

# Power Minimization in Decode-and-XOR-Forward Two-Way Relay Networks

Dong-Woo Lim, Chang-Jae Chun, Hyung-Myung Kim

**Abstract**—We consider a two-way relay network where two sources exchange information. A relay helps the two sources exchange information using the decode-and-XOR-forward protocol. We investigate the power minimization problem with minimum rate constraints. The system needs two time slots and in each time slot the required rate pair should be achievable. The power consumption is minimized in each time slot and we obtained the closed form solution. The simulation results confirm that the proposed power allocation scheme consumes lower total power than the conventional schemes.

**Keywords**—Decode-and-XOR-forward, power minimization, two-way relay

## I. INTRODUCTION

THE two-way relaying [1] has been emerged as a method to alleviate the spectral efficiency loss of half-duplex relaying systems. Two sources exchange information via a relay through two time slots. In the first time slot the two sources simultaneously transmit their symbols to the relay. Then, in the second time slot the relay forwards a symbol to the two sources. There are several protocols for the two-way relaying such as the AF (amplify-and-forward) protocol and DF (decode-and-forward) protocol.

In the AF protocol [1], the relay amplifies the received signal in the first time slot and transmits the amplified signal to the two sources in the second time slot. Then, the two sources subtract the back-propagating self-interference from the received signal and decode the symbols from each other. In the DF protocol [1], [2], the relay decodes the two symbols from the received signal in the first time slot and performs superposition encoding or XOR encoding [2].

Recently, there has been an increasing attention to the power allocation for two-way relay networks. The power allocation scheme to maximize the sum rate for two-way AF OFDM relay networks was proposed in [3] and [4]. The power minimization problem for the AF two-way relay networks with the constraint of outage probability was studied in [5]. In addition, the sum rate maximization problem for two-way DF relay networks with data rate fairness was studied in [6].

In this paper, we address the sum power minimization problem of two-way DF relay networks in which the relay performs XOR encoding. We call such system a DXF (decode-

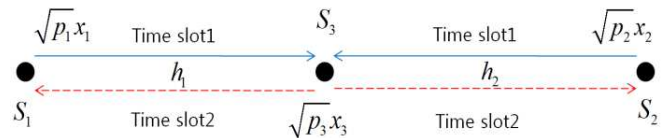


Fig. 1 System model

and-XOR-forward) relay network. In Section II, we present the system model and formulate the problem. In Section III, we obtain the closed form solution. Simulation results are presented in Section IV. Finally, Section V concludes the paper.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. Signal and Channel Models

We consider a two-way relay network in which two sources  $T_1$  and  $T_2$  exchange information with the help of a relay  $T_3$  (Fig. 1). All the terminals have a single antenna and are half-duplexed. It is assumed that there is no direct path between the two sources. In the first time slot (multiple access phase),  $T_1$  and  $T_2$  transmit  $\sqrt{p_1}x_1$  and  $\sqrt{p_2}x_2$ , respectively, where  $x_1$  and  $x_2$  are transmit symbols,  $p_i$  the transmit power of  $T_i$  and  $E[|x_i|^2] = 1$  for  $\forall i \in \{1, 2\}$ . The relay receives:

$$y_3 = h_1\sqrt{p_1}x_1 + h_2\sqrt{p_2}x_2 + n_3 \quad (1)$$

Where  $h_i$  for  $\forall i \in \{1, 2\}$  is the complex channel gain between the source terminal  $T_i$  and relay terminal  $T_3$  and  $n_3 \sim CN(0, 1)$  is additive white Gaussian noise at the relay. It is assumed that each of the two channels is reciprocal and all the terminals know the two channels. The relay decodes  $x_1$  and  $x_2$  from the received signal  $y_3$ . The multiple access channel (MAC) of the first time slot is well-known [1], [2]. The capacity region is given by:

$$C_{MAC} := \{(R_1, R_2) : \begin{aligned} &0 \leq R_1 \leq R_{1R}, \\ &0 \leq R_2 \leq R_{2R}, \quad R_1 + R_2 \leq R_R \end{aligned}\} \quad (2)$$

