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NonStationary CMA for Decision Feedback Equalization of Markovian Time Varying Channels

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Abstract— In this paper, we propose a modified version of the Constant Modulus Algorithm (CMA) tailored for blind Decision Feedback Equalizer (DFE) of first order Markovian time varying channels. The proposed NonStationary CMA (NSCMA) is designed so that it explicitly takes into account the Markovian structure of the channel nonstationarity. Hence, unlike the classical CMA, the NSCMA is not blind with respect to the channel time variations. This greatly helps the equalizer in the case of realistic channels, and avoids frequent transmissions of training sequences.

This paper develops a theoretical analysis of the steady state performance of the CMA and the NSCMA for DFEs within a time varying context. Therefore, approximate expressions of the mean square errors are derived. We prove that in the steady state, the NSCMA exhibits better performance than the classical CMA. These new results are confirmed by simulation.

Through an experimental study, we demonstrate that the Bit Error Rate (BER) is reduced by the NSCMA-DFE, and the improvement of the BER achieved by the NSCMA-DFE is as significant as the channel time variations are severe.

Keywords— Time varying channel, Markov model, Blind DFE, CMA, NSCMA.

I. Introduction

In wireless communications the channel time variations often conflict with the equalizer, and, as a result, impair communication efficiency. Classical blind equalizers designed for stationary channels are not robust with respect to the channel time variations. We present in this paper a solution to the particular problem of radio-mobile transmission. In this context, the channel time variations are often modeled by a first order Markov process [1], [2]. Furthermore, we are interested in Decision Feedback Equalizers (DFEs) that are preferred to transverse equalizers in the case of severe multi-path time varying channels [3]. Precisely, only the Feedback FIR filter is considered. As a matter of fact, we refer to the CMA tailored for a DFE (CMA-DFE) as proposed in [4], [5]. To guarantee better performance of the blind DFE, we propose a new adaptive algorithm that can identify the Markovian time variations of the actual channel impulse response. The proposed NonStationary CMA (NSCMA), which is based on the classical CMA [6], is designed to take into account the prior knowledge on the stochastic nonstationarity model of the channel to equalize. The adaptive identification of the unknown Markovian parameter of the nonstationarity model is carried out in a blind mode without using any training sequences. The update of the Markovian parameter is performed by the Recursive Least Square (RLS) algorithm chosen for its good convergence rate in presence of correlated inputs as it is the case here. The proposed approach is different from those presented in the literature [7], [8] in order to improve the performance of equalizers operating in fading propagation context and in semi blind or blind mode.

In this paper, we develop a theoretical analysis of the steady state performance of the CMA and the NSCMA for DFEs within a time varying context. Therefore, approximate expressions of the mean square errors are derived. These new results that are confirmed by simulation, have not been dealt with previously in the literature. However, in [9], [10], similar theoretical analysis are developed in the case of transversal CMA equalizer for a time constant channel.

In addition, an extensive experimental study of the NSCMA-DFE performance, under several realistic propagation conditions, is carried out. The results reported here, demonstrate the ability of the NSCMA to identify the actual parameter of the Markovian model of the channel time variations. Hence, it is shown that the NSCMA exhibits better tracking ability than the classical CMA. Moreover, the superiority of the NSCMA blind DFE over the CMA blind DFE becomes even more significant when the propagation conditions are severe.

This paper is organized as follows. Section II presents a formulation of the studied equalization problem. The design of the proposed NSCMA is presented in Section III. The analysis of the steady state performance of the NSCMA-DFE is presented in Section IV. Section V, presents the experimental study of the NSCMA performance.

II. Problem Formulation

A more typical and indeed more general DFE structure usually consists of a Feedforward FIR filter followed by a Feedback FIR filter. However, in this paper, only the Feedback FIR filter is considered for two reasons. Firstly, in the more general structure it is usually possible to separate the adaptation of the FIR section from the feedback one. Secondly, the aim of this paper is to highlight the good properties of the new proposed algorithm dedicated to the adaptation of the feedback section. The classical formulation of the studied DFE equalization problem is illustrated by Fig. 1.

The noisy received signal at the output of the channel is

$$y_n = x_n + \eta_n = a_n + \Xi_n^T A_n + \eta_n, \qquad (1)$$

where x_n is the noiseless channel output of power $P_x = E(x_n^2)$, $\Xi_n = \left(\xi_n^1, \xi_n^2, \ldots, \xi_n^N\right)^T$ is the time varying channel

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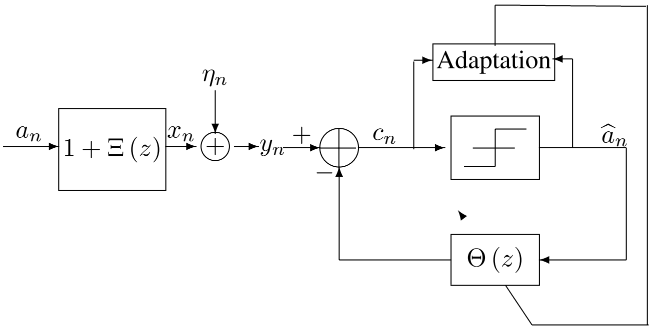

Fig. 1. Functional block diagram of the blind DFE

linear filter of length N, $A_n = (a_{n-1}, a_{n-2}, \ldots, a_{n-N})^T$ is the input data vector, and η_n is an additive white noise assumed to be Gaussian with zero mean value and independent of A_n. The noise power, $P_\eta = E(|\eta_n|^2)$, is fixed through the Signal to Noise Ratio,

$$\text{SNR(dB)} \stackrel{\triangle}{=} P_x(\text{dB}) - P_\eta(\text{dB}). \tag{2}$$

This equation shows the presence of a direct path in the transmission channel.

