \frac{\partial(u\tau_{xx} + u\tau_{xz} - q_x)}{\partial x} + \frac{\partial(w\tau_{xx} + w\tau_{xz} - q_x)}{\partial x} \tag{7}$$

where

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{c^2} Z_2 + \frac{1}{2}(Z_4 + Z_1) \\ \frac{1}{2\rho c}(Z_4 - Z_1) \\ Z_3 \\ \frac{1}{4}(Z_4 + Z_1) \end{pmatrix}, \tag{8}$$

and Z_i's are the amplitudes of the characteristic waves associated with each characteristic velocity λ_i. These velocities are given by [2]: $\lambda_1 = u - c$, $\lambda_2 = u$, $\lambda_3 = u$, $\lambda_4 = u + c$, where c is the speed of sound: $c^2 = \frac{\gamma P}{\rho}$, and Zi's are given by:

$$Z_1 = \lambda_1 \left(\frac{\partial P}{\partial x} - \rho c \frac{\partial u}{\partial x} \right), \qquad Z_2 = \lambda_2 \left(\frac{\partial P}{\partial x} - c^2 \frac{\partial \rho}{\partial x} \right), \qquad Z_3 = \lambda_3 \frac{\partial w}{\partial x},$$

$$Z_4 = \lambda_4 \left(\frac{\partial P}{\partial x} + \rho c \frac{\partial u}{\partial x} \right).$$

To specify the values of Z_i's of the incoming waves we use the LODI (the Local One-Dimensional Inviscid) relations [2]:

$$\frac{\partial \rho}{\partial t} + \frac{1}{c^2}\left[Z_2 + \frac{1}{2}(Z_4 + Z_1) \right] = 0 \tag{9}$$

$$\frac{\partial u}{\partial t} + \frac{1}{2\rho c}(Z_4 - Z_1) = 0 \tag{10}$$

$$\frac{\partial w}{\partial t} + Z_3 = 0 \tag{11}$$

$$\frac{\partial P}{\partial t} + \frac{1}{2}(Z_4 + Z_1) = 0 \tag{12}$$

For a subsonic inflow the parameters u, w, and T are constant so we need only to solve two equations (4) and (7). Since u= const. and from (10) in (11) and (12) we get $Z_1 = Z_4$ and $Z_2 = (\gamma - 1)Z_1$, and from (11) we have $Z_3 = 0$ by substituting in (4) with these values of Z_i's we get,

$$\frac{\partial \rho}{\partial t} = -\left[\frac{\gamma}{c^2}(u - c)\left(\frac{\partial P}{\partial x} - \rho c \frac{\partial u}{\partial x} \right) + \frac{\partial(\rho w)}{\partial z} \right] \tag{13}$$

Applying an approximation for time step with second-order of accuracy then we get,

$$\rho_{ij}^{n+1} = \frac{4}{3}\rho_{ij}^{n} - \frac{1}{3}\rho_{ij}^{n-1} - \frac{2\Delta t}{3}\left[\frac{\gamma}{c^2}(u - c)\left(\frac{\partial P}{\partial x} - \rho c \frac{\partial u}{\partial x} \right) + \frac{\partial(\rho w)}{\partial z} \right]_{ij} \tag{14}$$

For total energy E_t we consider the equation

$$\frac{\partial E_t}{\partial t} = \frac{h(T)}{\gamma M_\infty^2}\frac{\partial \rho}{\partial t} + \frac{\rho}{\gamma M_\infty^2}\frac{\partial h(T)}{\partial t} - \frac{\partial P}{\partial t} + \frac{1}{2}u^2 \frac{\partial \rho}{\partial t} + \rho u \frac{\partial u}{\partial t} \tag{15}$$

Where $h(T)$ is the enthalpy and. Since u and T are constants and from the state equation:

$$P = \frac{T\rho}{\gamma M_\infty^2 W} \tag{16}$$

We have:

$$\frac{\partial E_t}{\partial t} = -\left(\frac{h(T)}{\gamma M_\infty^2} + \frac{1}{2}u^2 - \frac{T}{\gamma M_\infty^2 W} \right)\left[\frac{\gamma}{c^2}(u - c)\left(\frac{\partial P}{\partial x} - \rho c \frac{\partial u}{\partial x} \right) + \frac{\partial(\rho w)}{\partial z} \right] \tag{17}$$

Again here we applying an approximation for time step with second-order of accuracy then we get,

$$(E_t)_{ij}^{n+1} = \frac{4}{3}(E_t)_{ij}^{n} - \frac{1}{3}(E_t)_{ij}^{n-1} - -\frac{2\Delta t}{3}\left(\frac{h(T)}{\gamma M_\infty^2} + \frac{1}{2}u^2 - \frac{T}{\gamma M_\infty^2 W} \right)_{ij}\left[\frac{\gamma}{c^2}(u - c)\left(\frac{\partial P}{\partial x} - \rho c \frac{\partial u}{\partial x} \right) + \frac{\partial(\rho w)}{\partial z} \right]_{ij} \tag{18}$$

In [1] for a subsonic non-reflection outflow the static pressure at the outflow $P = P_\infty$ was imposed to define the amplitude of incoming wave $Z_1 = K\left(P - P_\infty\right)$ corresponding to the negative velocity $\lambda_1 = u - c$, where $K = \sigma\left(1 - M_\infty^2\right)c / L$ (σ is constant and L is a characteristic size of the domain) and finding the other's (Z_2, Z_3, and Z_4) by using interior points [2].

