Improved Segmentation of Speckled Images Using an Arithmetic-to-Geometric Mean Ratio Kernel

J. Daba, and J. Dubois

Abstract—In this work, we improve a previously developed segmentation scheme aimed at extracting edge information from speckled images using a maximum likelihood edge detector. The scheme was based on finding a threshold for the probability density function of a new kernel defined as the arithmetic mean-to-geometric mean ratio field over a circular neighborhood set and, in a general context, is founded on a likelihood random field model (LRFM). The segmentation algorithm was applied to discriminated speckle areas obtained using simple elliptic discriminant functions based on measures of the signal-to-noise ratio with fractional order moments. A rigorous stochastic analysis was used to derive an exact expression for the cumulative density function of the probability density function of the random field. Based on this, an accurate probability of error was derived and the performance of the scheme was analysed. The improved segmentation scheme performed well for both simulated and real images and showed superior results to those previously obtained using the original LRFM scheme and standard edge detection methods. In particular, the false alarm probability was markedly lower than that of the original LRFM method with oversegmentation artifacts virtually eliminated. The importance of this work lies in the development of a stochastic-based segmentation, allowing an accurate quantification of the probability of false detection. Non visual quantification and misclassification in medical ultrasound speckled images is relatively new and is of interest to clinicians.

Keywords—Discriminant function, false alarm, segmentation, signal-to-noise ratio, skewness, speckle.

I. INTRODUCTION

SPECKLE noise is present in coherent imaging systems and is a form of object- or target-induced random noise. In the literature, segmentation algorithms applied to speckled images have not fully exploited the statistical nature of the underlying speckle formation process. Notable exceptions are [1 - 4]. A commonly used method employs the Laplacian of Gaussian edge detector.

In this paper, we employ a novel statistical segmentation scheme for speckled images. This scheme is an improvement

over a previously devised algorithm termed LRFM [5] by initially separating speckle areas from non speckle areas using simple elliptic discriminant functions. The segmentation algorithm is run on the discriminated speckle regions using an arithmetic mean-to-geometric mean ratio kernel applied to a circular window with a small radius. In the context of our work, segmentation can be considered as the first step of image analysis which aims at either a description of the image or a classification of the image content.

II. SPECKLE MODEL

The segmentation scheme is based on the physical and statistical model of coherent image formation, where the image intensities are exponentially distributed [6]:

$$p_I\left(y\right) = \frac{1}{\mu}\exp\left(-\frac{y}{\mu}\right), \; y \geq 0, \tag{1}$$

where μ is the mean intensity of a pixel. This is known as a single look speckle model.

For the case of multilook model, the intensity of a single pixel is the non-coherent sum of L statistically independent looks of that pixel:

$$I_L = \sum_{k=1}^{L} i_k. \tag{2}$$

I_L obeys a gamma distribution [6]:

$$p_{I_L}\left(y\right) = \frac{1}{\mu^L (L-1)!} y^{L-1} e^{-\frac{y}{\mu}}, \; y \geq 0. \tag{3}$$

The signal-to-noise ratio is increased by a factor of $\sqrt{L}$, however the resolution is decreased by the same amount [1-4].

In this paper, we consider binary images composed of pixels with reflectivity levels of either μ_0 or μ_1, and having a contrast ratio $r = \mu_1 / \mu_0 > 1$. Binary images in the context of this paper are reasonable since it has been shown in [7] that the mutual information extracted between the reflectivity and

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:1, No:10, 2007

a single speckle intensity is approximately equal to 1 bit per resolution cell.

III. IMPROVED SEGMENTATION SCHEME

A. Performance Improvement Using Speckle Discrimination

We demonstrate that using a simple elliptical discriminant function provides a very efficient way of reducing the segmentation's algorithm false alarm probability (or improving precision probability) by separating speckle areas from non speckle areas. Key to devising an effective discriminant function is the study of local statistical signal properties that discriminate the speckle areas from the background. Two signal-to-noise ratio measures are chosen based on fractional order moments [8]. The first one is the generalized inverse speckle contrast ratio defined as

$$R = \frac{E\left(I^{\eta}\right)}{\sqrt{E\left(I^{2\eta}\right) - E^2\left(I^{\eta}\right)}} \,. \tag{4}$$

When $\eta = 1$, this metric degenerates into the inverse SAR (standard deviation-to-average ratio) $E(I)/\sigma_I$ or the inverse Goodman's speckle contrast ratio. The second measure is the speckle skewness defined as

$$S = \frac{E\left[\left(I^{\eta} - E\left(I^{\eta}\right)\right)^3\right]}{\left(E\left(I^{2\eta}\right) - E^2\left(I^{\eta}\right)\right)^{\frac{3}{2}}} \,. \tag{5}$$

An elliptic discriminant function is designed in the (R, S) space to separate speckle from non-speckle areas. The orientation of the ellipse is determined by eigenvector analysis of the covariance matrix of an arbitrary sample of points corresponding to various kinds of speckle. The size of the ellipse is determined manually. The systematic algorithm to devise such a discriminant function is described in [9].