The input of the decision device is given by

$$c_n = y_n - \Theta_n^T \widehat{A}_n, \tag{3}$$

where $\Theta_n = \left(\theta_n^1, \theta_n^2, \ldots, \theta_n^N\right)^T$ is the adaptive feedback filter of length N that represents the equalizer impulse response, and $\widehat{A}_n = (\widehat{a}_{n-1}, \widehat{a}_{n-2}, \ldots, \widehat{a}_{n-N})^T$ the adaptive equalizer tap-input with $\widehat{a}_n$ being the estimation of the transmitted symbol a_n of power $P_a = E(|a_n|^2)$.

The time variations of the channel filter, Ξ_n, are represented by a first order Markovian model

$$\Xi_n = \alpha \Xi_{n-1} + \sqrt{1-\alpha^2}\,\Omega_n, \tag{4}$$

where $0 \leq \alpha < 1$ and $\Omega_n = \left(\omega_n^1, \omega_n^2, \ldots, \omega_n^N\right)^T$ is the nonstationarity noise vector. The noise components ω_n^i are assumed to be Gaussian, zero mean value, statistically independent and characterized by the power $P_\omega = E\left\{\left|\omega_n^i\right|^2\right\}\Big|_{i=1,\cdots,N}$. The power of the nonstationarity noise vector, $P_\Omega = E(\Omega_n^H \Omega_n) = NP_\omega$, is fixed by the Signal to Interference Ratio,

$$\text{SIR(dB)} \stackrel{\triangle}{=} P_a(\text{dB}) - E\left\{\left|\Xi_n^T A_n\right|^2\right\}(\text{dB}) = -P_\Omega(\text{dB}). \tag{5}$$

As a matter of fact, the Markov model is a widely used approximation model in radio mobile communication [2]. Moreover, if the channel components are assumed to fade independently following the same statistical model, the channel nonstationarity can be modeled, without loss of generality, by a first order Markov model [1]. The value of the Markovian parameter α is fixed through the value of parameter $f_d T_s$ according to the following relation, $\alpha = J_0(2\pi f_d T_s)$, where f_d is the maximum Doppler frequency, T_s is the sampling period of the digital communication system, and J_0 is the Bessel function of first kind of order zero.

III. Design of the NonStationary CMA

We propose in this paper to design an adaptive algorithm that takes into account the prior knowledge of the nonstationarity model structure (4). Hence, we keep the structure of the classical CMA 2-2 and include the constraints on the nature of the nonstationarity as follows

$$\Theta_{n+1} = \beta\Theta_n + \mu\epsilon_n^G \widehat{A}_n^*, \tag{6}$$

where μ is the step size that controls the adaptation of Θ_n, β is an estimate of the unknown Markovian parameter α, and ϵ_n^G is the error signal described by

$$\epsilon_n^G = c_n\left(|c_n|^2 - R_2\right), \quad R_2 = \frac{E\left\{|a_n|^4\right\}}{E\left\{|a_n|^2\right\}}. \tag{7}$$

The estimation of the Markovian parameter at every instant n, is performed by the RLS algorithm. It is given by the minimization of the cost function $J_n(\beta) = \sum_{i=1}^{n} |e_i|^2$, where

$$e_i = c_i - \widehat{a}_i, \tag{8}$$

is the instantaneous error relating to the DFE output. Therefore, the estimate at time n, β_n, is the solution of

$$\frac{\partial}{\partial\beta}J(\beta) = 2e_{nR}\frac{\partial e_{nR}}{\partial\beta} + 2e_{nI}\frac{\partial e_{nI}}{\partial\beta} = 0, \tag{9}$$

where $e_{nR} = \text{Re}\{e_n\}$ and $e_{nI} = \text{Im}\{e_n\}$. The partial derivative of e_{nR} with respect to β is given by

$$\begin{aligned}\frac{\partial}{\partial\beta}e_{nR} &= \frac{\partial}{\partial\beta}\left(\frac{e_n + e_n^*}{2}\right)\\ &= \frac{1}{2}\frac{\partial}{\partial\beta}\left[y_n - \Theta_n^T\widehat{A}_n - \widehat{a}_n + y_n^* - \Theta_n^H\widehat{A}_n^* - \widehat{a}_n^*\right]\\ &= -\text{Re}\left\{\Theta_{n-1}^T\widehat{A}_n\right\} - \beta\frac{\partial}{\partial\beta}\text{Re}\left\{\Theta_{n-1}^T\widehat{A}_n\right\} \\ &\quad - \mu\frac{\partial}{\partial\beta}\text{Re}\left\{\epsilon_{n-1}^G\widehat{A}_{n-1}^H\widehat{A}_n\right\},\end{aligned} \tag{10}$$

whose complexity is due to the recursive nature of the DFE. In fact, an infinite memory is needed to calculate the two last terms of (10). Basing on classical approximations, one can write that

$$\frac{\partial}{\partial\beta}e_{nR} \simeq -\text{Re}\left\{\Theta_{n-1}^T\widehat{A}_n\right\}. \tag{11}$$

In the same manner, one obtain

$$\frac{\partial}{\partial\beta}e_{nI} = -\text{Im}\left\{\Theta_{n-1}^T\widehat{A}_n\right\}. \tag{12}$$