V. Numerical Results and Discussions

In The computations were done on a staggered spatial grid with range of parameters: $2 \le M_\infty \le 4$, $M_0 = 1$, $Pr = 0.7$, $2 \le n \le 15$, $T_0 = 642$, $T_\infty = 800 D = 0.1cm$, $H = 3.0cm$, $L_b = 5cm$ and $L = 10cm$, with Pressure ratio n = 10.26, $M_\infty = 3.75$ and $P_\infty = 1000\,Pa$ and the computed results after applying the non-reflection boundaries condition at the entrance boundary are compared with that computed results before applying the non-reflection boundaries condition at the entrance boundary. We can see that from the Pressure profiles on the wall Figure 2, and Pressure distribution Figures 3 and 4 the implementation of the NSCBC method on the entrance boundary contributed to improve the numerical solution on the boundaries.

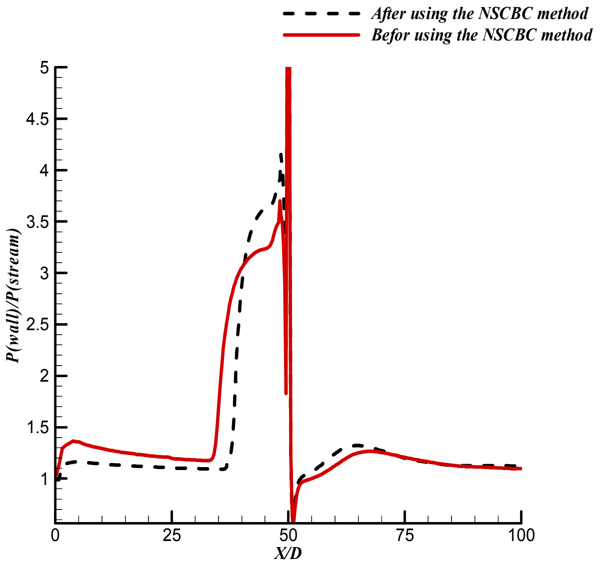

Fig. 2 Pressure profiles on the wall

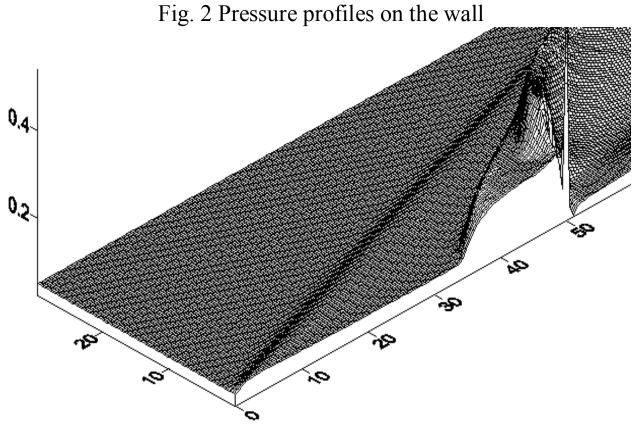

Fig. 3 Pressure distribution before applying the NSCBC method

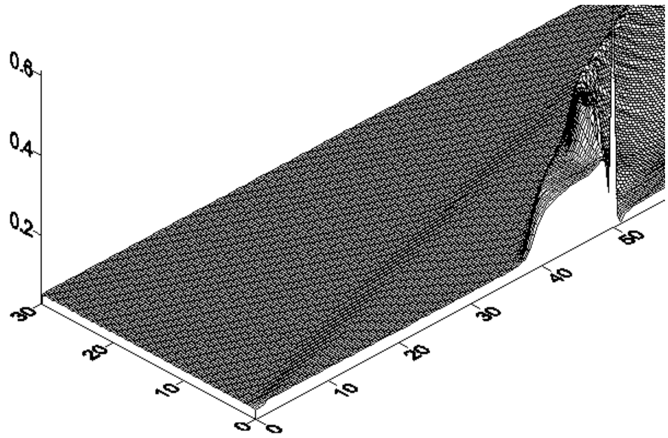

Fig. 4 Pressure distribution after applying the NSCBC method

VI. Conclusions

Numerical boundary conditions for the problem of turbulent supersonic gas flow in a plane channel with a perpendicular injection jets are constructed without any extrapolation. The NSCBC method is based on a local inviscid one-dimensional analysis of the waves crossing the boundary. The amplitude variations of the waves entering the domain are estimated from an analysis of the local one-dimensional inviscid equations. These amplitude variations are then used in a reduced set of conservation equations to determine boundary variables which were not specified by the physical boundary conditions.

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