Conditions on Blind Source Separability of Linear FIR-MIMO Systems with Binary Inputs

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Abstract—In this note, we investigate the blind source separability of linear FIR-MIMO systems. The concept of *semi-reversibility* of a system is presented. It is shown that for a semi-reversible system, if the input signals belong to a binary alphabet, then the source data can be blindly separated. One sufficient condition for a system to be semi-reversible is obtained. It is also shown that the proposed criteria is weaker than that in the literature which requires that the channel matrix is irreducible/invertible or reversible.

Keywords—Blind source separable, FIR-MIMO system, Binary input, Bezout equality.

I. INTRODUCTION

BLIND source separation and blind system identification have recently received much attention in both of the theoretical area and the application field. The main objective of the blind signal separation is to recover the source signals only using the received signals without knowing or knowing a little of the transmitted data and the channels. Due to the advantage of blind source separation, it has wide range of applications, which includes: data communications, speech recognition and reverberation cancellation, image restoration, seismic signal processing and *etc*. The investigation of the blind techniques has attracted much attention since the work pioneered by Jutten, Herault and Ans in 1985([1]). Among the research work finished during the last two decades, the main methods can be roughly categorized into two groups: the method based on high-order statistics (HOS) and that based on the second-order statistics (SOS).

HOS based methods were proposed in 1980's and three subgroups have been developed which are: hidden Markov model-based methods, polyspectra methods and Bussgang methods. We refer the readers to see the tutorial paper by Mendel in 1991([2]) and a survey paper by Abed-meraim *el al.* in 1997([3]) and references therein for more details. One disadvantage of these approaches is that in order to obtain certain performance much more data are needed, the other drawback is that the noise can only be an additive one.

SOS based methods began from the work by Tong *et al.* in 1991([4] and [5]). Since then, there are many algorithms have been proposed, such as, subspace based algorithm (SSA), linear prediction algorithm (LPA), outer-product decomposition algorithm (OPDA) and *etc*. We refer the readers to see [3] and [6] for more details. The most benefit of these approaches is that not only the requirement that the noise has to be an

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additive one in the HOS based methods has been removed, but also fewer data can give better performance. However, since the implementation of these algorithms is based on the oversampling at the received terminal. The multiple channels can not share any common zeros.

One may notice that there is one identical characteristic of both the HOS based methods and the SOS based ones. The common point is that both of these two methods are based on two steps to obtain the final result. The first step is to identify the channels and the second one is to design a filter to detect the source signals. To our knowledge, the fundamental theorem for the identifiability and/or equalibility of a channel is the Bezout equality, which is a famous formula in the linear system theory. In the systematic theory, since no assumption is made on the input data, in order to estimate the source signals, more restrictions have to be imposed on the system channels. For example, the channel is always assumed to be irreducible. This assumption means that all source signals sent by different users have to be received simultaneously by the receival antenna array. It is evidently that this request is a strong restriction and it limits the applications of the algorithm in the areas such as mobile communications.

Recently, a system-theoretic foundation for the blind equalization of an FIR MIMO channels system has been obtain in [7], see also the review paper [8], in which the condition for a channel to be identifiable is relaxed from irreducible to reversible. However, the obtained results still relies on the generalized Bezout equality.

The main goal of signal processing is to recover the transmitted signals. In a practical communication system, the source signals always belong to a finite alphabet. As a matter of fact, the finiteness of the source alphabet can provide some extra information which can be used to investigate the system (see, for example, [9]). In this paper, by taking into account the finite alphabet property of the transmitted signals, we proposed a new identifiable condition for a linear FIR-MIMO communication system. It is shown that the proposed condition is weaker than that in the literature which requires that the channel matrix is irreducible/invertible or reversible. The rest of the paper is organized as follows. In section II, we give some preliminaries. In section III, we give the main results and its proofs. Some concluding remarks and possible future work have been addressed in the final section IV.