Finally, in view of (11) and (12) and by replacing e_{nR} and e_{nI} by their expressions as functions of e_n and e_n^* in (9), we can write that

$$\begin{aligned}\frac{\partial}{\partial\beta}J(\beta)\Big|_{\beta=\beta_n} &= -2\sum_{i=1}^{n}\text{Re}\left\{e_i\Theta_{i-1}^H\widehat{A}_i^*\right\}\\ &= -2\textstyle\sum_{i=1}^{n}\text{Re}\left\{\begin{array}{c}\left(\begin{array}{c}y_i - \widehat{a}_i\\ -\mu\epsilon_{i-1}^G\widehat{A}_{i-1}^H\widehat{A}_i\end{array}\right)\\ \Theta_{i-1}^H\widehat{A}_i^*\end{array}\right\}\\ &\quad +2\beta_n\textstyle\sum_{i=1}^{n}\left|\Theta_{i-1}^T\widehat{A}_i\right|^2.\end{aligned} \tag{13}$$

Thus, it follows from (9) and (13) that

$$\beta_n = \frac{\sum_{i=1}^{n} \mathrm{Re}\left\{ \left(y_i - \mu\epsilon_{i-1}^G \widehat{A}_{i-1}^H \widehat{A}_i - \widehat{a}_i \right) \Theta_{i-1}^H \widehat{A}_i^* \right\}}{\sum_{i=1}^{n} \left| \Theta_{i-1}^T \widehat{A}_i \right|^2}. \quad (14)$$

The NSCMA is then described by the following equations where a recursive implementation of β_n is deduced from the optimal solution given by (14):

$$\Theta_{n+1} = \beta_n \Theta_n + \mu\epsilon_n^G \widehat{A}_n^*, \quad (15)$$

$$\beta_n = \frac{\mathrm{num}_n}{\mathrm{den}_n}, \quad (16)$$

where

$$\begin{aligned} \mathrm{num}_n &= \mathrm{num}_{n-1} + \mathrm{Re}\left\{ \begin{pmatrix} y_n - \widehat{a}_n \\ -\mu\epsilon_{n-1}^G \widehat{A}_{n-1}^H \widehat{A}_n \end{pmatrix} \Theta_{n-1}^H \widehat{A}_n^* \right\}; \\ & \quad \mathrm{num}_0 = 1, \\ \mathrm{den}_n &= \mathrm{den}_{n-1} + \left| \Theta_{n-1}^T \widehat{A}_n \right|^2 ; \mathrm{den}_0 = 1. \end{aligned} \quad (17)$$

IV. Performance analysis of the NSCMA adaptive filter

The aim here is to study the ability of the NSCMA adaptive filter to track, in the steady state, the time variations of the channel. Given that the adaptation process is nonlinear, the theoretical analysis is carried out under the following considerations:

- The channel is noiseless.
- The equalizer performs in a training mode; therefore, $\widehat{A}_n = A_n$.
- The adaptive parameter β_n has already converged to a constant mean value denoted by $\beta = \lim_{n\to\infty} E\{\beta_n\}$. The time variations of the NSCMA adaptive filter vector are then described by,

$$\Theta_{n+1} = \beta\Theta_n + \mu\epsilon_n^G A_n^*. \quad (18)$$

The steady state performance of the NSCMA adaptive filter is measured by its mean square error (MSE), which is defined as

$$\mathrm{MSE} = \lim_{n\longrightarrow\infty} E\left\{ |e_n|^2 \right\}. \quad (19)$$

If $V_n = \Xi_n - \Theta_n$ denotes the deviation vector, then the instantaneous error (8) is rewritten as

$$e_n = y_n - \Theta_n^T A_n - a_n = V_n^T A_n. \quad (20)$$

By combining (18) and (4) one can show that

$$V_{n+1} = \beta V_n + (\alpha - \beta)\,\Xi_n - \mu\epsilon_n^G A_n^* + \sqrt{1-\alpha^2}\,\Omega_{n+1}. \quad (21)$$

Assumption 3.1 The transmitted symbols a_n are assumed i.i.d. and zero mean valued. Moreover, for complex valued data $E\left\{a_n^2\right\} = 0$ is assumed, which is most often verified by modulation scatter diagrams.

Assumption 3.2 In steady state, $|e_n|^2$ is reasonably small, therefore e_n components of power ≥ 2 are neglected.

Assumption 3.3 The deviation vector V_n is independent of a_n that is valid for small values of the step size. So, we can write the following:

$$E\left\{ \|V_n\|^2 \right\} = \frac{E\left\{ |e_n|^2 \right\}}{P_a}. \quad (22)$$

By squaring (21) and taking expectations of both sides, we get under the assumptions 3.1, 3.2, and 3.3 which are used in [9], the following relation (the computation steps are detailed in Appendix):

$$\begin{aligned} \lim_{n\longrightarrow\infty} E\left\{ \|V_{n+1}\|^2 \right\} &= \beta^2 \lim_{n\longrightarrow\infty} E\left\{ \|V_n\|^2 \right\} \\ &\quad + \lim_{n\longrightarrow\infty} E\left\{ |e_n|^2 \right\} \begin{pmatrix} \mu^2 N P_a T_1 \\ -\mu\beta T_3 \end{pmatrix} \\ &\quad + \gamma P_\Omega + \mu^2 N P_a T_2 \end{aligned} \quad (23)$$

where,

$$\gamma = 1 - \alpha^2 + (\alpha-\beta)^2 + 2(\alpha-\beta)\frac{\beta - \mu U}{1+Z},$$

$$Z = \frac{\mu\alpha U}{1-\alpha\beta},$$

and U, T_1, T_2 and T_3 are given by the following :

