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Nonlinear Time-History Analysis of 3-Dimensional Semi-rigid Steel Frames

Phu-Cuong Nguyen, and Seung-Eock Kim

Abstract**—**This paper presents nonlinear elastic dynamic analysis of 3-D semi-rigid steel frames including geometric and connection nonlinearities. The geometric nonlinearity is considered by using stability functions and updating geometric stiffness matrix. The nonlinear behavior of the steel beam-to-column connection is considered by using a zero-length independent connection element comprising of six translational and rotational springs. The nonlinear dynamic equilibrium equations are solved by the Newmark numerical integration method. The nonlinear time-history analysis results are compared with those of previous studies and commercial SAP2000 software to verify the accuracy and efficiency of the proposed procedure.

Keywords**—**Geometric nonlinearity, nonlinear time-history analysis, semi-rigid connection, stability functions.

I. Introduction

BEAM-TO-COLUMN joints of steel frames are usually assumed to be rigid or pinned connections in structural design. This assumption causes an inaccurate estimation of the response of frames since real beam-to-column joints are between fully rigid and pinned connections.

In this paper, an independent zero-length connection element with six different translational and rotational springs connecting two different nodes with zero distance is developed. This is efficient because modification of the beam-column stiffness matrix considering the semi-rigid connections is unnecessary and the connection is ready to integrate with any element types. The dynamic behavior of rotational springs is captured through the independent hardening model employing the Richard-Abbott four-parameter model [1] and the Chen-Lui exponential model [2]. The translational springs with constant stiffness are used to model linear semi-rigid connections.

The Newmark numerical integration method combined with the Newton-Raphson iterative algorithm is adopted to solve the nonlinear dynamic equilibrium equations. The results of the second-order elastic dynamic response are compared with those of previous studies and commercial SAP2000 software [3] to demonstrate the accuracy and computational efficiency.

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II. Nonlinear Element Modelling

A. Nonlinear Beam-column Element

To capture the effect of axial force acting through the lateral displacement of the beam-column element ($P-\delta$ effect), the stability functions reported by Chen and Lui [4] are used to minimize the modeling and solution time. Only one element per member is generally needed to accurately capture the $P-\delta$ effect. The material is assumed to be elastic. The incremental force-displacement equation of a 3-D beam-column element can be expressed in accordance with Kim and Thai [5]:

$$\begin{Bmatrix} \Delta P \\ \Delta M_{yA} \\ \Delta M_{yB} \\ \Delta M_{zA} \\ \Delta M_{zB} \\ \Delta T \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{1y}\frac{EI_y}{L} & S_{2y}\frac{EI_y}{L} & 0 & 0 & 0 \\ 0 & S_{2y}\frac{EI_y}{L} & S_{1y}\frac{EI_y}{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{1z}\frac{EI_z}{L} & S_{2z}\frac{EI_z}{L} & 0 \\ 0 & 0 & 0 & S_{2z}\frac{EI_z}{L} & S_{1z}\frac{EI_z}{L} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} \end{bmatrix} \begin{Bmatrix} \Delta \delta \\ \Delta \theta_{yA} \\ \Delta \theta_{yB} \\ \Delta \theta_{zA} \\ \Delta \theta_{zB} \\ \Delta \phi \end{Bmatrix} \tag{1}$$

where E and G are the elastic and shear modulus of material; A and L are the area and length of beam-column element; J is the torsional constant; I_n is the moment of inertia with respect to the n axes $(n=y,z)$; ΔP, ΔM_{yA}, ΔM_{yB}, ΔM_{zA}, ΔM_{zB}, and ΔT are the incremental axial force, A and B end moments with respect to y and z axes, and torsion respectively; $\Delta\delta$, $\Delta\theta_{yA}$, $\Delta\theta_{yB}$, $\Delta\theta_{zA}$, $\Delta\theta_{zB}$, and $\Delta\phi$ are the incremental axial displacement, joint rotations, and angle of twist; S_{1n} and S_{2n} are the stability functions [4] with respect to the n axis.

