Regional Stability Analysis of Rotor-Ball Bearing and Rotor- Roller Bearing Systems Considering Switching Phenomena

Jafar Abbaszadeh Chekan, Kaveh Merat, Hassan Zohoor

***Abstract*—**In this study the regional stability of a rotor system which is supported on rolling bearings with radial clearance is studied. The rotor is assumed to be rigid. Due to radial clearance of bearings and dynamic configuration of system, each rolling elements of bearings has the possibility to be in contact with both of the races (under compression) or lose its contact. As a result, this change in dynamic of the system makes it to be known as switching system which is a type of Hybrid systems. In this investigation by adopting Multiple Lyapunov Function theorem and using Hamiltonian function as a candidate Lyapunov function, the stability of the system is studied. The purpose of this study is to inspect the regional stability of rotor-roller bearing and rotor-ball bearing systems.

***Keywords*—**Stability analysis, Rotor-rolling bearing systems, Switching systems, Multiple Lyapunov Function Method

I. INTRODUCTION

ROTORDYNAMICS is an important field of study in dynamical systems. Because of demands for rotary machines with high speeds such as turbines, pumps, fans, spindle machines and so on, the dynamics and stability analysis of this type of systems are considered over a hundred years. As some of the rotor systems are supported by rolling bearings with radial clearances, the stability analysis of such systems can be helpful in the corresponding areas of the industry. The dynamic equations of rotor-bearing systems are mostly complicated with high nonlinearity; as a result, the analytical solution for these equations is sophisticated and in some cases is impossible to obtain; therefore, it is necessary to use the stability criteria such as Lyapunov theory, Routh-Hurwitz method or Floquet theory to analyze the stability of rotor-bearing systems.

The bearings play an instrumental role in the dynamics of rotor bearing systems. Hence, investigations are done in analyzing the effect of the bearings' parameters such as stiffness and damping coefficients on the stability region of the rotor-bearing systems. [1] studied the effect of bearing parameters on the stability of the rigid rotor system. In their study they applied Routh-Hurwitz criterion to study the influence of principal and cross-coupling stiffness and damping coefficients on the stability region of the system. In addition, [2] studied the stability of a rigid rotating shaft, supported by bearings identical to ref [1] but using the Lyapunov criterion, and obtained the similar results to their previous work. Furthermore, in another investigation [3] studied the stability of a rotor system which includes elastic shaft and is supported on the bearings with nonlinear parameters. Lyapunov criterion is employed in his investigation and the Hamiltonian function is considered as Lyapunov candidate function. Finally, the influence of shaft parameters such as Yung Elastic modulus and bearing parameters are studied in [3]. In the above mentioned researches the bearings are assumed as ideal boundary conditions such as simple springs and dampers which are not appropriate models for rolling bearings. In a work by [4], a more precise mathematical model of rolling bearings is developed. In their work the bearing is modeled by Hertzian contact theory for each of rolling elements, considering the radial clearance, the bearing's forces and moments. In some other studies such as [5, 6] the nonlinear dynamic of a rotor with ball bearing having radial clearance is also investigated. As the rotor systems with the rolling bearings which have radial clearance exhibit switching behavior associated with the contact of the rolling elements with the races of the bearing, it falls into the category of hybrid systems. The hybrid systems combine continuous dynamics, represented by differential or difference equations, with finite state dynamics usually called finite automaton. A special class of these systems is switched systems which are considered in our model. The switched system models consist of finite differential equations where some rule designates which one is the governing differential equation at any time interval. So in order to investigate the stability of this bearing model, one should extend stability theorems for switched systems which are briefly discussed below.