This scheme proved superior to the original LRFM method by markedly lowering the false alarm probability and reducing "oversegmentation" artifacts.

B. Arithmetic Mean-to-Geometric Mean Ratio Kernel

Let $N(j,k)$ represent an arbitrary pixel in the original image and is the center of a neighborhood set $\mathcal{R}$. At this stage, we assume that $\mathcal{R}$ is a homogeneous set. When selecting a window of pixels to perform an arithmetic operation, we are introduction strong prior knowledge that there is only one edge within the window. If this condition is violated, the segmentation scheme will result in missed edges. To minimize the miss probability, the window size should be sufficiently small that only a single edge is present. Instead of the 3 x 3 square window which was proposed in the original LRFM scheme [5], a circular window (with sufficiently small radius) is used in order to conform with the geometry of the newly proposed discriminant function for speckle separation. The

orientation of the circle's diameter is determined by means of exhaustive simulations.

The speckle intensity image is scanned with a sliding circular window made of two semi-circular regions $\mathcal{R}_1$ and $\mathcal{R}_2$ of the same size. At each pixel coordinate $N(j,k)$, we form the average intensity values $m_1(j,k)$ and $m_2(j,k)$ over the sets $\mathcal{R}_1$ and $\mathcal{R}_2$, respectively, and construct in a new field $Y(j,k)$ using a kernel defined by the arithmetic mean-to-geometric mean ratio:

$$Y(j,k) = \frac{\left(m_1(j,k) + m_2(j,k)\right)/2}{\sqrt{m_1(j,k)m_2(j,k)}} \,. \tag{6}$$

The likelihood function $Z(j,k) = p_Y(y(j,k))$ is formed based on the statistical distribution of the new kernel $Y(j,k)$. For a pixel to belong to the homogeneous region, $Z(j,k)$ must be greater than an appropriate threshold value λ. The edge detection rule is thus: $Z(j,k) \underset{\text{edge}}{\overset{\text{no edge}}{\gtrless}} \lambda$. There are two ways of choosing this threshold: (1) through simulation, by starting with a high value of λ and lowering it, and (2) analytically, by determining the P_e and fixing the value of λ which will result in the desired P_e (provided that an analytical form for the cumulative distribution function of the probability density function $p_Y(y(j,k))$ can be derived).

IV. RESULTS AND DISCUSSIONS

A. Edge Magnitude Estimation

Since the underlying noise process is exponentially distributed, the following ratio is optimal in the maximum likelihood sense for estimating the edge magnitude M:

$$\hat{M} = \frac{E(Y)}{\left|E(I_D) - E(I_B)\right|} = \frac{D\mu_0 + B\mu_1}{\left|D\mu_0 - B\mu_1\right|} \tag{7}$$

where B is the number of bright pixels μ_1 and D is the number of the dark pixels μ_0. When $B = 0$, the edge magnitude $\widehat{M} = 1$, indicating no edge. Similarly, when $D = 0$, $\widehat{M} = 1$.

The probability of error is given by

$$P_e = \left(1 - P_{edge}\right)P_F + P_{edge}P_M \tag{8}$$

where P_{edge} is the apriori probability of an edge, P_M is the probability of missed edges, and P_F is the probability of false alarms. We can show that:

$$P_F(\delta) = P_M\left(\frac{1}{\delta}\right) = P_U(\delta), \; U = \frac{Z}{M}, \; \delta = \frac{\lambda}{M} > 1 \,. \tag{9}$$

B. On the Use of Multi-look Processing

This is equivalent to up-sampling the image by a factor of L thus improving the detector performance. The larger P_F is set, the smaller P_M becomes. As the threshold-to-edge ratio $\delta =$

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:1, No:10, 2007

λ/M increases, the number of false edges increases and consequently the number of missed edges decreases. Conversely, when δ decreases, the number of false edges decreases and the number of missed edges increases. Thus, when choosing the threshold value, the trade off between P_M and P_F must be considered.

A tradeoff also exists between L and the size of the neighborhood set. If the image contains reasonably sized well defined objects with little detail, then it is possible to use a large size neighborhood to accurately detect edges without the need of multi-look speckle reduction. On the other hand, if the image contains objects with fine details, then using a large neighborhood size would tend to smear a number of edges of interest. This requires pre-image enhancement techniques to reduce speckle without adversely reducing the resolution and degrading the image.

C. Worst Case Scenario for the False Alarm Probability

The worst case scenario for P_M corresponds to the circular window running along the object contours with a diameter that is perfectly aligned with the edge's orientation. Such regions consist of approximately the same amount of dark and bright pixels. Assuming the circle's radius to be r, the edge magnitude can be estimated according to Eq. (7) with $D = 0.5\pi r^2 - 1$ and $B = 0.5\pi r^2 + 1$. The miss probability

$$P_M = 1 - P_U(\delta), 0 \leq \delta \leq \min\left(1, \frac{p_Y(\text{mode}(Y))}{\widehat{M}}\right) \qquad (10)$$

By setting a value for P_M and extending it, the corresponding values for δ and $\lambda = \delta\widehat{M}$ can be found. In turn, the worse case scenario P_F can be determined for $\widehat{M}$ approximately equal to one. We also note that as the image contrast ratio $r = \mu_1/\mu_0$ increases, the error probability P_e decreases.