The tangent stiffness matrix of a beam-column element considering both the $P-\delta$ and $P-\Delta$ effects is obtained as follows:

$$[K]_{12\times12} = [T]^T_{6\times12}[K_e]_{6\times6}[T]_{6\times12} + [K_g]_{12\times12} \tag{2}$$

where $[K_e]$ is the elemental stiffness matrix including the $P-\delta$ effect, and $[T]$ is the transformation matrix, and $[K_g]$ is the geometric stiffness matrix accounting for the $P-\Delta$ effect, shown as [5].

B. Nonlinear Spring Element

1) Spring Element

An independent zero-length element with three translational and three rotational springs is developed to simulate the steel beam-to-column connection. The multi-spring elements connect two nodes with identical coordinates. The translational spring has linear stiffness, while the rotational one has linear or nonlinear stiffness. The coupling effects between the six spring elements of a connection are neglected.

The relation between the incremental force vector $\{\Delta F_s\}$ and displacement vector $\{\Delta U_s\}$ of the spring element corresponding to six degrees of freedom is as follows:

$$\{\Delta F_S\} = [K_S]\{\Delta U_S\} \tag{3}$$

where $[K_S]$ is the diagonal tangent stiffness matrix for each spring. The tangent stiffness for the linear springs k_n^{lin} or the nonlinear springs k_n^{non} is

$$k_n^{lin} = R_{k,n}^{lin} \tag{4a}$$

$$k_n^{non} = R_{kt,n}^{non} \tag{4b}$$

where $R_{k,n}^{lin}$ is a constant scalar of a linear spring, $R_{kt,n}^{non}$ is the tangent stiffness of a nonlinear spring with respect to the n axis $(n = x, y, z)$.

2) Nonlinear Semi-rigid Connection Models

In this study, the Richard-Abbott model [1] and the Chen-Lui exponential model [2] are used to evaluate the nonlinear behavior of semi-rigid connections. The independent hardening model is used to predict the cyclic behavior of the connections.

Richard and Abbott [1] proposed a four-parameter model. The moment-rotation relationship of the connection is defined by

$$M = \frac{(R_{ki} - R_{kp})|\theta_r|}{\left\{1 + \left|\frac{(R_{ki} - R_{kp})|\theta_r|}{M_0}\right|^n\right\}^{\frac{1}{n}}} + R_{kp}|\theta_r| \tag{5}$$

where M and θ_r are the moment and the rotation of the connection, n is the parameter defining the shape, R_{ki} is the initial connection stiffness, R_{kp} is the strain-hardening stiffness and M_0 is the reference moment.

Lui and Chen [2] proposed the following exponential model:

$$M = M_0 + \sum_{j=1}^{n} C_j \left(1 - \exp^{-\frac{|\theta_r|}{2j\alpha}}\right) + R_{kf}|\theta_r| \tag{6}$$

in which M and $|\theta_r|$ are the moment and the absolute value of the rotational deformation of the connection, α is the scaling factor, R_{kf} is the strain-hardening stiffness of the connection, M_0 is the initial moment, C_j is the curve-fitting coefficient, and n is the number of terms considered.

3) Cyclic Behavior of Semi-rigid Connections

The independent hardening model shown in Fig. 1 is used to trace the cyclic behavior of semi-rigid connections because of its simple application [6]. The instantaneous tangent stiffness of the connections is determined by taking derivative of (5) and (6).