The stability of switched system has been widely been investigated, but only some of the more relevant are summarized. [7, 8] introduced Multiple Lyapunov functions to study the stability of switched systems. In [9] Hespanha studied uniform stability for linear switched systems by extending the LaSalle's Invariance Principle which is also based on Multiple Lyapunov function method. Also finding one common Lyapunov function for proof of stability is implemented in [10] by using Lie-Algebra. The stability of switched linear system having dissipative Hamiltonian are also

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investigated under certain switching sequence related to power-converters with ideal switchings in Gerritsen, K., A. Van der Schaft, and W. Heemels's investigation [11]. Similarly Zhu, L. and Y. Wang [12], inspect the stability of switched dissipative Hamiltonian systems under arbitrary switching paths. Both articles [11, 12] chose their dissipative Hamiltonian function as their respective Multi-Lyapunov functions to consider the asymptotic stability of system. Our stability proof is similar to those in [11, 12] but slightly different because of lack of dissipation in our system which leads to investigation of regional stability for such systems. Here, the stability region is defined as a region that system states never leave if the initial condition is in predefined set.

In this study the rotor includes a shaft which is assumed to be rigid and is supported on two rolling bearings (either roller bearings or ball bearings). Also, in dynamic modeling, it is assumed that the system has two translational Degrees of Freedom (2-DOF). The mathematical formulation is presented in Section II.

In Section III by adopting the Multiple Lyapunov Function theorems and using Hamiltonian function as a candidate Lyapunov function, the regional stability of the system is studied considering switching in system's dynamic. Furthermore in Section IV the effects of load-stiffness parameter and rotor speed on regional stability of roller bearing and ball bearing will be investigated, and some numerical solutions are presented to show the relation between the solution trajectory and the deduced regional stability.

II. Procedure for Paper Submission

As it can be seen in Fig. 1, the rotor system under study contains rigid shaft which is supported by two rolling bearing. Here, two cases are considered for modeling, in the first case the system's bearings are considered to be ball bearing type and in the second case, the bearings are assumed to be roller bearing.

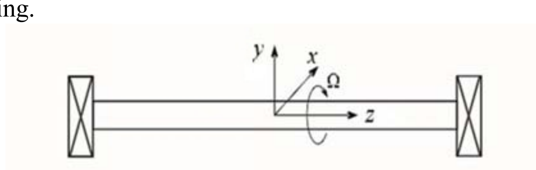


Fig. 1 The schematic diagram of rotor bearing system

Some assumptions which are included in this study are: the shaft is considered to be rigid, straight, symmetric and balance, the bearings are supposed to be frictionless. Furthermore, it is assumed that the shaft has two translational degrees of freedom in x, y direction depicted in Fig. 1.

First, in this section the kinetic energy terms of the system will be evaluated and at the next step the elastic deformation energy of both types of bearings will be calculated, to extract the Hamiltonian Function of rotor bearing system.

A. Kinematic Analysis

In this study two coordinate systems are considered. The first coordinate system, XYZ is global coordinate system which is located at mass center of the rotor and its Z axis is along with the center line of the bearings' outer race and shaft rotates with rotation speed **Ω** around the Z axes. The second coordinate system xyz is considered to be attached to the rotor at the mass center point and its unit vectors are introduced by i_c, j_c, and k_c. Because of radial clearance and elastic characteristic of the rolling elements of bearings, the shaft can have displacements in perpendicular direction, so this displacements x and y are considered to be along i_c, j_c directions so, the position of mass center of rotor, ***r***, can be written as:

$$\vec{r} = x\,\hat{i}_c + y\,\hat{j}_c \tag{1}$$

Differentiating (1) with respect to time, the velocity vector $\dot{\vec{r}}$ is:

$$\dot{\vec{r}} = \left(\dot{x} - y\Omega\right)\hat{i}_c + \left(\dot{y} + x\Omega\right)\hat{j}_c \tag{2}$$

By considering that m is the mass of the rotor and I_p is the rotor's moment of inertia along the Z axes, the kinetic energy of the system can be computed as:

$$T = \frac{1}{2}m\left(\left(\dot{x} - y\Omega\right)^2 + \left(\dot{y} + x\Omega\right)^2\right) + \frac{1}{2}I_p\Omega^2 \tag{3}$$

B. Modeling Bearing

In this study, the system is modeled with two type of bearings; roller bearing and ball bearing. The schematic figures of the bearings are shown in Figs. 2, where Fig.2 (a) shows the angular position of *j*-th rolling elements for both type of bearing, Figs. 2(b) and 2(c) represent the elastic deformation of *j*-th roller element in the roller bearing and ball element in the ball bearing respectively.