II. PRELIMINARIES

Let Q and K be two positive integers, we denote $C[z]^{Q \times K}$ the set of all $Q \times K$ matrices with each entry being a

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polynomial in z with complex coefficients in C. We assume that z is a delayed operator, which means

$$z^\ell x(t) = x(t - \ell) \qquad \text{for} \qquad \ell \geq 0.$$

In this case, each polynomial is a finite impulse response (FIR) filter.

Let $H(z) \in C[z]^{Q \times K}$ be a FIR system represented by

$$\{H_k \in C^{Q \times K} | k = 0, \cdots, M\}$$

and

$$H(z) = \sum_{k=0}^{M} H_k z^k.$$

Then a linear FIR MIMO channel-filter system can be written as

$$X(k) = H(z)S(k) + \mathcal{N}(k), \qquad k \in \mathbb{Z},$$

where we understand the vector $S(k)$ as a K-dimensional source signal; the vector $X(k)$ a Q-dimensional channel output signal; the vector $\mathcal{N}(k)$ a Q-dimensional noises. $Q \geq K$ is always supposed to be satisfied.

In the above model, if we take $K = 1$, then the corresponding system is called a SIMO system, and if we take $K = 1$ and $Q = 1$, the corresponding system is called a SISO system.

Without loss of generality, in the procedure of the analysis carried out in the following, we remove the effect of the noise. In this case, the above model can be rewrited as the following

$$\begin{pmatrix} x_1(k) \\ \vdots \\ x_Q(k) \end{pmatrix} = \begin{pmatrix} H_{11}(z) & \cdots & H_{1K}(z) \\ \vdots & \ddots & \vdots \\ H_{Q1}(z) & \cdots & H_{QK}(z) \end{pmatrix} \begin{pmatrix} s_1(k) \\ \vdots \\ s_K(k) \end{pmatrix}. \quad (1)$$

In the related references in the literature, the transition matrix, $H(z)$, is always supposed to be irreducible, which means that the rank of $H(\lambda)$ is K for any $\lambda \in C$. It is easy to see that if $\lambda = 0$, then $H(0)$ should be full column rank. Since $H(0)$ is the leading coefficient matrix of $H(z)$, it implies that all signals transmitted from different users have to arrive at the destination antenna array at the same time, which is very restrictive, especially, for mobile communications.

We should note that the reason to request the transition matrix to be irreducible is that if $H(z)$ is irreducible, then it implies from the Bezout equality that there exists an irreducible FIR filter $W(z) \in C^{Q \times K}[z]$ such that

$$W^T(z)H(z) = \mathbf{I},$$

from which a filter can be designed by certain methods to estimate the source signals.

In order to extend the application area limited by the condition of irreducibility imposed on the transition matrix, a relative weaker condition has been given in [7]. For the sake of giving the statement of the result, let $\Lambda(z)$ be a diagonal matrix with monic monomial diagonal elements. A square matrix $G(z) \in C[z]^{K \times K}$ is said to be *transparent* if it has a decomposition as the following

$$G(z) = PD\Lambda(z),$$

where P is a permutation matrix and D a regular diagonal constant matrix. It is easy to see that if two signals are related

by a transparent matrix, then they are essentially the same except for a permutation ambiguity, an amplitude ambiguity and a delay ambiguity. A matrix $H(z) \in C[z]^{Q \times K}$ is said to be *reversible* if there exists $W(z) \in C[z]^{Q \times K}$ such that the product

$$G(z) := W^T(z)H(z)$$

is transparent. It is proved, in [7] (see also [8]), that if a transition channel $H(z)$ is reversible, then the channel can be blindly equalized using certain methods. The characterization of such kind of channel has been obtained in those papers.

In this paper, by using the finite property of the alphabet from which the source signals are taken, we will give a weaker condition on a channel such that the source signals can be separated.

III. MAIN RESULTS

Before giving the description of the main results, let's give a particular case to show the idea. For the sake, we introduce the following definition.

