Simulation of Dynamics of a Permanent Magnet Linear Actuator

Ivan Yatchev and Ewen Ritchie

Abstract—Comparison of two approaches for the simulation of the dynamic behaviour of a permanent magnet linear actuator is presented. These are full coupled model, where the electromagnetic field, electric circuit and mechanical motion problems are solved simultaneously, and decoupled model, where first a set of static magnetic filed analysis is carried out and then the electric circuit and mechanical motion equations are solved employing bi-cubic spline approximations of the field analysis results. The results show that the proposed decoupled model is of satisfactory accuracy and gives more flexibility when the actuator response is required to be estimated for different external conditions, e.g. external circuit parameters or mechanical loads.

Keywords—Coupled problems, dynamic models, finite element analysis, linear actuators, permanent magnets.

I. INTRODUCTION

IN recent years dynamic simulation of electromagnetic actuators has been a subject of continued interest to researchers. The variety of actuator constructions and their loads is a factor that stimulates this interest. Dynamic simulation requires the solution of a coupled problem consisting of electromagnetic field, electric/electronic circuit and mechanical motion problems.

The most widespread method for electromagnetic field modelling of linear actuators is the finite element method in its different formulations [1]-[10].

Two principal approaches are possible to solve the problem – coupled and decoupled. The coupled approach (for example [3]) requires solution of all the problems simultaneously. The decoupled model (e.g. [2]) involves separate solutions of the magnetostatic field problem, where a set of solutions is obtained for a wide range of current and displacement, and of the electric circuit and mechanical motion problems. The main drawback of the decoupled model is that the eddy currents are not taken into account.

Solution of the dynamics for different types of actuators is presented in [4], [5], [7-15].

In previous paper [16], we have presented a coupled model

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of a linear actuator with moving permanent magnet.

In this paper, we propose a decoupled dynamic model that employs bi-cubic spline approximations of the results from the magnetostatic field analysis and compare it with the coupled model.

II. ACTUATOR GEOMETRY

The principal geometry of the actuator is shown in Fig. 1. It is axi-symmetrical and comprises inner and outer core, coil, and moving permanent magnet. The magnet is magnetised radially and moves up or down depending on the direction of the current in the coil.
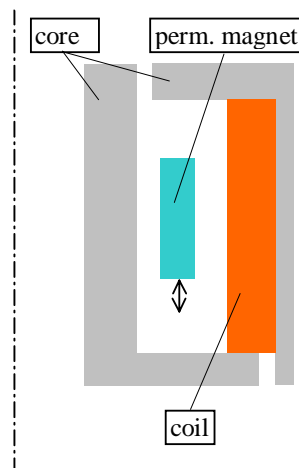

Fig. 1 Principal geometry of the actuator

III. COUPLED MODEL

The coupled model [16] consists of the equations of the above mentioned three problems: The governing equations for the three problems are:

- Electromagnetic field equation:

$$\nabla \times \left(\nu \nabla \times \vec{A}\right) - \vec{J}_e + \sigma \frac{\partial \vec{A}}{\partial t} - \sigma \vec{v} \times \vec{B} - \nabla \times \left(\nu \mu_0 \vec{M}\right) = 0 , \qquad (1)$$

where $\vec{A}$ is the magnetic vector potential having only one nonzero component (A_φ); ν is the reluctivity; $\vec{J}_e$ is the current density of the external sources (i.e. in the coil); σ is the electrical conductivity; $\vec{v}$ is the velocity of the moving part; $\vec{B}$ is the flux density; μ_0 is the magnetic permeability of free space; $\vec{M}$ is the magnetisation vector;

- Electric circuit equation:

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$$u = Ri + \frac{d\Psi}{dt} \; , \qquad (2)$$

where u is the supply voltage; R is the resistance of the coil; Ψ is the flux linkages of the coil; i is the current in the coil; t is time.

- Mechanical motion equation (force balance equation):

$$m\frac{d^2 x}{dt^2} + \beta\frac{dx}{dt} = F_{em} - F_{load} \; , \qquad (3)$$

where m is the mass of the moving part; x is the displacement of the moving part; β is the damping coefficient; F_{em} is the electromagnetic force; F_{load} is the load force. In general, the load force may be of various categories, e.g., mass, spring, hydraulic or pneumatic force. In the case studied we have used a spring force as load.