In order to obtain potential energy of the system, Hertzian contact theory is applied. By considering the kinematic shown in Fig. 2(a) the angular position of *j*-th rolling element, ψ_j, for both type of bearings can be obtained by:

$$\psi_j = \omega_c t + \varphi_j \qquad j = 1,2,\dots N$$
$$in\,which\ \ \varphi_j = \frac{2\pi j}{N} \tag{4}$$

where N represents the number of rolling elements and ω_c is the rotation speed of bearing's cage and is [5, 6]:

$$\omega_c = A\,\Omega\ ,\ \ A = \frac{r_i}{r_i + r_o} \tag{5}$$

where r_i and r_o are inner race and outer race radius.

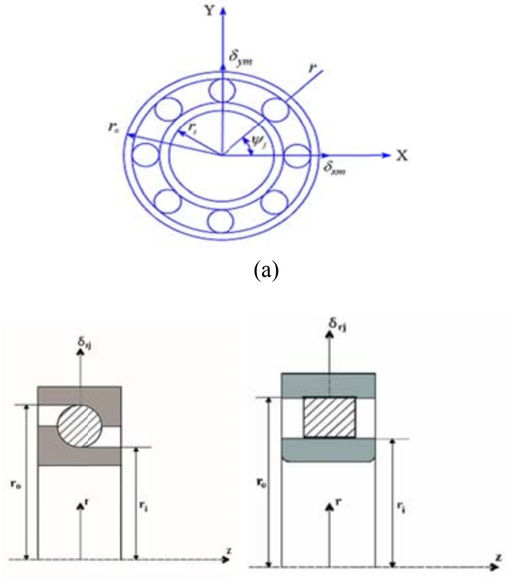
Due to relative displacement of inner and outer race of bearing, each rolling element of bearing can have elastic deformation. In this study the outer race is fixed on a rigid support and inner race is fixed on the shaft, so it has the same displacement as the shaft. So, the inner race displacement is:

$$\delta_{xm} = x\,Cos\Omega t - y\,Sin\Omega t$$
$$\delta_{ym} = x\,Sin\Omega t + y\,Cos\Omega t \tag{6}$$

where δ_{xm} and δ_{ym} are inner race displacement in global coordinate system.

The effective displacement of *j*-th rolling element for both roller bearing [4] and ball bearing can be written as[5, 6]:

$$\delta_{rj} = \delta_{xm} Cos\psi_j + \delta_{ym} Sin\psi_j - \delta_c \tag{7}$$



(a)

(b) (c)

Figs. 2 (a).Description of the bearings, location of the j-th rolling element, (b) The elastic deformation of the j-th ball element, (c) The elastic deformation of the j-th ball element

Substituting Eqs. (4)-(6) into Eq. (7), we have:

$$\delta_{rj} = x\,Cos\left(B\,t+\varphi_j\right) + y\,Sin\left(B\,t+\varphi_j\right) - \delta_c \tag{8}$$

in which $B = (A-1)\Omega$

In order to omit the explicit term of time from the system formulation for stability analysis, the coordinate transformation is considered as:

$$\begin{aligned} x &= -X\,Sin\left(B\,t\right) + Y\,Cos\left(B\,t\right) \\ y &= X\,Cos\left(B\,t\right) + Y\,Sin\left(B\,t\right) \end{aligned} \tag{9}$$

If above coordinate transformation is applied to the system, the terms T and δ_{rj} can be rewritten in new coordinate system as:

$$\begin{aligned} T &= \frac{1}{2} m\left(\dot{X}^2 + \dot{Y}^2 + 2A\Omega(\dot{X}\,Y - \dot{Y}\,X) + A^2\Omega^2(X^2+Y^2)\right) \\ &+ \frac{1}{2} I_p \Omega^2 \\ \delta_{rj} &= X\,Sin\left(\varphi_j\right) + Y\,Cos\left(\varphi_j\right) - \delta_c \end{aligned} \tag{10}$$