	Real valued data
U	$P_a(3P_a - R_2)$
T_1	$9E\left(\|a_n\|^4\right) - 12R_2E\left\{\|a_n\|^2\right\} + R_2^2$
T_2	$E\left\{\|a_n\|^6\right\} - 2R_2E\left\{\|a_n\|^4\right\} + R_2^2E\left\{\|a_n\|^2\right\}$
T_3	$6E\left\{\|a_n\|^2\right\} - 2R$

Complex valued data ($E\left\{a_n^2\right\} = 0$)
$P_a(2P_a - R_2)$
$5E\left(\|a_n\|^4\right) - 8R_2E\left\{\|a_n\|^2\right\} + R_2^2$
$E\left\{\|a_n\|^6\right\} - 2R_2E\left\{\|a_n\|^4\right\} + R_2^2E\left\{\|a_n\|^2\right\}$
$4E\left\{\|a_n\|^2\right\} - 2R_2$

As in the steady state $\lim_{n\longrightarrow\infty} E\left\{ \|V_{n+1}\|^2 \right\} = \lim_{n\longrightarrow\infty} E\left\{ \|V_n\|^2 \right\}$, by substituting (22) into (23) the following expression of the MSE is deduced:

$$MSE = \frac{\gamma P_a P_\Omega + \mu^2 N P_a^2 T_2}{1 - \beta^2 - \mu^2 N P_a^2 T_1 + \mu\beta T_3 P_a}. \quad (24)$$

The analytical expression of the MSE, which is valid for small values of the step size, is interesting for a preliminary study of the tracking ability of the NSCMA adaptive filter. Furthermore, from the result (24), one can easily deduce the misadjustment related to the CMA-DFE that corresponds to $\beta = 1$.

Based on (24), the tracking ability of the NSCMA is compared with that of the CMA. The superiority of the NSCMA over the CMA is noticed in several transmission contexts. To illustrate this superiority, we choose the two transmitted signals that are presented in [9]: s_1 a complex 16 QAM signal with $E\left\{|a_n|^6\right\} = 1950$, $E\left\{|a_n|^4\right\} = 132$, $P_a = E\left\{|a_n|^2\right\} = 10$, and $R_2 = 13.2$; s_2 a real values 6-PAM constellated $a_n \in \{5, 3, 1, -1, -3, -5\}$ with $E\left\{|a_n|^6\right\} =$

5451.7, $E\left\{|a_n|^4\right\} = 235.7$, $P_a = E\left\{|a_n|^2\right\} = 11.67$, and $R_2 = 20.2$.

The Markovian parameter is $\alpha = 0.9$, the value of P_Ω is fixed by a SIR $= 11$ dB, and $N = 5$ as in radio mobile communication. Fig. 2 and Fig. 3 illustrate, for the two transmitted signals s_1 and s_2, the variations of the theoretical MSE (24) over μ, for various values of β: $\beta = 1$ (CMA-DFE), $\beta = 0.95$ and $\beta = \alpha = 0.9$. They show that for all values of β, the variation of the MSE exhibits a minimum corresponding to an optimal μ value. This behavior that can be deduced from (24), is appropriate to the time variation of the equalized channel. In fact, when the channel is constant over time, the MSE is an increasing function of μ.

For the input signals, the NSCMA outperforms the CMA not only for $\beta = \alpha$ but also for $\beta = 0.95$. Hence, the NSCMA-DFE is expected to outperform the classical CMA-DFE even if the convergence of the adaptive Markovian parameter, β_n, to the actual Markovian parameter, α, occurs with a bias.

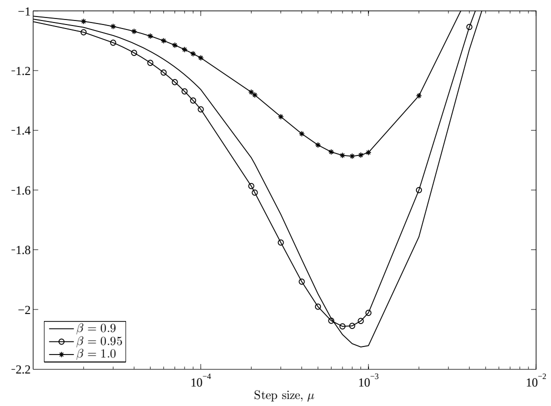

Fig. 2. Variations of the theoretical MSE over the step size μ (16 QAM signal; $N = 5$, $\alpha = 0.9$, SIR = 11 dB).

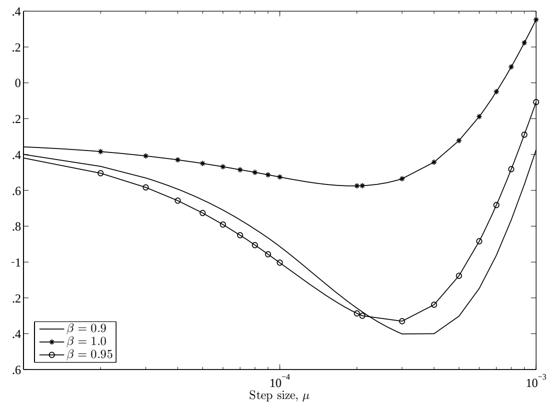

Fig. 3. Variations of the theoretical MSE over the step size μ (6-PAM signal; $\alpha = 0.9$, SIR = 11 dB, $N = 5$).