The approach for the solution to the coupled problem is described in more detail in [16]. It employs the backward Euler method for integration in time, a moving finite element mesh, and the electromagnetic force obtained using Maxwell stress tensor is averaged over several integration paths, similar to the eggshell method.

IV. DECOUPLED MODEL

The idea for the decoupled model was proposed in [2]. Here, we extend this idea by obtaining more results from the magnetostatic finite element analysis and treating them using bi-cubic spline approximations. The essence of decoupling is that the field analysis is separated from the rest of the problem solution.

Instead of (1), the magnetic field is described by the Poissonian type equation, employing the magnetic vector potential. Equations (2) and (3) are solved simultaneously. Bearing in mind that the flux linkages may be presented as $\Psi = Li$, where L is the coil inductance and i is the coil current, (2) is substituted by the form

$$U = Ri + \frac{\partial \Psi}{\partial x}\frac{dx}{dt} + \left(L + i\frac{\partial L}{\partial i} \right)\frac{di}{dt} \qquad (4)$$

After introducing the velocity v as an unknown and reducing the order of the force equation, the final set of equations becomes

$$\frac{di}{dt} = \frac{1}{L + i\frac{\partial L}{\partial i}}\left(U - Ri - \frac{\partial \Psi}{\partial x}v \right) \qquad (5)$$

$$\frac{dx}{dt} = v \qquad (6)$$

$$\frac{dv}{dt} = \frac{1}{m}\left(F_e - \beta v \pm mg \right). \qquad (7)$$

Depending on the actuator position, the last term in (7) may be positive, negative, or zero.

The following functions must be determined to solve the system (5)-(7): $\frac{\partial \Psi}{\partial x}(x,i)$, $L(x,i)$, $\frac{\partial L}{\partial i}(x,i)$, $F_e(x,i)$. The results of these functions were obtained from the finite element analysis of the magnetostatic field.

In house computer code was used for the finite element analysis. In order to obtain the desired functions both current and displacement must be varied. A grid displacement-current was defined by varying the current from zero to a value greater than the steady-state one and the displacement varying between the two end positions of the mover. At each point of this grid two magnetic field analyses were performed. The first was with all material properties data from manufacturers' catalogues. From this analysis, the total flux linkages and the electromagnetic force were obtained. In the second analysis the permanent magnet was replaced by a ferromagnetic body of the same magnetic permeability, i.e. the coercive force of the magnet was set to zero. The result for the coil inductance is obtained from this analysis. All these results were stored in an intermediate data file.

To obtain the desired functions first the data from the intermediate file were used for creating bi-cubic spline approximations of the functions $\Psi(x,i)$, $L(x,i)$ and $F_e(x,i)$. The necessary derivatives $\frac{\partial \Psi}{\partial x}(x,i)$ and $\frac{\partial L}{\partial i}(x,i)$ were obtained using bi-cubic spline approximations. Subsequently, the set of ordinary differential equations, (5)-(7), was solved numerically. Here the Runge-Kutta method of order 4-5, implemented in the *ode45* function of the Matlab® package [17] was used. The approach is illustrated in Fig. 2.
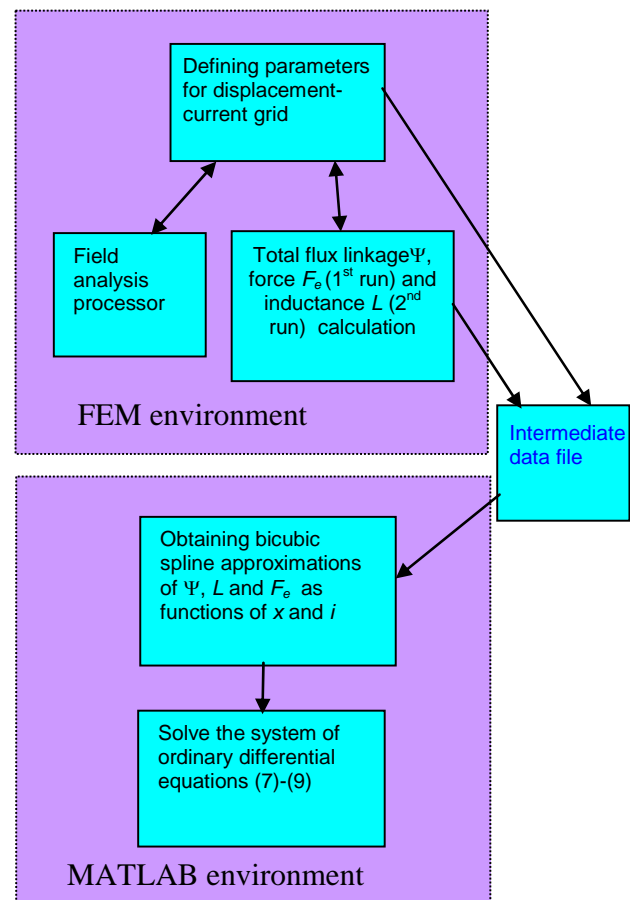

