Power Minimization in Decode-and-XOR-Forward Two-Way Relay Networks

Dong-Woo Lim, Chang-Jae Chun, Hyung-Myung Kim

***Abstract*—**We consider a two-way relay network where two sources exchange information. A relay helps the two sources exchange information using the decode-and-XOR-forward protocol. We investigate the power minimization problem with minimum rate constraints. The system needs two time slots and in each time slot the required rate pair should be achievable. The power consumption is minimized in each time slot and we obtained the closed form solution. The simulation results confirm that the proposed power allocation scheme consumes lower total power than the conventional schemes.

***Keywords*—**Decode-and-XOR-forward, power minimization, two-way relay

I. Introduction

THE two-way relaying [1] has been emerged as a method to alleviate the spectral efficiency loss of half-duplex relaying systems. Two sources exchange information via a relay through two time slots. In the first time slot the two sources simultaneously transmit their symbols to the relay. Then, in the second time slot the relay forwards a symbol to the two sources. There are several protocols for the two-way relaying such as the AF (amplify-and-forward) protocol and DF (decode-and-forward) protocol.

In the AF protocol [1], the relay amplifies the received signal in the first time slot and transmits the amplified signal to the two sources in the second time slot. Then, the two sources subtract the back-propagating self-interference from the received signal and decode the symbols from each other. In the DF protocol [1], [2], the relay decodes the two symbols from the received signal in the first time slot and performs superposition encoding or XOR encoding [2].

Recently, there has been an increasing attention to the power allocation for two-way relay networks. The power allocation scheme to maximize the sum rate for two-way AF OFDM relay networks was proposed in [3] and [4]. The power minimization problem for the AF two-way relay networks with the constraint of outage probability was studied in [5]. In addition, the sum rate maximization problem for two-way DF relay networks with data rate fairness was studied in [6].

In this paper, we address the sum power minimization problem of two-way DF relay networks in which the relay performs XOR encoding. We call such system a DXF (decode-and-XOR-forward) relay network. In Section II, we present the system model and formulate the problem. In Section III, we obtain the closed form solution. Simulation results are presented in Section IV. Finally, Section V concludes the paper.

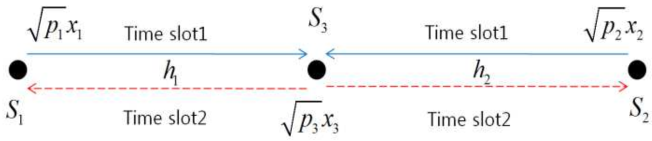

Fig. 1 System model

II. System Model and Problem Formulation

A. Signal and Channel Models

We consider a two-way relay network in which two sources T_1 and T_2 exchange information with the help of a relay T_3 (Fig. 1). All the terminals have a single antenna and are half-duplexed. It is assumed that there is no direct path between the two sources. In the first time slot (multiple access phase), T_1 and T_2 transmit $\sqrt{p_1}x_1$ and $\sqrt{p_2}x_2$, respectively, where x_1 and x_2 are transmit symbols, p_i the transmit power of T_i and $E[|x_i|^2]=1$ for $\forall i\in\{1,2\}$. The relay receives:

$$y_3 = h_1\sqrt{p_1}x_1 + h_2\sqrt{p_2}x_2 + n_3 \tag{1}$$

Where h_i for $\forall i\in\{1,2\}$ is the complex channel gain between the source terminal T_i and relay terminal T_3 and $n_3 \sim CN(0,1)$ is additive white Gaussian noise at the relay. It is assumed that each of the two channels is reciprocal and all the terminals know the two channels. The relay decodes x_1 and x_2 from the received signal y_3. The multiple access channel (MAC) of the first time slot is well-known [1], [2]. The capacity region is given by:

$$\begin{aligned} C_{MAC} := \{(R_1,R_2):\quad & 0 \le R_1 \le R_{1R}, \\ 0 \le R_2 \le R_{2R},\quad & R_1 + R_2 \le R_R\} \end{aligned} \tag{2}$$