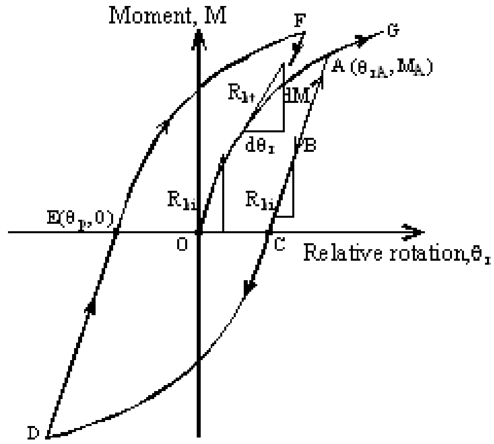

Fig. 1 Independent hardening model

III. Nonlinear Analysis Algorithm

The Newmark's method has been chosen for the numerical integration of the equation of motion because of its simplicity [7]. The residual forces in each time step can be eliminated by using the Newton-Raphson iterative procedure [8]. The incremental equation of motion of a structure can be written as

$$[M]\{\Delta\ddot{D}\} + [C]\{\Delta\dot{D}\} + [K]\{\Delta D\} = \{\Delta F\} \tag{7}$$

where $[\Delta\ddot{D}]$, $[\Delta\dot{D}]$, and $[\Delta D]$ are the vectors of incremental acceleration, velocity, and displacement, respectively; $[M]$, $[C]$, and $[K]$ are the mass, damping, and tangent stiffness matrices, respectively; $\{\Delta F\}$ is the external load increment vector. The viscous damping matrix $[C]$ can be defined as

$$[C] = \alpha_M [M] + \beta_K [K] \quad (8)$$

where α_M and β_K are mass- and stiffness-proportional damping factors, respectively. If both modes are assumed to have the same damping ratio ξ, then

$$\alpha_M = \xi \frac{2\omega_1\omega_2}{\omega_1 + \omega_2} \quad ; \quad \beta_K = \xi \frac{2}{\omega_1 + \omega_2} \quad (9)$$

where ω_1 and ω_2 are the natural radian frequencies of the first and second modes of the considered frame, respectively.

IV. Verification

A. Single-bay Two-story Semi-rigid Steel Frame

A single-bay two-story steel frame with flexible beam-to-column connections was studied by Chan and Chui [8]. The geometry and loading of the frame are given in Fig. 2. All the frame members are W8x48 with Young's modulus E of 205×10^6 kN/m^2. An initial geometric imperfection ψ of 1/438 is considered. The vertical static loads are applied on the frame to consider the second-order effects followed by the horizontal forces applied suddenly at each floor during 0.5 second, as shown in Fig. 2. The lumped masses of 5.1 and 10.2 Ton are modeled at the top of the columns and the middle of beams, respectively. The material is assumed to be elastic, and the viscous damping is ignored. A time step Δt of 0.001 second is chosen in the dynamic analysis. The four parameters of the Richard-Abbott model are: $R_{ki} = 23{,}000\, kN \cdot m / rad$, $R_{kp} = 70\, kN \cdot m / rad$, $M_o = 180\, kN \cdot m$, and $n = 1.6$. The time-displacement responses at the second floor predicted by the proposed analysis for the rigid, linear semi-rigid, and nonlinear semi-rigid frames match well with those of Chan and Chui [8], as shown in Fig. 3. In addition, the moment-rotation curves at connection C also agree well with the results of Chan and Chui as shown in Fig. 4.