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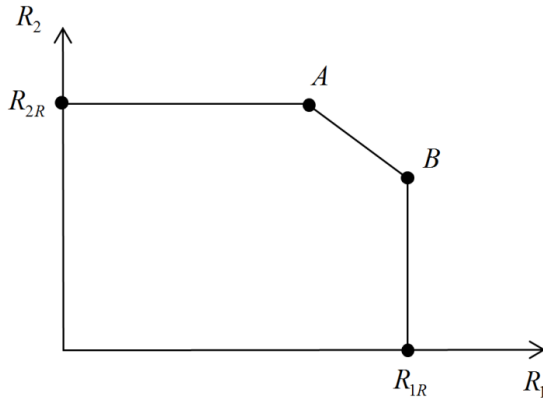


Fig. 2 The capacity region of the first time slot

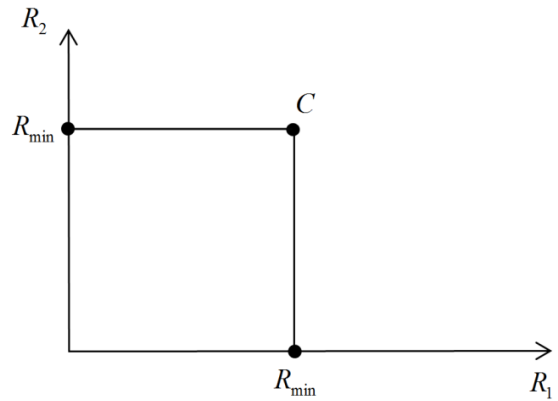


Fig. 3 The achievable rate region of the second time slot

with the boundaries

$$R_{1R} := \frac{1}{2} \log(1 + p_1 |h_1|^2), \quad (3)$$

$$R_{2R} := \frac{1}{2} \log(1 + p_2 |h_2|^2), \quad (4)$$

$$R_R := \frac{1}{2} \log(1 + p_1 |h_1|^2 + p_2 |h_2|^2) \quad (5)$$

The capacity region is a pentagon as shown in Fig. 2.

Then, in the second time slot (broadcast phase)  $T_3$  transmits  $\sqrt{p_3}x_3$  where  $p_3$  is the transmit power of  $T_3$ . The transmit symbol  $x_3$  is obtained by performing an XOR operation on the two decoded messages at the relay  $T_3$  [7], [8]. The two source terminals  $T_1$  and  $T_2$  receive:

$$y_1 = h_1 \sqrt{p_3} x_3 + n_1 \quad (6)$$

and

$$y_2 = h_2 \sqrt{p_3} x_3 + n_2 \quad (7)$$

Where  $n_i \sim CN(0,1)$  at the source terminal  $T_i$  for

$\forall i \in \{1, 2\}$ . Since the terminals  $T_1$  and  $T_2$  know their own transmitted symbols, each source obtains the message from the other source by performing an XOR operation on the decoded message and their own transmitted message. The achievable rate region of the second time slot is given by

$$R_{BC} := \{(R_1, R_2) : 0 \leq R_1 \leq R_{\min}, 0 \leq R_2 \leq R_{\min}\} \quad (8)$$

with

$$R_{\min} = \min \left\{ \frac{1}{2} \log(1 + p_3 |h_1|^2), \frac{1}{2} \log(1 + p_3 |h_2|^2) \right\} \quad (9)$$

The achievable rate region is a square as shown in Fig. 3.

In order to achieve the rate  $R_1$  from  $T_1$  to  $T_2$  and the rate  $R_2$  from  $T_2$  to  $T_1$ , the rate pair  $[R_1, R_2]$  has to be achievable in the first time slot as well as in the second time slot.

### B. Problem Formulation

Given instantaneous  $h_1$  and  $h_2$  at the two source terminals  $T_1$  and  $T_2$ , we formulate the total power minimization problem subject to the constraints on the data rates. We can formulate the optimization problem as:

$$\begin{aligned} \min \quad & p_1 + p_2 + p_3 \\ \text{s.t.} \quad & [R_{1,th}, R_{2,th}] \in C_{MAC} \cap R_{BC}, \end{aligned} \quad (10)$$

Where  $[R_{1,th}, R_{2,th}]$  is a required rate pair. The data rate constraint mean that the required rate pair should be achievable in the first time slot as well as in the second time slot. It is assumed that  $R_{1,th} > 0$  and  $R_{2,th} > 0$ , because the two sources  $T_1$  and  $T_2$  communicate with each other. In the following section, we minimize the sum power.