Fig. 2 Flowchart of the decoupled approach

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V. RESULTS

Results were obtained for a linear actuator with ferrite permanent magnet. The direction of magnetisation of the magnet was radial, the remanent flux density was 0.37 T. The ferromagnetic core was massive and eddy currents flowing in it were taken into account in the coupled model. The number of turns of the coil was 5900. The coil resistance is 602 Ω.

The number of finite elements used in simulations was about 9 000. As the accuracy of the force calculation is of major importance for both the static and dynamic cases, a comparison with experiment was performed for the static case. The computed and measured electromagnetic forces are shown in Fig. 3. The mass of the mover is 0.5 kg.
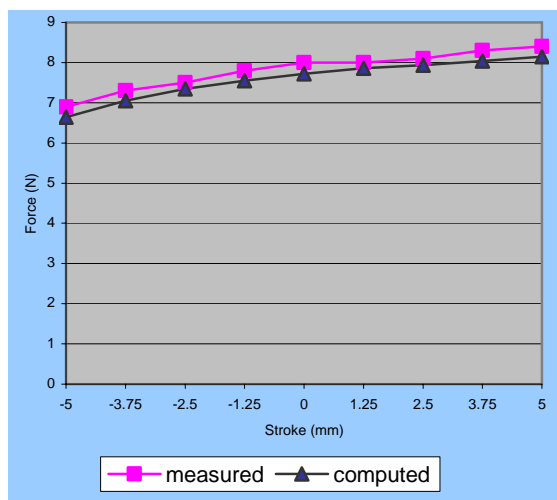

Fig. 3 Measured and computed electromagnetic force for the static case. The value of the current is 105 mA

As seen, the agreement between computed and measured force is satisfactory (the relative error is less than 4%). The maximum working stroke of the actuator is 10 mm (±5 mm from the symmetry position of the permanent magnet).

A typical field plot is shown in Fig. 4 for the static and dynamic cases. It can be seen that the field is pushed out to the surface of the core due to the effect of the eddy currents.

The actuator dynamic response was simulated for a sinusoidal supply voltage. The current in the coil is a solution of the equation system. The load force is a spring exerting zero force at the symmetry position of the mover (permanent magnet) and with a stiffness of 1 N/mm for movement in either direction. In Figs. 5-8, time variations of the voltage, current, electromagnetic force, displacement and velocity of the mover are shown for both the coupled and decoupled models for a frequency of 10 Hz.
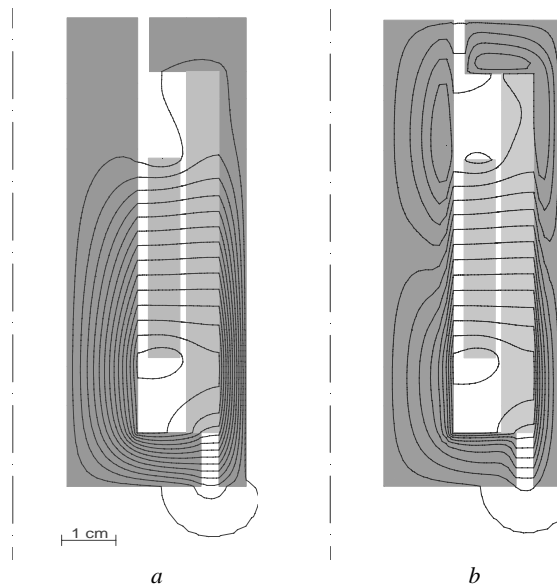