A comparison between the theoretical MSE and the experimental results is shown in Fig. 4 and Fig. 5 which correspond respectively to the transmitted signals s_1 and s_2. Here, we consider the NSCMA with $\alpha = \beta$. These figures show an agreement between theory and simulation for small values of the step size.

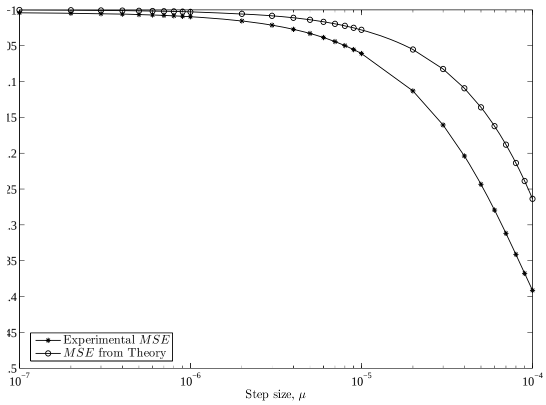

Fig. 4. Evolution of theoretical MSE over the step size μ (16 QAM signal; $\alpha = 0.95$, SIR = 11 dB, SNR = 20 dB)

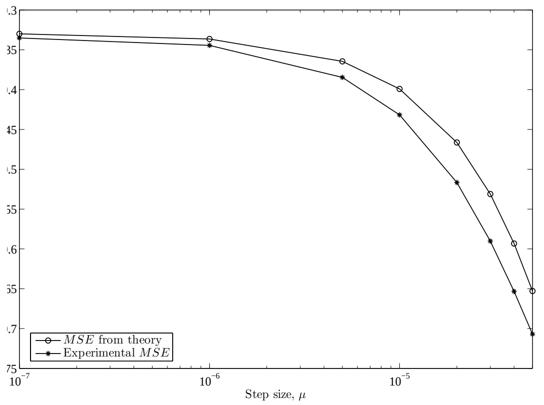

Fig. 5. Evolution of theoretical MSE over the step size μ (6 PAM signal; $\alpha = 0.95$, SIR = 11 dB, SNR = 20 dB)

V. Experimental study of the NSCMA-DFE performance

We show in this section the results of an experimental study that demonstrate the good properties of the proposed NSCMA. Our simulations are carried out under the following considerations. The transmitted signal a_n is assumed i..i.d. and belongs to the QPSK alphabet $\{\pm 1 \pm j\}$. The equalized Rice type channel is constituted of a LOS (Line Of Sight) path plus five ($N = 5$) supplementary paths modeling interference as in radio mobile (cost 207) or WLAN channel models.

A. Convergence of the adaptive Markovian parameter

To study the mean convergence behavior of the adaptive Markovian parameter, we consider the case of a noiseless channel ($P_\eta = 0$). Fig. 6 shows the time variations of β_n in the case of a quite severe time varying channel as $\alpha = 0.94$ and SIR $= 8$ dB. The value of $\alpha = 0.94$ ($f_d T_s = 0.08$) indicates a fast time varying channel and the small value of the SIR (8 dB) corresponds to a high intersymbol interference. The step size μ of the adaptive equalizer is fixed to the optimal value that provides the lowest value of the MSE. This figure shows that the adaptive estimate β_n converges, in almost 150 samples, to an average value close to the actual value of the Markovian parameter α. This significant convergence rate is due to the good convergence properties of the RLS algorithm.

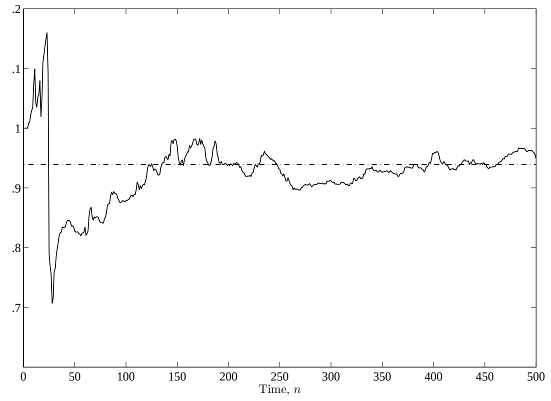

Fig. 6. Time variations of β_n ($\alpha = 0.94$, SIR $= 8$ dB).

To go further into the analysis, the variations of convergence bias, $Bias = \left(\frac{|E\{\beta_n\}-\alpha|}{\alpha}\right)(\%)$, and the standard deviation $\sigma_\beta = \sqrt{E\{\beta_n^2\} - (E\{\beta_n\})^2}$, of the adaptive Markovian parameter are evaluated. Therefore, Tab. I recaps the variations of $Bias$ and σ_β over the SNR. Two types of the channel time variations are considered: ($\alpha = 0.95$, SIR $= 8$ dB) and ($\alpha = 0.9$, SIR $= 8$ dB). The results reported in Tab. I, show that the values of the convergence bias are relatively low for the two cases of α. Indeed, the values of $Bias$ vary in the range $[0:3](\%)$. Therefore, the proposed NSCMA exhibits a good ability to identify the Markovian parameter.

B. Tracking ability of the NSCMA

Fig. 7 shows the variations over μ of the MSEs relating to the CMA, the NSCMA, and the NSCMA with a fixed Markovian parameter ($\beta_n = \alpha$). Two values of α are considered: 0.95 and 0.9 (SIR $= 8$ dB; SNR $= 20$ dB).