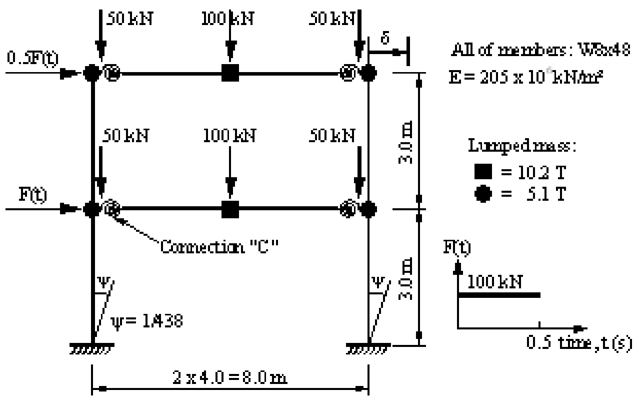

Fig. 2 Single-bay two-story semi-rigid steel frame

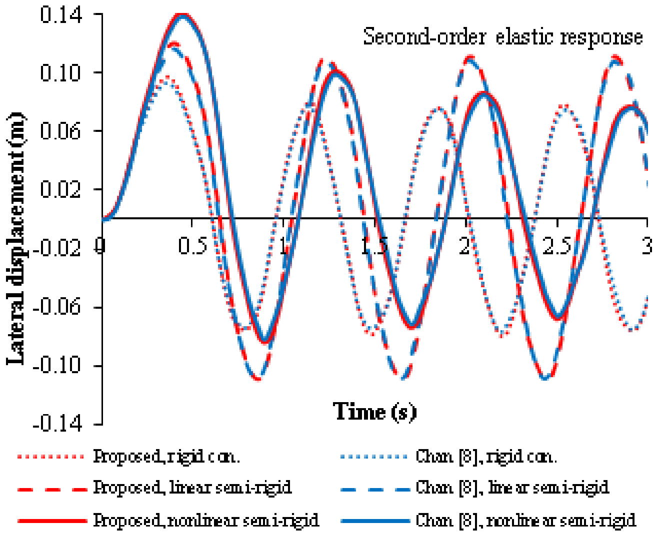

Fig. 3 Time-displacement response at top of the two-story frame

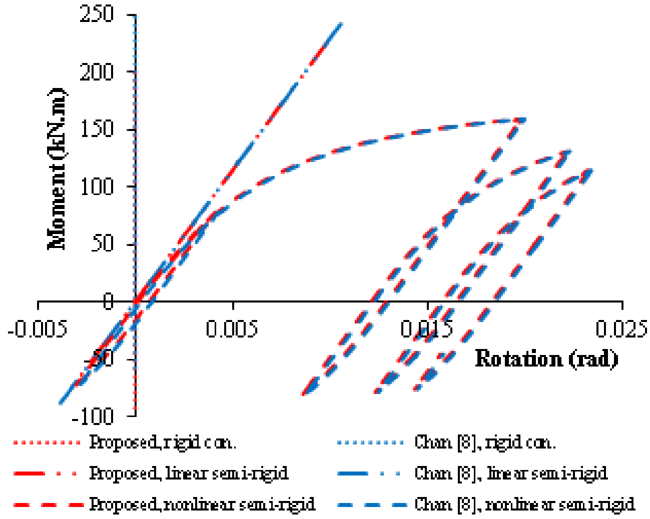

Fig. 4 Hysteresis loops at connection C of the two-story frame

B. Two-story Space Frame Subjected to Impulse Force

The nonlinear dynamic response of two-story space frames with various connection types (fully rigid, linear semi-rigid, and nonlinear semi-rigid connections) subjected to an impulse force of 100kN is studied. The member sizes and properties of the frame are shown in Fig. 5, [8]. Static vertical loads of 36.9 and 46.1 kN are applied in order to consider the $P-\Delta$ and $P-\delta$ effects. These static loads are considered as lumped masses at nodes. The Chen-Lui exponential model is used for the flush end plate connection of semi-rigid joints. The parameters of the model are: R_{ki} = 12,340.198kN.m/rad; R_{kf} = 108.924kN.m/rad; M_o = 0.0kN.m; α = 0.00031783; C_1 = -28.286; C_2 = 573.189; C_3 = -3,433.98; C_4 = 8,511.3; C_5 = -9,362.567; and C_6 = 3,832.899 (unit of C_i is kN.m) [9]. The connection stiffness about the weak-axis of the sections assumes to be one fifth of the stiffness about the strong-axis. A time step Δt of 0.005 second is chosen.

The time-displacement response at point A and the hysteresis loops of moment-rotation at joint C are shown in Figs. 6-7, respectively. The results of both rigid and linear semi-rigid connection cases compare well with those of the SAP2000 software.