By considering Fig.2(c) for a roller bearing the elastic deformation of roller elements δ_{Rj} is:

$$\begin{aligned} \delta_{Rj} &= \delta_{rj} \\ \delta_R\left(\psi_j\right) &= \begin{cases} \delta_{Rj} & \delta_{Rj} > 0 \\ 0 & \delta_{Rj} \le 0 \end{cases} \end{aligned} \tag{11}$$

where above equation shows that for $\delta_{Rj} \le 0$ the *j*-th roller element is not under compression.

By adopting similar condition for ball elements, the *j*-th ball element's elastic deformation can be written as:

$$\begin{aligned} \delta_{Bj} &= \delta_{rj} \\ \delta_B\left(\psi_j\right) &= \begin{cases} \delta_{Bj} & \delta_{Bj} > 0 \\ 0 & \delta_{Bj} \le 0 \end{cases} \end{aligned} \tag{12}$$

By applying Hertzian contact theory for *j*-th rolling element (for both of roller and ball elements) the corresponding resultant normal load can be assumed as:

$$Q_j = K_n \delta^n\left(\psi_j\right) \tag{13}$$

where $\delta\left(\psi_j\right)$ can be either δ_{Rj} or δ_{Bj} .

The elastic deformation energy of *j*-th rolling elements could be written as:

$$U_j = \frac{K_n}{n+1} \delta^{n+1}\left(\psi_j\right) \tag{14}$$

By applying superposition principal on all of rolling elements, the potential energy of the system, *U*, is derived from Eq. (14) as:

$$U = \sum_{j=1}^{N} \frac{K_n}{n+1} \delta^{n+1}\left(\psi_j\right) \tag{15}$$

Due to similarity of both left and right bearings in the case studies (rotor-roller bearing and rotor-ball bearing systems), it can be concluded that the potential energy for both of rotor bearing systems is:

$$U = \sum_{j=1}^{N} 2\frac{K_n}{n+1} \delta^{n+1}\left(\psi_j\right) \tag{16}$$

By substituting Eqs. (6)-(10) into Eq. (16), and considering exponent 'n' is equal to 3/2 for ball elements with elliptical contact and is 10/9 for roller type with rectangular contact [4], the potential energy for the first case study with roller bearing, U_R, and the second one with ball bearings, U_B, are:

$$\begin{aligned} U_R &= \sum_{j=1}^{N} 18\frac{K_R}{19} \delta_R^{\frac{19}{9}}\left(\psi_j\right) \\ U_B &= \sum_{j=1}^{N} 4\frac{K_B}{5} \delta_B^{\frac{5}{2}}\left(\psi_j\right) \end{aligned} \tag{17}$$

where K_R and K_B are effective stiffness constant for roller elements' contact and ball elements' contact respectively.

III. Regional Stability Analysis

In this article by considering bearing's radial clearance and dynamic configuration in both of the rotor systems, each rolling elements can be compressed between the races (turn into active rolling element) or lose its contact and become stress free element (turn in to non-active rolling element). Hence, considering changes in contact status of roller elements and consequently, the change in the dynamic equations of system these rotor systems could be categorized as switching systems. In other words the system has finite different dynamics as:

$$\dot{\underline{x}} = f_{i(t)}\left(\underline{x}\right) \qquad 1 \le i\left(t\right) \le \Upsilon \tag{18}$$

where $\underline{x} \in \Re^n$ are the states of the dynamical system, the index *i(t)* represents that system is in which mode of contact and this index is changed with respect to contact condition defined in

(11), (12) and Υ indicates the number of all possible modes of switching in the system that can occur.