This figure shows that the NSCMA outperforms the CMA (gain $\simeq 1$ dB). Moreover, the similarity of the performance of the NSCMA with that of the NSCMA with fixed Markovian parameter ($\beta_n = \alpha$) indicates that the adaptive Markovian parameter β_n has converged to its true value α. Therefore, one can conclude that, for this case, the convergence of the Markovian parameter is achieved with an insignificant bias.

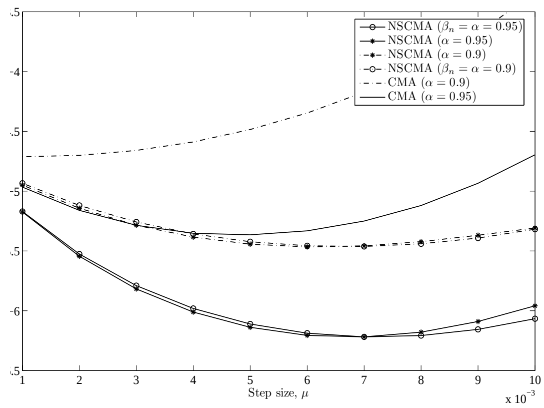

Fig. 7. Variations of the MSE over μ for SIR $= 8$ dB, SNR $= 20$ dB.

C. Performance of the blind DFE

The ability of the NSCMA to reduce the BER is firstly checked through the scatter diagrams of the equalizer output. Fig.8 shows the scatter diagrams of the NSCMA equalizer outputs and those of the CMA equalizer outputs for respectively two cases of channel time variations: $\alpha = 0.95$, SIR $= 8$ dB and $\alpha = 0.9$, SIR $= 8$ dB. The first represents a moderate time varying channel, however the latter represents fast variations. In both cases, the SNR $= 20$ dB and the step size μ is fixed to its optimal value for the two algorithms. Fig. 8 shows that, for these two considered propagation contexts, the NSCMA equalizer outputs are more closer to the transmitted QPSK alphabet ($\{\pm 1 \pm j\}$) than the CMA equalizer outputs. This results in an enhancement of the BER by the NSCMA-DFE.

To go further into the evaluation of the NSCMA performance, the variations of the BER over the SNR are evaluated for the above two considered values of α (0.95 and 0.9) and for two different values of SIR: 8dB and 11dB. Fig. 9 and Fig. 10 display the experimental results relating to the CMA-DFE and the NSCMA-DFE.

TABLE I

VARIATIONS OF THE CONVERGENCE BIAS OVER THE SNR

	SIR (dB)	10	11	12	13	14	15	16	17	18	19	20
$\alpha = 0.95$	$Bias$ (%)	0.27	0.89	0.26	0.6	1.17	0.11	0.67	0.90	1.15	0.03	0.24
	σ_β ($\times 10^{-3}$)	2.3	1.8	2.0	1.7	1.5	1.6	1.5	1.4	1.4	1.5	1.5
$\alpha = 0.9$	$Bias$ (%)	1.56	0.90	2.5	0.04	1.25	2.17	2.83	0.37	0.95	1.38	1.71
	σ_β ($\times 10^{-3}$)	4.5	3.5	3.0	3.2	2.9	2.6	2.4	2.8	2.6	2.5	2.4

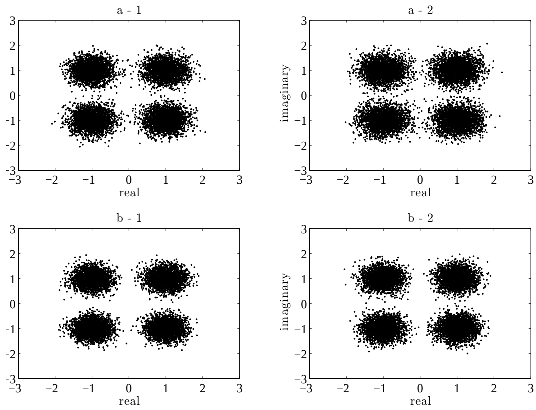

Fig. 8. Scatter diagram of the equalizer output: **a -** CMA, **b -** NSCMA **1 -** $\alpha = 0.95$, **2 -** $\alpha = 0.9$ for QPSK constellation

Fig. 9 that corresponds to SIR : 11dB, shows that in both cases of α, the NSCMA outperforms the CMA for all considered values of the SNR.

Particularly, when the channel time variations are fast, ($\alpha = 0.9$), the gain achieved is very important. For example by setting the BER at 5×10^{-4}, the gain in SNR is equal to 3.59 dB for $\alpha = 0.9$, while it is only equal to 2.44 dB when $\alpha = 0.95$.

Fig. 10 corresponds to SIR 8 dB (a more severe transmission context). Unsurprisingly, the NSCMA presents lower BER than the CMA. The gain in SNR is more important when SIR $= 8$ dB than when SIR $= 11$ dB. For $\alpha = 0.95$ and BER $= 5 \times 10^{-3}$, the gain in SNR is equal to 4.35 dB compared to 0.8 dB realized for SIR $= 11$ dB.

In all studied cases, the BER is remarkably reduced by the use the proposed NSCMA-DFE. The results confirm the ability of the NSCMA to identify the Markovian parameter in a blind DFE. Thus, the NSCMA presents better performance than CMA especially in severe propagation conditions. The gap between performances of the two algorithms increases as the channel time variations become fast.