a   *b*
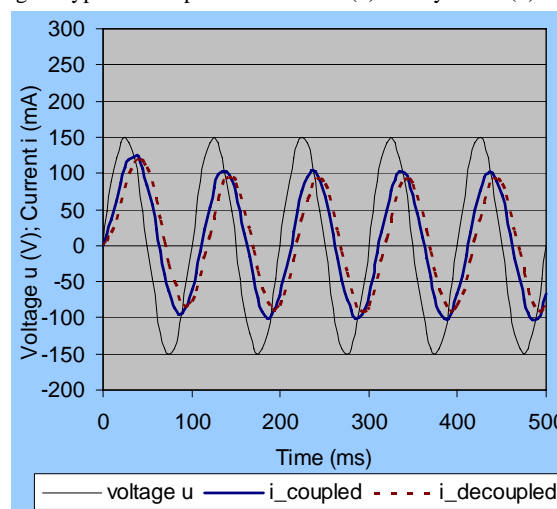Fig. 4 Typical field plot for the static (*a*) and dynamic (*b*) case

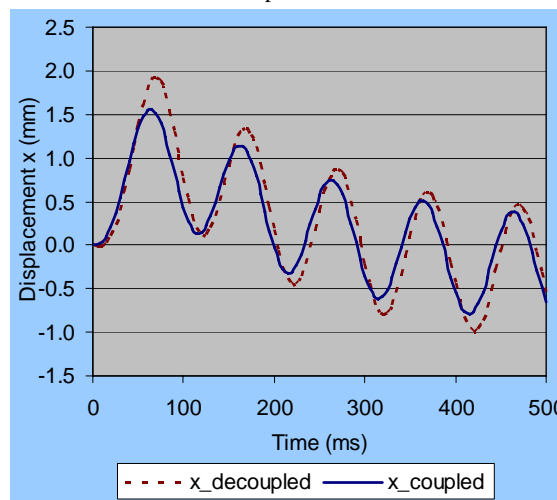Fig. 5 Voltage *u* and current *i* versus time for coupled and decoupled model

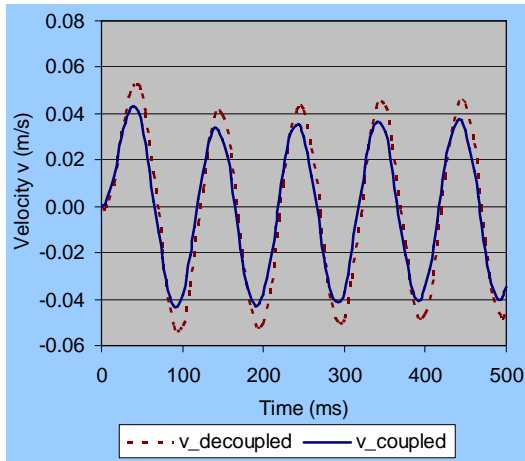Fig. 6 Electromagnetic force *F* versus time for the coupled and decoupled models

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Fig. 7 Displacement x versus time for the coupled and decoupled models
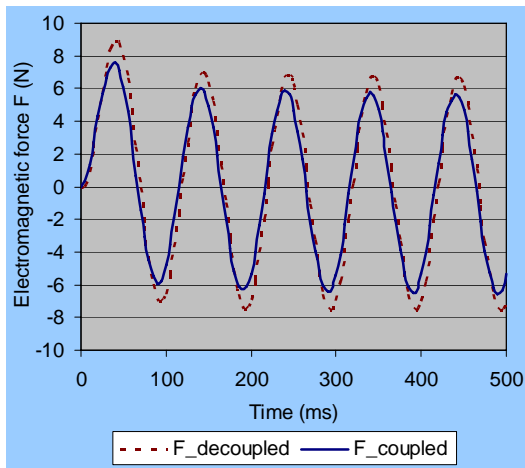
Fig. 8 Velocity v versus time for the coupled and decoupled models

Fig. 9 Current i after the transient process

Fig. 10 Displacement x after the transient process

Fig. 11 Displacement x for 20 Hz frequency

Due to the absence of longitudinal symmetry, oscillations of the mover are not symmetrical around its longitudinal symmetry position. As seen, there is a slight difference between the results obtained using the two models due to the eddy currents not being accounted for in the decoupled model. The decoupled model is much more flexible i.e. easier to change values for the model constants when the actuator response has to be estimated for a number of different external conditions. These may be the parameters of the circuit, of the actuator coil, or the mechanical parameters of the load mechanism. These condition changes will lead to corresponding changes in the constant coefficients of the system of equations (5)-(7) but there will be no need for additional field analysis or bicubic spline approximation.