Definition 1: A polynomial

$$A(z) = a_0 + a_1 z + \cdots + a_n z^n$$

is called an *item dominant polynomial*, if there exists an $0 \leq i \leq n$ such that

$$|a_i| > \sum_{j=0, j \neq i}^{n} |a_j|.$$

Consider a FIR-SISO communication system, the order of the transition channel, $H(z)$, is M. If there exists a FIR filter $W(z)$ of order L such that the product,

$$\begin{aligned} G(z) &= W(z)H(z) \\ &= g_0 + g_1 z + \cdots + g_{M+L} z^{M+L}, \end{aligned}$$

is an item dominant polynomial, then there exists an $0 \leq i \leq M + L$ such that

$$\begin{aligned} & W(z)X(k) \\ &= W(z)H(z)s(k) \\ &= G(z)s(k) \\ &= g_0 s(k) + g_1 s(k-1) + \cdots g_{M+L} s(K-M-L) \\ &= g_i s(k-i) \left(1 + \frac{1}{g_i} \sum_{j=0, j \neq i}^{M+L} g_j \frac{s(k-j)}{s(k-i)} \right). \end{aligned}$$

If the input signals are taken from a binary set $\{\pm 1\}$, then it is easy to see that

$$\left| \sum_{j=0, j \neq i}^{M+L} g_j \frac{s(k-j)}{s(k-i)} \right| \leq \sum_{j=0, j \neq i}^{M+L} |g_j| < |g_i|,$$

which implies that

$$\begin{aligned} & \hat{s}(k) \\ &= \text{sgn}(W(z)X(k)) \\ &= \text{sgn}\left(g_i s(k-i) \left(1 + \frac{1}{g_i} \sum_{j=0, j \neq i}^{M+L} g_j \frac{s(k-j)}{s(k-i)} \right) \right) \\ &= \text{sgn}(g_i s(k-i)) \\ &= \text{sgn}(g_i) s(k-i). \end{aligned}$$

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This means that the source signals can be separated successfully. The idea has been used to investigate the case of a SIMO channel in [10]. In this paper, we will consider the case of a MIMO system. In order to apply the above idea to this situation, we need to introduce some more notation stated as the following.

Definition 2: Let $B(z) = (b_{ij}(z)) \in C[z]^{Q \times K}$ be a polynomial matrix of order M, an entry

$$b_{ij_0}(z) = \sum_{k=0}^{M} b_{ij}^k z^k$$

is said to be a dominant element in the i-th row , if there exists a $0 \le k_0 \le M$ such that

$$|b_{ij_0}^{k_0}| > \sum_{k=0, k \neq k_0}^{M} |b_{ij_0}^k| + \sum_{j=1, j \neq j_0}^{K} \sum_{k=0}^{M} |b_{ij}^k|.$$

$B(z)$ is called an *entry-dominant polynomial matrix* if there is only one dominant element in each row and in each column.

For the convenience in the rest of the paper, we denote $\Xi[z]$ the set of all entry-dominant polynomial square matrixes.

Definition 3: A transition matrix $H(z) \in C[z]^{Q \times K}$ is said to be *semi-reversible* if there exists $W(z) \in C[z]^{Q \times K}$ such that the product

$$W^T(z)H(z) \in \Xi[z].$$

Here are our main results.

Theorem 4: Consider the FIR-MIMO channel system in (1), and the input alphabet is assumed to be $\{\pm 1\}$, if the channel $H(z)$ is semi-reversible, then the source signals can be separated uniquely up to a permutation ambiguity, an amplitude ambiguity and a delay ambiguity.

Proof: Since $H(z)$ is semi-reversible, it implies from Definition 3 that there exists $W(z)$ such that $W^T(z)H(z) \in \Xi[z]$. Denote the global system by

$$G(z) = W^T(z)H(z)$$
$$= (g_{ij}(z))_{K \times K}.$$

According to the definition of the set $\Xi[z]$, we know that there is only one entry-dominant polynomial in each row and in each column.