### III. OPTIMAL POWER ALLOCATION

The objective here is to find the solution of the problem in (8). The capacity region of the first time slot depends only on the powers  $p_1$  and  $p_2$  and the achievable rate region of the second time slot depends only on  $p_3$ . Therefore we can independently minimize  $p_3$  and  $p_1 + p_2$  as the following two subsections. In the first subsection, we minimize the power of the second time slot  $p_3$ . We can get a unique power allocation that locates the required rate pair  $[R_{1,th}, R_{2,th}]$  on the boundary of  $R_{BC}$ . Since the achievable rate region of the second time slot is a square, the power allocation is optimal to minimize the power  $p_3$ . In the second subsection, we minimize the sum power of the first time slot  $p_1 + p_2$ .

#### A. Power Minimization in the Second Time Slot

Since during the second time slot the achievable rate region

$R_{BC}$  does not depend on  $p_1$  and  $p_2$ , in this subsection, we minimize  $p_3$  under the condition that the rate pair  $[R_{1,th}, R_{2,th}]$  is achievable in the second time slot. In order to achieve the rate pair  $[R_{1,th}, R_{2,th}]$ , the achievable rate region  $R_{BC}$  should include the rate pair  $[R_{1,th}, R_{2,th}]$ . As shown in Fig. 3, the achievable rate region of the second time slot is a square. It means that the data rates  $R_1$  and  $R_2$  of the second time slot depend only on the power  $p_3$ . Therefore we can reduce  $p_3$  until  $[R_{1,th}, R_{2,th}]$  is located on the boundary of  $R_{BC}$  and there exists a unique  $p_3^*$  that locates  $[R_{1,th}, R_{2,th}]$  on the boundary. If  $R_{1,th} \geq R_{2,th}$ , then  $[R_{1,th}, R_{2,th}]$  is located on the right side of the square in Fig. 3, and otherwise  $[R_{1,th}, R_{2,th}]$  is located on the top side of the square. Hence, the optimal value for  $p_3$  becomes

$$p_3^* = \frac{2^{\max(2R_{1,th}, 2R_{2,th})} - 1}{\min(|h_1|^2, |h_2|^2)} \quad (11)$$

From the above solution, it can be observed that the optimal power  $p_3^*$  locates  $[R_{1,th}, R_{2,th}]$  on the boundary.

#### B. Power Minimization in the First Time Slot

In this subsection, we minimize  $p_1 + p_2$  under the condition that the rate pair  $[R_{1,th}, R_{2,th}]$  is achievable in the first time slot. Since the capacity region  $C_{MAC}$  of the first time slot is a pentagon as shown in Fig. 2, it is not possible to minimize  $p_1 + p_2$  as we did in the second time slot. The data rate  $R_1$  of the first time slot depends on  $p_1$  and  $p_2$ , and the data rate  $R_2$  of the first time slot also depends on  $p_1$  and  $p_2$ . We can optimize  $p_1$  and  $p_2$  by the following two lemmas.

Lemma 1: If the required rate pair  $[R_{1,th}, R_{2,th}]$  is not located on the boundary of the capacity region  $C_{MAC}$ , then the power allocation in the first time slot is not optimal.

Proof: In order to achieve the rate  $[R_{1,th}, R_{2,th}]$ , the capacity region  $C_{MAC}$  should include the rate pair. Then, the rate pair is located in the interior of  $C_{MAC}$  or on the boundary of  $C_{MAC}$ . If a power allocation locates the rate pair  $[R_{1,th}, R_{2,th}]$  in the interior of  $C_{MAC}$ , we can reduce  $p_1$  or  $p_2$  until  $[R_{1,th}, R_{2,th}]$  located on the boundary of  $C_{MAC}$ . Therefore the power allocation that locates  $[R_{1,th}, R_{2,th}]$  in the interior of  $C_{MAC}$  is not optimal. ■

We can now consider only the power allocations that locate

$[R_{1,th}, R_{2,th}]$  on the boundary of  $C_{MAC}$ . As shown in Fig. 2, The boundary is composed of the line segments  $\overline{AR_{2R}}$ ,  $\overline{BR_{1R}}$  and  $\overline{AB}$ . Lemma 2 shows that the optimal power allocation locates  $[R_{1,th}, R_{2,th}]$  on the line segment  $\overline{AB}$ .