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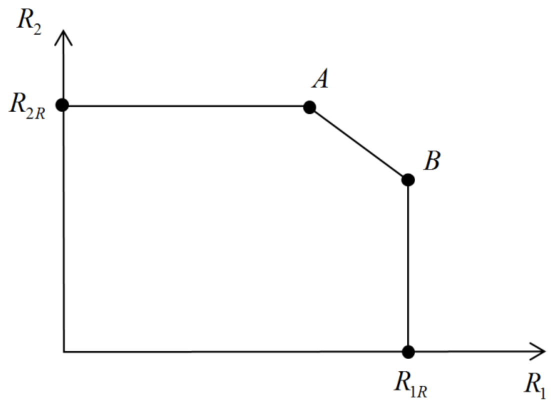

Fig. 2 The capacity region of the first time slot

with the boundaries

$$R_{1R} := \frac{1}{2}\log(1+p_1|h_1|^2), \tag{3}$$

$$R_{2R} := \frac{1}{2}\log(1+p_2|h_2|^2), \tag{4}$$

$$R_R := \frac{1}{2}\log(1+p_1|h_1|^2+p_2|h_2|^2) \tag{5}$$

The capacity region is a pentagon as shown in Fig. 2.

Then, in the second time slot (broadcast phase) T_3 transmits $\sqrt{p_3}x_3$ where p_3 is the transmit power of T_3. The transmit symbol x_3 is obtained by performing an XOR operation on the two decoded messages at the relay T_3 [7], [8]. The two source terminals T_1 and T_2 receive:

$$y_1 = h_1\sqrt{p_3}x_3 + n_1 \tag{6}$$

and

$$y_2 = h_2\sqrt{p_3}x_3 + n_2 \tag{7}$$

Where $n_i \sim CN(0,1)$ at the source terminal T_i for $\forall i \in \{1,2\}$. Since the terminals T_1 and T_2 know their own transmitted symbols, each source obtains the message from the other source by performing an XOR operation on the decoded message and their own transmitted message. The achievable rate region of the second time slot is given by

$$R_{BC} := \{(R_1,R_2):\ \ 0 \le R_1 \le R_{\min},\ \ 0 \le R_2 \le R_{\min}\} \tag{8}$$

with

$$R_{\min} = \min\{\frac{1}{2}\log(1+p_3|h_1|^2), \frac{1}{2}\log(1+p_3|h_2|^2)\} \tag{9}$$

The achievable rate region is a square as shown in Fig. 3.

In order to achieve the rate R_1 from T_1 to T_2 and the rate R_2 from T_2 to T_1, the rate pair $[R_1, R_2]$ has to be achievable in the first time slot as well as in the second time slot.

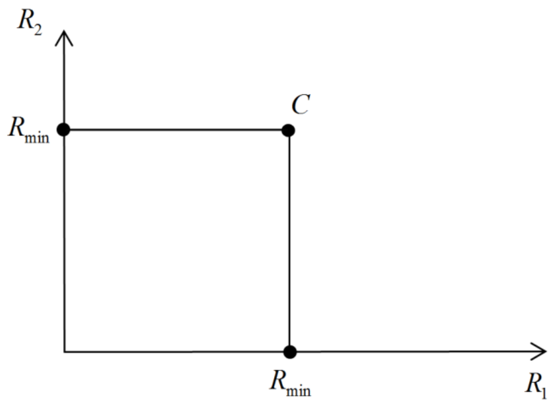

Fig. 3 The achievable rate region of the second time slot

B. Problem Formulation

Given instantaneous h_1 and h_2 at the two source terminals T_1 and T_2, we formulate the total power minimization problem subject to the constraints on the data rates. We can formulate the optimization problem as:

$$\begin{aligned} &\min \quad p_1+p_2+p_3 \\ &\ s.t. \\ &\quad [R_{1,th}, R_{2,th}] \in C_{MAC} \cap R_{BC}, \\ &\quad p_1, p_2, p_3 \ge 0 \end{aligned} \tag{10}$$

Where $[R_{1,th}, R_{2,th}]$ is a required rate pair.The data rate constraint mean that the required rate pair should be achievable in the first time slot as well as in the second time slot. It is assumed that $R_{1,th} > 0$ and $R_{2,th} > 0$, because the two sources T_1 and T_2 communicate with each other. In the following section, we minimize the sum power.