Results for the current and the displacement after finishing the transient process are shown in Fig. 9 and Fig. 10.

In Fig. 11, the results for the displacement at frequency of 20 Hz are shown. The difference between the amplitudes of the two displacements is less than 10%. Note that such correspondence between results obtained using the two methods may be expected only at low frequencies. For higher frequencies the effects of induced eddy currents will be more significant.
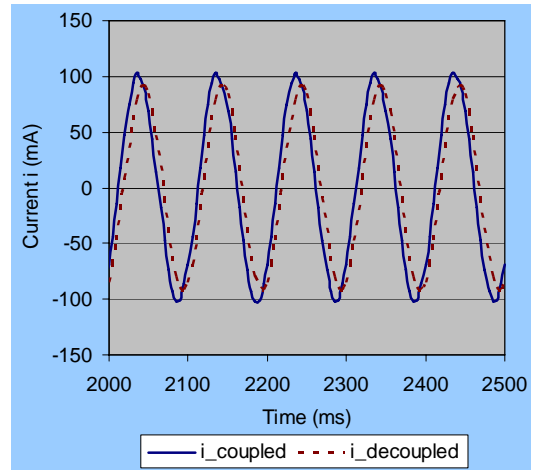
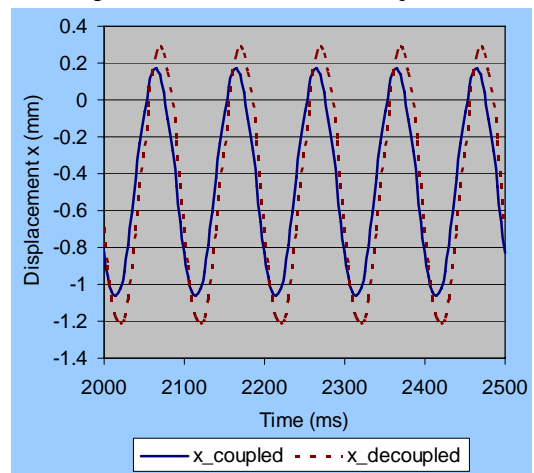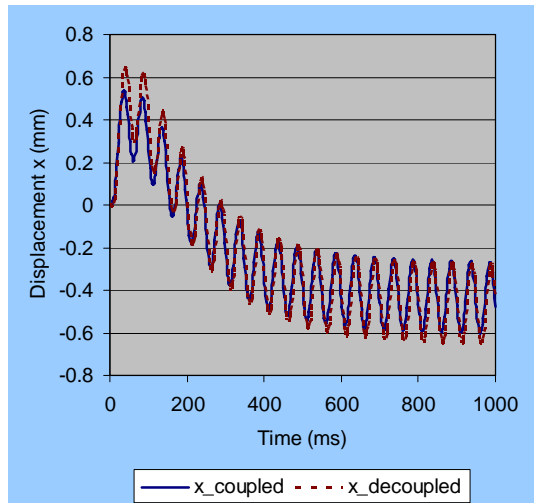It can also be noticed that bigger deflections between the coupled and the decoupled model are observed in the beginning of the transient process. After that the two models give closer results.

VI. CONCLUSION

The presented decoupled dynamic model of a linear actuator with moving permanent magnet employs bicubic spline approximations of the magnetic field analysis results. The decoupled model may be more flexible than the coupled model for study of the actuator behaviour at different external conditions, e.g., different circuit conditions and mechanical parameters of the load mechanism. It is also suitable for modelling of the actuator performance in a larger system. Coupling with additional dynamic equations may be easily implemented.

Further work may include a study of the benefits accrued, and the magnitude of effects, and errors caused by neglecting eddy currents in the decoupled version for wider range of applications.

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Ivan Yatchev (M'94) was born in Sofia, Bulgaria, in 1958. He received his M.Sc. degree in Electrical Engineering and his PhD degree from the Technical University of Sofia in 1983 and 1988, respectively.

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