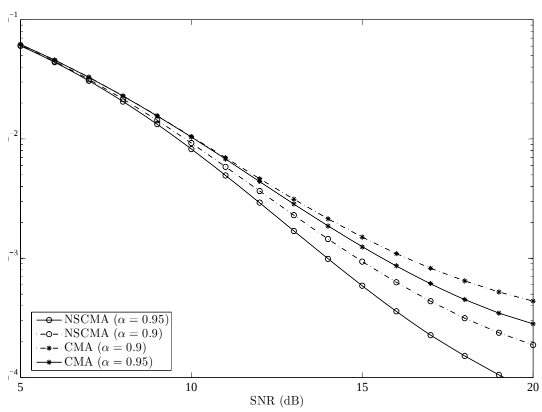

Fig. 9. Variations of the BER over the SNR for SIR = 11 dB.

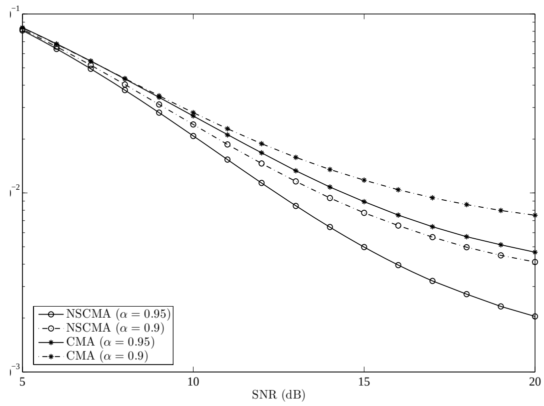

Fig. 10. Variations of the BER over the SNR for SIR = 8 dB.

All the reported results highlight the good tracking proprieties of the proposed NSCMA. Indeed, it outperforms the classical CMA and it reduces considerably the BER when it is used in a blind DFE. Moreover, the performance improvement is as significant as the channel time variations are fast.

VI. Conclusion

A new modified CMA tailored for first order Markovian time varying radio mobile channels was introduced. The proposed NSCMA is obtained by modifying the classical CMA, according to the *a priori* knowledge of the channel time variations model. The performance of the NSCMA was compared with the one of the CMA through a theoretical and an experimental study. Therefore, it is shown that the NSCMA exhibits better tracking ability than the CMA. Indeed, several experimental results, display the ability of the NSCMA to identify the Markovian parameter of the channel time varying model and to improve significantly the BER.

Appendix

From the recurrence (21), it is easy to deduce the following

$$\begin{aligned} \|V_{n+1}\|^2 = \; & \beta^2 \|V_n\|^2 + (\alpha-\beta)^2 \|\Xi_n\|^2 \\ & +(1-\alpha^2)\|\Omega_{n+1}\|^2 + \mu^2 \left\|\epsilon_n^G A_n^*\right\|^2 \\ & -2\mu\beta\Re e\left(V_n^H A_n^* \epsilon_n^G\right) + 2\beta(\alpha-\beta)\Re e\left[V_n^H \Xi_n\right] \\ & -2\mu(\alpha-\beta)\Re e\left(\Xi_n^H A_n^* \epsilon_n^G\right) \\ & -2\mu\beta\sqrt{1-\alpha^2}\Re e\left(\Omega_{n+1}^H A_n^* \epsilon_n^G\right) \\ & +2\beta(\alpha-\beta)\sqrt{1-\alpha^2}\Re e\left[V_n^H \Omega_{n+1}\right] \end{aligned}$$

Since the components of $\Omega_{n+1}$are independent of each other, and as Ω_{n+1} is a zero mean vector which is independent of the input vector A_n, therefore $E\left\{\Omega_{n+1}^H A_n^* \epsilon_n^G\right\} = 0$ and

$E\left\{V_n^H\Omega_{n+1}\right\}=0$. So, one can write that

$$E\left\{\|V_{n+1}\|^2\right\} = \beta^2 E\left\{\|V_n\|^2\right\} + (\alpha-\beta)^2 E\left\{\|\Xi_n\|^2\right\} +(1-\alpha^2)E\left\{\|\Omega_{n+1}\|^2\right\} +\mu^2 \underbrace{E\left\{\left\|\epsilon_n^G A_n^*\right\|^2\right\}}_{Term1} -2\mu\beta\underbrace{\Re e E\left(V_n^H A_n^*\epsilon_n^G\right)}_{Term2} +2\beta(\alpha-\beta)\underbrace{\Re e E\left(V_n^H\Xi_n\right)}_{Term3} -2\mu(\alpha-\beta)\underbrace{\Re e E\left(\Xi_n^H A_n^*\epsilon_n^G\right)}_{Term4}. \quad (25)$$

• *Term1*: Under the assumption 3.3 and since the input signal a_n is assumed independent, one can write that $E\left\{\left\|\epsilon_n^G A_n^*\right\|^2\right\} = NP_aE\left\{\left\|\epsilon_n^G\right\|^2\right\}$.