Consider the K-dimensional vector

$$Y(k) = W^T(z)X(k)$$
$$= W^T(z)H(z)S(k)$$
$$= G(z)S(k).$$

For each $1 \le i \le K$, we assume that the dominant entry in the i-th row is

$$g_{ij}(z) = g_{ij}^0 + g_{ij}^1 z + \cdots + g_{ij}^M z^M$$

and the dominant item of which is g_{ij}^d, where $0 \le d \le M$. It's easy to see that,

$$y_i(k)$$
$$= g_{i1}(z)s_1(k) + \cdots + g_{iK}(z)s_K(k)$$
$$= \sum_{m=0}^{M} g_{i1}^m s_1(k-m) + \cdots + \sum_{m=0}^{M} g_{iK}^m s_K(k-m)$$
$$= g_{ij}^d s_j(k-d) \left(1 + \frac{1}{g_{ij}^d} \left(\sum_{m=0, m \neq d}^{M} g_{ij}^m \frac{s_j(k-m)}{s_j(k-d)} \right. \right.$$
$$\left. \left. + \sum_{n=0, n \neq j}^{K} \sum_{m=0,}^{M} g_{in}^m \frac{s_n(k-m)}{s_j(k-d)} \right) \right)$$

It implies from the definition of the entry-dominant matrix that

$$\left| \sum_{m=0, m \neq d}^{M} g_{ij}^m \frac{s_j(k-m)}{s_j(k-d)} + \sum_{n=0, n \neq j}^{K} \sum_{m=0,}^{M} g_{in}^m \frac{s_n(k-m)}{s_j(k-d)} \right|$$
$$< |g_{ij}^d|,$$

which implies that

$$\hat{y}_i(k) = \text{sgn}(y_i(k))$$
$$= \text{sgn}(g_{ij}^d s_j(k-d))$$
$$= \text{sgn}(g_{ij}^d) s_j(k-d).$$

This completes the proof. ∎

The following theorem gives a sufficient condition for a channel to be semi-reversible.

Theorem 5: If the channel $H(z)$ can be decomposed as

$$H(z) = H_I(z)B(z),$$

where $H_I(z)$ is a $Q \times K$ irreducible matrix and $B(z) \in \Xi[z]$, then $H(z)$ is semi-reversible.

Proof: Since $H_I(z)$ is an irreducible matrix, according to Theorem 1 in [8], we know that there exists an irreducible matrix $W(z)$ such that

$$W^T(z)H_I(z) = \mathbf{I}.$$

Therefore,

$$W^T(z)H(z) = W^T(z)H_I(z)B(z)$$
$$= B(z) \in \Xi[z],$$

which means that $H(z)$ is semi-reversible. ∎

Let $\Theta(z)$ be a diagonal matrix with each entry being an item dominant polynomial. we have that,

Corollary 6: If the channel can be decomposed as

$$H(z) = H_I(z)P\Theta(z),$$

where P is a $K \times K$ permutation matrix, then $H(z)$ is semi-reversible.

We note that the channel considered in [10] is a special case of Corollary 6, which can be stated as

Corollary 7: For a SIMO channel, if

$$H(z) = H_I(z)A(z),$$

where $A(z)$ is an item dominant polynomial, then $H(z)$ is semi-reversible, moreover, the zeros of $A(z)$ are the common zeros of all the sub-channels.

Let's give a remark to end this section.

Remark 8: Consider a FIR-SIMO channel system, let $H(z)$ be the transition matrix. If $H(z)$ is a reversible matrix, then it implies from [8] that

$$H(z) = H_I(z)z^{\ell}$$

for some $\ell \geq 0$, which is a special case of Corollary 7. This means that the concept of semi-reversible is a significant extension of that of reversible.

IV. CONCLUSION

In this paper, we consider the blind source separation of a FIR-MIMO channel, under the condition that the input signals belong to a binary set $\{\pm 1\}$, we give a weak condition, under which, the source signals can be blindly estimated. The new developed *semi-reversible* includes *irreducible* and *reversible* as its special cases. Moreover, in the case of *semi-reversible*, different channels can share common zeros.

In a practical situation, an interesting question is to give a design of the filter to blindly recover the source signals of a FIR-MIMO communication system. For the case of a SIMO system, some work has been done in [10]. However, much more work does not possess a positive answer.

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