III. Optimal Power Allocation

The objective here is to find the solution of the problem in (8). The capacity region of the first time slot depends only on the powers p_1 and p_2 and the achievable rate region of the second time slot depends only on p_3. Therefore we can independently minimize p_3 and p_1+p_2 as the following two subsections. In the first subsection, we minimize the power of the second time slot p_3. We can get a unique power allocation that locates the required rate pair $[R_{1,th}, R_{2,th}]$ on the boundary of R_{BC}. Since the achievable rate region of the second time slot is a square, the power allocation is optimal to minimize the power p_3. In the second subsection, we minimize the sum power of the first time slot p_1+p_2.

A. Power Minimization in the Second Time Slot

Since during the second time slot the achievable rate region

R_{BC} does not depend on p_1 and p_2, in this subsection, we minimize p_3 under the condition that the rate pair $[R_{1,th}, R_{2,th}]$ is achievable in the second time slot. In order to achieve the rate pair $[R_{1,th}, R_{2,th}]$, the achievable rate region R_{BC} should include the rate pair $[R_{1,th}, R_{2,th}]$. As shown in Fig. 3, the achievable rate region of the second time slot is a square. It means that the data rates R_1 and R_2 of the second time slot depend only on the power p_3. Therefore we can reduce p_3 until $[R_{1,th}, R_{2,th}]$ is located on the boundary of R_{BC} and there exists a unique p_3^* that locates $[R_{1,th}, R_{2,th}]$ on the boundary. If $R_{1,th} \geq R_{2,th}$, then $[R_{1,th}, R_{2,th}]$ is located on the right side of the square in Fig. 3, and otherwise $[R_{1,th}, R_{2,th}]$ is located on the top side of the square. Hence, the optimal value for p_3 becomes

$$p_3^* = \frac{2^{\max(2R_{1,th}, 2R_{2,th})} - 1}{\min(|h_1|^2, |h_2|^2)} \tag{11}$$

From the above solution, it can be observed that the optimal power p_3^* locates $[R_{1,th}, R_{2,th}]$ on the boundary.

B. Power Minimization in the First Time Slot

In this subsection, we minimize $p_1 + p_2$ under the condition that the rate pair $[R_{1,th}, R_{2,th}]$ is achievable in the first time slot. Since the capacity region C_{MAC} of the first time slot is a pentagon as shown in Fig. 2, it is not possible to minimize $p_1 + p_2$ as we did in the second time slot. The data rate R_1 of the first time slot depends on p_1 and p_2, and the data rate R_2 of the first time slot also depends on p_1 and p_2. We can optimize p_1 and p_2 by the following two lemmas.

Lemma 1: If the required rate pair $[R_{1,th}, R_{2,th}]$ is not located on the boundary of the capacity region C_{MAC}, then the power allocation in the first time slot is not optimal.

Proof: In order to achieve the rate $[R_{1,th}, R_{2,th}]$, the capacity region C_{MAC} should include the rate pair. Then, the rate pair is located in the interior of C_{MAC} or on the boundary of C_{MAC}. If a power allocation locates the rate pair $[R_{1,th}, R_{2,th}]$ in the interior of C_{MAC}, we can reduce p_1 or p_2 until $[R_{1,th}, R_{2,th}]$ located on the boundary of C_{MAC}. Therefore the power allocation that locates $[R_{1,th}, R_{2,th}]$ in the interior of C_{MAC} is not optimal. ■

We can now consider only the power allocations that locate $[R_{1,th}, R_{2,th}]$ on the boundary of C_{MAC}. As shown in Fig. 2, The boundary is composed of the line segments $\overline{AR_{2R}}$, $\overline{BR_{1R}}$ and $\overline{AB}$. Lemma 2 shows that the optimal power allocation locates $[R_{1,th}, R_{2,th}]$ on the line segment $\overline{AB}$.