Lemma 2: If the required rate pair  $[R_{1,th}, R_{2,th}]$  is not located on the line segment  $\overline{AB}$  of the capacity region  $C_{MAC}$ , then the power allocation in the first time slot is not optimal.

Proof: First, if a power allocation locates the rate pair  $[R_{1,th}, R_{2,th}]$  on the line segment  $\overline{AR_{2R}}$ , we can reduce  $p_1$  until  $[R_{1,th}, R_{2,th}]$  is located on the point  $A$ . Second, if a power allocation locates the rate pair  $[R_{1,th}, R_{2,th}]$  on the line segment  $\overline{BR_{1R}}$ , we can reduce  $p_2$  until  $[R_{1,th}, R_{2,th}]$  is located on the point  $B$ . Therefore the optimal power allocation locates  $[R_{1,th}, R_{2,th}]$  on the line segment  $\overline{AB}$ . ■

By the Lemma 1 and Lemma 2, we can consider only the power allocations that locate  $[R_{1,th}, R_{2,th}]$  on the line segment  $\overline{AB}$ . Then, we can easily obtain the set of the power allocations by substituting  $[R_{1,th}, R_{2,th}]$  into (2):

$$P_{BC} := \{(p_1, p_2) : \frac{2^{2R_{1,th}} - 1}{|h_1|^2} \leq p_1 \leq \frac{2^{2R_{1,th} + 2R_{2,th}} - 2^{2R_{2,th}}}{|h_1|^2}, p_2 = \frac{2^{2R_{1,th} + 2R_{2,th}} - p_1 |h_1|^2 - 1}{|h_2|^2}\} \quad (12)$$

When the channel is symmetric ( $|h_1| = |h_2|$ ),  $p_1 + p_2$  becomes:

$$p_1 + p_2 = \frac{2^{2R_{1,th} + 2R_{2,th}} - 1}{|h_2|^2} \quad (13)$$

Therefore all the power allocations in  $P_{BC}$  are optimal in the symmetric channel case. When the channel is asymmetric ( $|h_1| \neq |h_2|$ ), we assume without loss of generality  $|h_2| > |h_1|$ . Let us consider the sum power  $p_1 + p_2$  in  $P_{BC}$ .  $p_1 + p_2$  is a function whose domain is  $P_{BC}$ . Then, we can obtain the directional derivative of  $p_1 + p_2$  in  $P_{BC}$ . The directional derivative  $D_{\mathbf{a}}f$  of a function  $f$  in the direction of a nonzero vector  $\mathbf{a}$  is given as [9]:

$$D_{\mathbf{a}}f = \frac{1}{|\mathbf{a}|} \mathbf{a} \cdot \text{grad } f \quad (14)$$

Where  $\text{grad } f$  is the gradient of  $f$  and " $\cdot$ " represents inner product. The directional derivative  $D_{\mathbf{a}}(p_1 + p_2)$  in the direction of  $\mathbf{a} = (-\frac{1}{|h_1|^2}, \frac{1}{|h_2|^2})$  is negative because of the

channel assumption ( $|h_2| > |h_1|$ ):

$$D_{\mathbf{a}}(p_1 + p_2) = \frac{|h_1|^2 - |h_2|^2}{|\mathbf{a}| |h_1|^2 |h_2|^2} < 0 \quad (15)$$

The set  $P_{BC}$  is one-dimensional and the directional derivative  $D_{\mathbf{a}}(p_1 + p_2)$  is negative. Therefore, for an arbitrary power allocation in  $P_{BC}$  we can move the power allocation in the

direction of  $\mathbf{a}$  until  $p_1 = \frac{2^{2R_{1,th}} - 1}{|h_1|^2}$ . Then, the sum power

$p_1 + p_2$  always reduces. The optimal powers  $p_1^*$  and  $p_2^*$  follow.

$$\begin{cases} p_1^* = \frac{2^{2R_{1,th}} - 1}{|h_1|^2} \\ p_2^* = \frac{2^{2R_{1,th} + 2R_{2,th}} - 2^{2R_{1,th}}}{|h_2|^2} \end{cases} \quad (16)$$

#### IV. SIMULATION RESULTS

In this section, we present the numerical results to compare the power minimization scheme with the three-phase DXF strategy and the one-way DF relay strategy (the four-phase). In the first time slot of the three-phase DXF strategy, one of the two sources transmits its symbol and the other source transmits its symbol in the second time slot. Then, the relay in the third time slot acts as in the second time slot of our system model. The one-way relay strategy needs four time slots for both direction transmissions.