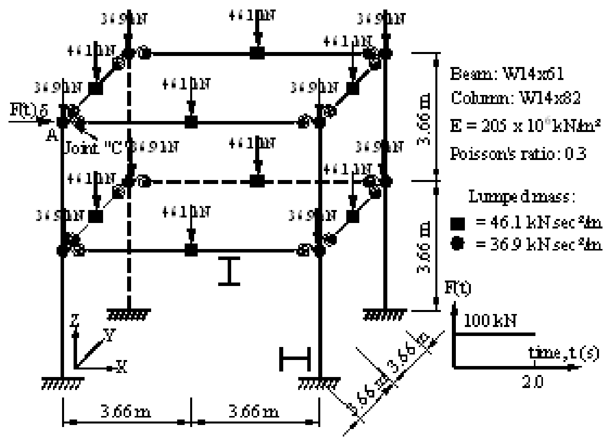

Fig. 5 Dimensions and properties of a two-story space frame

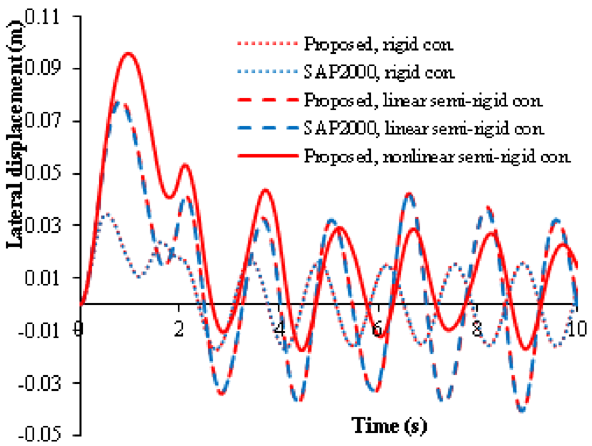

Fig. 6 Time-displacement response at node A in nonlinear elastic analysis

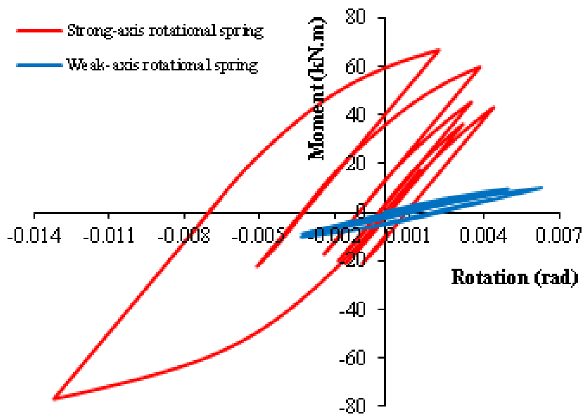

Fig. 7 Moment-rotation curve of nonlinear strong-axis and weak-axis springs at connection C

It is noted that the SAP2000 software could not analyze the nonlinear semi-rigid frame. The nonlinear semi-rigid frame has a larger displacement and a longer period, as shown in Fig. 6, because it has greater flexibility than the rigid one due to the presence of the semi-rigid connections. In the nonlinear semi-rigid connection case, the response shows a displacement shift due to permanent rotational deformation at connections. It was found that the nonlinear connections dampened the deflection due to energy dissipation.

V. Conclusion

A simple efficient numerical procedure is developed for the nonlinear elastic dynamic analysis of three-dimensional steel frames with semi-rigid connections. The geometric nonlinearity is considered by using the stability functions. An independent zero-length connection element comprising of six translational and rotational springs is proposed to simulate the steel beam-to-column connection. The independent hardening model is used to trace the hysteresis loops of the nonlinear semi-rigid connections. The proposed method is verified to be accurate through two numerical examples. It is found that the nonlinear semi-rigid connections dampen the deflection due to energy dissipation.

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