Lemma 2: If the required rate pair $[R_{1,th}, R_{2,th}]$ is not located on the line segment $\overline{AB}$ of the capacity region C_{MAC}, then the power allocation in the first time slot is not optimal.

Proof: First, if a power allocation locates the rate pair $[R_{1,th}, R_{2,th}]$ on the line segment $\overline{AR_{2R}}$, we can reduce p_1 until $[R_{1,th}, R_{2,th}]$ is located on the point A. Second, if a power allocation locates the rate pair $[R_{1,th}, R_{2,th}]$ on the line segment $\overline{BR_{1R}}$, we can reduce p_2 until $[R_{1,th}, R_{2,th}]$ is located on the point B. Therefore the optimal power allocation locates $[R_{1,th}, R_{2,th}]$ on the line segment $\overline{AB}$. ■

By the Lemma 1 and Lemma 2, we can consider only the power allocations that locate $[R_{1,th}, R_{2,th}]$ on the line segment $\overline{AB}$. Then, we can easily obtain the set of the power allocations by substituting $[R_{1,th}, R_{2,th}]$ into (2):

$$P_{BC} := \{(p_1, p_2): \frac{2^{2R_{1,th}} - 1}{|h_1|^2} \leq p_1 \leq \frac{2^{2R_{1,th}+2R_{2,th}} - 2^{2R_{2,th}}}{|h_1|^2}, p_2 = \frac{2^{2R_{1,th}+2R_{2,th}} - p_1|h_1|^2 - 1}{|h_2|^2}\} \tag{12}$$

When the channel is symmetric ($|h_1| = |h_2|$), $p_1 + p_2$ becomes:

$$p_1 + p_2 = \frac{2^{2R_{1,th}+2R_{2,th}} - 1}{|h_2|^2} \tag{13}$$

Therefore all the power allocations in P_{BC} are optimal in the symmetric channel case. When the channel is asymmetric ($|h_1| \neq |h_2|$), we assume without loss of generality $|h_2| > |h_1|$. Let us consider the sum power $p_1 + p_2$ in P_{BC}. $p_1 + p_2$ is a function whose domain is P_{BC}. Then, we can obtain the directional derivative of $p_1 + p_2$ in P_{BC}. The directional derivative $D_{\mathbf{a}} f$ of a function f in the direction of a nonzero vector $\mathbf{a}$ is given as [9]:

$$D_{\mathbf{a}} f = \frac{1}{|\mathbf{a}|} \mathbf{a} \bullet grad\, f \tag{14}$$

Where $grad\ f$ is the gradient of f and "$\bullet$" represents inner product. The directional derivative $D_{\mathbf{a}}(p_1 + p_2)$ in the direction of $\mathbf{a} = (-\frac{1}{|h_1|^2}, \frac{1}{|h_2|^2})$ is negative because of the channel assumption ($|h_2|>|h_1|$):

$$D_{\mathbf{a}}(p_1+p_2)=\frac{|h_1|^2-|h_2|^2}{|\mathbf{a}||h_1|^2|h_2|^2}<0 \qquad (15)$$

The set P_{BC} is one-dimensional and the directional derivative $D_{\mathbf{a}}(p_1+p_2)$ is negative. Therefore, for an arbitrary power allocation in P_{BC} we can move the power allocation in the direction of $\mathbf{a}$ until $p_1=\frac{2^{2R_{1,th}}-1}{|h_1|^2}$. Then, the sum power p_1+p_2 always reduces. The optimal powers p_1^* and p_2^* follow.

$$\begin{cases} p_1^*=\dfrac{2^{2R_{1,th}}-1}{|h_1|^2} \\ p_2^*=\dfrac{2^{2R_{1,th}+2R_{2,th}}-2^{2R_{1,th}}}{|h_2|^2} \end{cases} \qquad (16)$$

IV. Simulation Results

In this section, we present the numerical results to compare the power minimization scheme with the three-phase DXF strategy and the one-way DF relay strategy (the four-phase). In the first time slot of the three-phase DXF strategy, one of the two sources transmits its symbol and the other source transmits its symbol in the second time slot. Then, the relay in the third time slot acts as in the second time slot of our system model. The one-way relay strategy needs four time slots for both direction transmissions.