We assume a linear one-dimensional network geometry, where the distance between  $T_1$  and  $T_2$  is normalized to 1, and  $d$  is the distance between  $T_1$  and  $T_3$ . The two channels have path loss and Rayleigh fading. It is assumed that  $h_1 \sim CN(0, 1/d^\alpha)$  and  $h_2 \sim CN(0, 1/(1-d)^\alpha)$ , where  $\alpha$  is the path loss exponent. In our simulation, we take  $\alpha = 4$ , and consider two scenarios in which the sources have different rate requirements. The sum power consumption against relay location, where the required rate pair is  $[R_{1,th}, R_{2,th}] = [1, 1]$ , is plotted in Fig. 4. The required rate pair is symmetric and the proposed scheme always has lower sum power consumption. The more asymmetric the channels are, the more efficient the proposed scheme is. In other words, when the relay is close to the sources, the power consumption gap is larger than that of the center location. In Fig. 5, the sum power consumption against relay location, where the required rate pair is  $[R_{1,th}, R_{2,th}] = [0.5, 1.5]$ , is plotted. The required rate pair is asymmetric. The comparison between Fig. 4 and Fig. 5 shows:

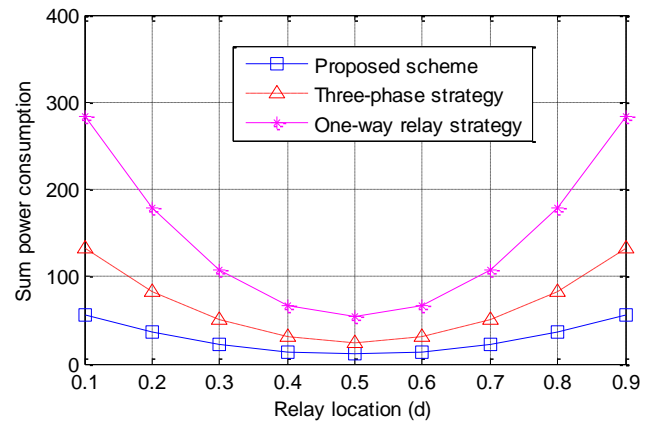


Fig. 4 Sum power consumption against relay location ( $[R_{1,th}, R_{2,th}] = [1, 1]$ )

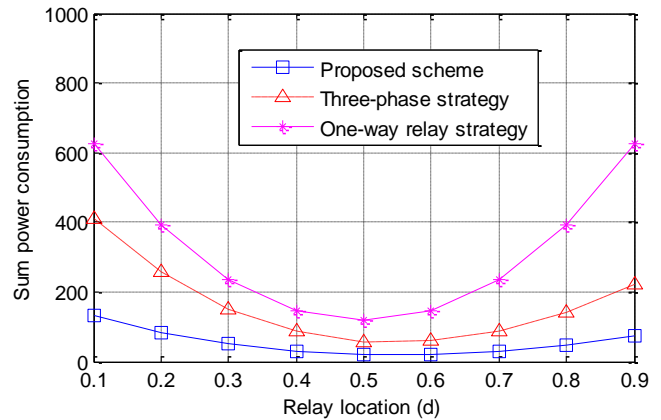


Fig. 5 Sum power consumption against relay location ( $[R_{1,th}, R_{2,th}] = [0.5, 1.5]$ )

that the proposed scheme of the asymmetric rate requirements is more efficient.

#### V. CONCLUSION

In this paper, we proposed the total power minimization scheme for DXF two-way relay networks while satisfying the minimum rate requirements. The simulation results show that the proposed scheme is more efficient when the channels and the minimum rate requirements are asymmetric. The power minimization problem with the outage probability constraint remains for further work.

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