According to (8), we obtain

$$\begin{aligned} c_n &= e_n + a_n,\\ |c_n|^2 - R_2 &= |e_n|^2 + |a_n|^2 + e_n a_n^* + e_n{}^* a_n - R_2,\\ \epsilon_n^G &= (e_n+2a_n)|e_n|^2 + (2e_n+a_n)|a_n|^2 \\ &\quad + e_n\left(e_n a_n^* - R_2\right) + \left(e_n{}^*a_n - R_2\right)a_n. \end{aligned} \quad (26)$$

Under the assumption 3.2, on can write the following:

$$E\left\{\Re e\left\{\left\|\epsilon_n^G\right\|^2\right\}\right\} = T_1 E\left\{|e_n|^2\right\} + T_2,$$

where

$$T_1 = \begin{cases} 9E\left(|a_n|^4\right) - 12R_2E\left\{|a_n|^2\right\} \\ +R_2^2 \text{ (Real input)} \\ 5E\left(|a_n|^4\right) - 8R_2E\left\{|a_n|^2\right\} \\ +R_2^2 \text{ (Complex input } E\left\{a_n^2\right\}=0) \end{cases}$$

$$T_2 = \begin{cases} E\left\{|a_n|^6\right\} - 2R_2E\left\{|a_n|^4\right\} \\ +R_2^2E\left\{|a_n|^2\right\} \text{ (Real input)} \\ E\left\{|a_n|^6\right\} - 2R_2E\left\{|a_n|^4\right\} \\ +R_2^2E\left\{|a_n|^2\right\} \text{ (Complex input } E\left\{a_n^2\right\}=0\text{)} \end{cases}.$$

• *Term2*: From (20), one can write that, $E\left\{V_n^H A_n^*\epsilon_n^G\right\} = E(e_n^*\epsilon_n^G)$, where

$$\begin{aligned} \Re e\left\{\epsilon_n^G e_n{}^*\right\} &= |e_n|^4 \\ &+ |e_n|^2\left(2|a_n|^2 + 2e_n{}^*a_n + e_n a_n^* - R_2\right) \\ &+ e_n{}^*a_n\left(|a_n|^2 + e_n{}^*a_n - R_2\right). \end{aligned}$$

Assuming that when the adaptive filter reaches the steady state e_n^2 is reasonably small to neglect the statistics of order higher than 2, on can write the following

$$2E\left\{\Re e\left\{e_n{}^*\epsilon_n^G\right\}\right\} = T_3E\left\{|e_n|^2\right\},$$

where

$$T_3 = \begin{cases} \left(6E\left\{|a_n|^2\right\} - 2R_2\right)P_a \text{ (Real input)} \\ \left(4E\left\{|a_n|^2\right\} - 2R_2\right)P_a \text{ (Complex input)} \end{cases}.$$

• *Term3*: Combining (4) and (21) one can write the following

$$\begin{aligned} V_{n+1}^H\Xi_{n+1} &= \beta\alpha V_n^H\Xi_n + \alpha(\alpha-\beta)\Xi_n^H\Xi_n \\ &- \mu\alpha\left(\epsilon_n^G A_n^*\right)^H\Xi_n + (1-\alpha^2)\Omega_{n+1}^H\Omega_{n+1} \\ &+ \sqrt{1-\alpha^2}\underbrace{\begin{pmatrix} \beta\alpha V_n^H + \alpha(\alpha-\beta)\Xi_n^H \\ -\mu\alpha\left(\epsilon_n^G A_n^*\right)^H + \alpha\Xi_n^H \end{pmatrix}}_{\Upsilon}\Omega_{n+1}. \end{aligned}$$

As $\Upsilon = 0$, we obtain the following:

$$\begin{aligned} E\left\{V_{n+1}^H\Xi_{n+1}\right\} &= \beta\alpha E\left\{V_n^H\Xi_n\right\} \\ &+ \alpha(\alpha-\beta)E\left\{\Xi_n^H\Xi_n\right\} \\ &- \mu\alpha E\left\{\left(\epsilon_n^G A_n^*\right)^H\Xi_n\right\} \\ &+ (1-\alpha^2)E\left\{\Omega_{n+1}^H\Omega_{n+1}\right\}. \end{aligned}$$

Consequently,

$$\begin{aligned} \lim_{n\longrightarrow\infty} E\left\{V_{n+1}^H\Xi_{n+1}\right\} &= \beta\alpha \lim_{n\longrightarrow\infty} E\left\{V_n^H\Xi_n\right\} \\ &+ (1-\alpha\beta)P_\Omega \\ &- \mu\alpha \quad \mathcal{C}. \end{aligned}$$

where $\mathcal{C} = \lim_{n\longrightarrow\infty} E\left\{\Re e\left(\left(\epsilon_n^G A_n^*\right)^H\Xi_n\right)\right\}$.

Assuming that at the steady state $\lim_{n\longrightarrow\infty} E\left\{V_{n+1}^H\Xi_{n+1}\right\}$ is finite, we can deduce the following:

$$\lim_{n\longrightarrow\infty} E\left\{\Re e\left(V_n^H\Xi_n\right)\right\} = P_\Omega - \frac{\mu\alpha}{(1-\alpha\beta)} \quad \mathcal{C}.$$

• *Term4*: From (4) and (21) we show the following:

- Case of real valued input:
$E\left\{\Xi_n^H A_n^*\epsilon_n^G\right\} = \underbrace{P_a(3P_a - R_2)}_{U}E\left\{\Xi_n^H V_n\right\}$.

- Case Complex Valued input ($E\left\{a_n^2\right\}=0$):
$E\left\{\Xi_n^H A_n^*\epsilon_n^G\right\} = \underbrace{P_a(2P_a - R_2)}_{U}E\left\{\Xi_n^H V_n\right\}$.

From the three above equations we deduce that $\lim_{n\longrightarrow\infty} E\left\{\Re e\left(V_n^H\Xi_n\right)\right\} = \frac{P_\Omega}{1+Z}$, where $Z = \frac{\mu\alpha U}{(1-\alpha\beta)}$. By substituting all the computed terms into (25) one can obtain the result (23).

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