We assume a linear one-dimensional network geometry, where the distance between T_1 and T_2 is normalized to 1, and d is the distance between T_1 and T_3. The two channels have path loss and Rayleigh fading. It is assumed that $h_1 \sim CN(0,1/d^{\alpha})$ and $h_2 \sim CN(0,1/(1-d)^{\alpha})$, where α is the path loss exponent. In our simulation, we take $\alpha=4$, and consider two scenarios in which the sources have different rate requirements. The sum power consumption against relay location, where the required rate pair is $[R_{1,th}, R_{2,th}]=[1, 1]$, is plotted in Fig. 4. The required rate pair is symmetric and the proposed scheme always has lower sum power consumption. The more asymmetric the channels are, the more efficient the proposed scheme is. In other words, when the relay is close to the sources, the power consumption gap is larger than that of the center location. In Fig. 5, the sum power consumption against relay location, where the required rate pair is $[R_{1,th}, R_{2,th}]=[0.5, 1.5]$, is plotted. The required rate pair is asymmetric. The comparison between Fig. 4 and Fig. 5 shows: that the proposed scheme of the asymmetric rate requirements is more efficient.

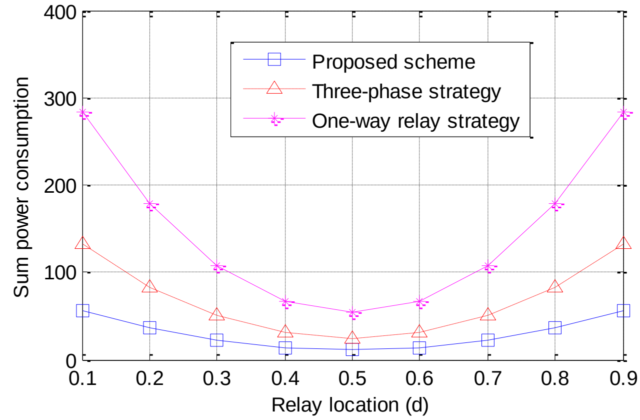

Fig. 4 Sum power consumption against relay location ($[R_{1,th}, R_{2,th}]=[1, 1]$)

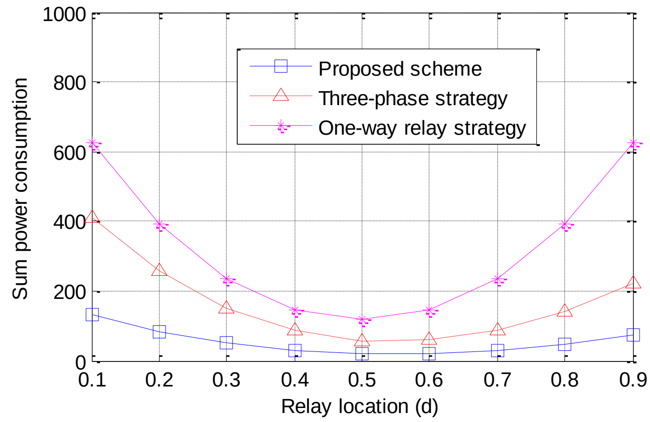

Fig. 5 Sum power consumption against relay location ($[R_{1,th}, R_{2,th}]=[0.5, 1.5]$)

V. Conclusion

In this paper, we proposed the total power minimization scheme for DXF two-way relay networks while satisfying the minimum rate requirements. The simulation results show that the proposed scheme is more efficient when the channels and the minimum rate requirements are asymmetric. The power minimization problem with the outage probability constraint remains for further work.

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