D. Simulation

For the image set of Figs. 1-4, P_e was around 1.5 % for P_M = 12% and approximately 2.4% for P_M = 26%. To calculate the overall experimental P_e, an edge image was created by a gradient operator and then XOR with the resulting segmented images to give the number of mismatches: P_e = number of mismatches/total number of pixels. Overall, P_e was reasonably low. We note that this is an improvement over our previously developed method where the likelihood random field consisted only of the arithmetic mean of intensity pixels over a square neighborhood set.

This was markedly evident for $P_F = (P_e - P_{edge}P_M)/(1 - P_{edge})$.

Fig. 1 Original speckled ultrasound image

Fig. 2 Pre-processed binary image with reflectivity levels $\mu_0 = 10$ and $\mu_1 = 35$

Fig. 3 Segmented image for a large threshold value equal to 0.01

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
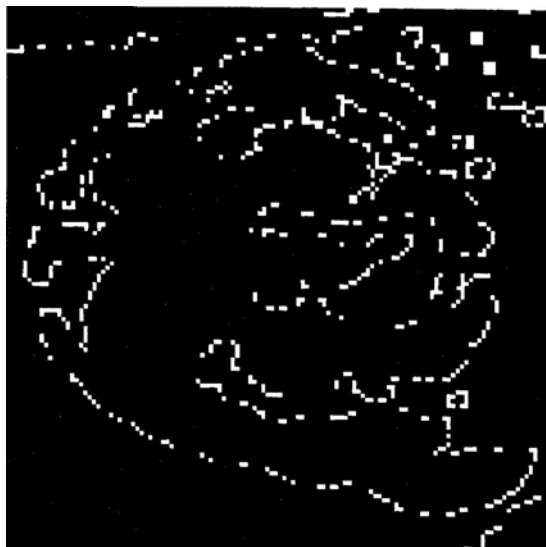Vol:1, No:10, 2007

Fig. 4 Segmented image for a low threshold value equal to 0.008

V. CONCLUSION

In this paper, we have implemented an improved segmentation scheme which aims to extract edge information from speckled images. The new scheme was based on finding a threshold for the probability density function of the arithmetic mean-to-geometric mean kernel over a circular neighborhood set. In order to reduce over-segmentation, the algorithm was run only speckle regions that were discriminated using an elliptic discriminant function. The probability of error was also quantified. The algorithm performed reasonably well for clinical ultrasound images and was superior to the results obtained using the original LRFM method using a pure arithmetic mean kernel over a square rectangular neighborhood set. The performance improvement was especially significant for the false alarm probability. The improved LRFM scheme was also shown to significantly outperform standard edge detection. We have thus demonstrated an accurate segmentation method based on reasonable assumptions which allow the probability of false detection to be quantified. Quantification and misclassification in speckled images is relatively new and should be of interest to ultrasound clinicians and remote sensing human observers.

REFERENCES

[1] A. Bovik, D. Munson, "Optimal Detection of Object Boundaries in Uncorrelated Speckle," *Optical Engineering*, Vol. 25, No. 11, Nov. 1986.
[2] A. Bovik, "On Detecting Edges in Speckle Imagery," *IEEE Transactions on Acoustics Speech and Signal Processing*, Vol. 36, No. 10, Oct. 1988.
[3] H. Arsenault, "Information Extraction from Images Degraded by Speckle," *Proceedings of IGARSS 87 Symposium*, 1987, pp. 1317-1322.
[4] P. Kelly, H. Derin, "Adaptive Segmentation of Speckled Images Using a Hierarchical RFM," *IEEE Transactions on Acoustics, Speech, and Signal Processing,* Vol. 36, No. 10, Oct. 88.
[5] J. S. Daba and M. R. Bell, "Segmentation of Speckled Images Using a Likelihood Random Field Model," *Optical Engineering*, accepted, to appear in 2007.
[6] J. Goodman, "Statistical Properties of Laser Speckle Patterns," *Speckle and Related Phenomena*, 2nd Edition, J. C. Dainty Ed., Springer Verlag, NY, 1984.
[7] V. Frost and K. Shanmugan, "The Information Content of SAR Images of Terrain," *IEEE Transactions on Aerospace Electron. Syst.*, Vol. AES-19, No. 5, pp. 768-774, 1993.
[8] V. Dutt and J. F. Greenleaf, "Speckle Analysis Using Signal to Noise Ratios Based on Fractional Order Moments," *Ultrasonic Imaging*, Vol. 17, pp. 251-268, 1995.
[9] R. W. Prager, A. H. Gee, G. M. Treece, and L. Berman, "*Speckle Detection in Ultrasound Images Using First Order Statistics*," *Technical Report CUED/F-INFENG/TR* 415, University of Cambridge, Dept. of Engineering, July 2001.