As a result of geometric arrangement of rolling elements in the bearing, and due to the radial clearance of bearing the maximum number of rolling elements which could be in contact (under compression) is $\left[\frac{N+1}{2}\right]$ where brackets represents the floor function. Hence, Υ represents the number of all subsets of I={1, … , N} which have at most $\left[\frac{N+1}{2}\right]$ members.

When the system is in the *i(t)* mode, we can use Hamilton Function as an eligible Lyapunov Function. Since the system is a Hamiltonian system, Hamilton Function in each mode can be written as:

$$H = T_2 + U - T_0 \tag{19}$$

in which the subscripts of T denote the degree of homogeneity in the generalized velocity variables, $\dot{q}_i$.

Due to frictionless assumption of roller elements contact, there is no damping in this dynamical system. As a result, in each mode of system, *i(t)*, and when no switching occurs, the value of Hamilton Function is constant [2,3], i.e., $\dot{H} = 0$.

It is notable that at the instance of contact (switching time), there is no compression on the rolling element, so there will be no change in the potential energy in switching. Also in small infinitesimal time switching happens, the kinetic energy will remain the same. Hence at switching instant the value of Hamilton Function doesn't jump and remains the same. Therefore, it can be concluded that the Hamilton value of this hybrid system remains constant in the total time interval including all the switchings. This property will be later used in stability analysis.

After each switching phenomena in the system, the number of active rolling elements (rolling elements under compression) could alter; as a result of these changes, different dynamic modes will be activated. The Multiple Lyapunov Function approach, proposed in [8], is applied here for stability analysis as declared in the following theorem:

Theorem Suppose there are candidate Lyapunov functions $V_i(\underline{x})$, $i = 1, \ldots, \Upsilon$, for respective vector fields $\dot{\underline{x}} = f_i(\underline{x})$, and each V_i has following properties

1. For each possible x in mode i , there is a real constant *c*, that $V_i(\underline{x}) \geq c$,and the boundedness of $V_i(\underline{x})$ should lead to boundedness of states, x

2. All V_i are non-increasing in their corresponding modes [$\dot{V}_i(\underline{x}) \leq 0$]

3. $V_{i(t^-)} = V_{i(t^+)}$ at each switching time

Then, it can be inferred the switching system is stable in Lyapunov sense.

Hamilton function in each mode *i(t)* is an appropriate Lyapunov function V_i . As it was mentioned before chosen Lyapunov functions (Hamilton Functions) have no change at the switching time; i.e, a moment before and after the switch, $V_{i(t^-)} = H_{i(t^-)} = H_{i(t^+)} = V_{i(t^+)}$, therefore third condition is satisfied. Also, the second condition is fulfilled due to constant value of Hamilton Functions in each mode of system as mentioned before. Therefore, the only remaining condition that assures the stability of the system is the first condition. Regarding the Hamilton function in (19), the terms T_2 is dependent on generalized velocity and $(U - T_0)$ depends on generalized coordinate, so in order to verify the first criteria, the terms T_2 and $U - T_0$ are analyzed separately.

Due to constant value of Hamilton function, which is equal to initial value, H_0, and the positive definiteness of term T_2 ($T_2 \geq 0$), we could write:

$$U - T_0 = H_0 - T_2 < H_0 \tag{20}$$

So, it can be inferred that the term $U - T_0$ is bounded from above. Also because of higher power of generalized coordinates in positive definite potential term U with respect to centrifugal term T_0 the lower boundness of the term $U - T_0$ is easily confirmed; $U - T_0 > -u$ where $u > 0$. So the term $(U - T_0)$ is completely bounded from above and below. Furthermore $(U - T_0)$ is a non-fractional function in poltnmial format, which leads to boundedness of generalized coordinates that comprise closed region in (X, Y) domain. It is obvious that system cannot leave the mentioned region. So this region, namely; Γ_S is called the stability region of the system in (X,Y) domain. It is notable that the boundary of region Γ_S could be found by condition:

$$U - T_0 = H_0 \tag{21}$$

Also the upper bound on T_2 could be obtained from the lower bound of the term $U - T_0$ as:

$$T_2 < H_0 + u \tag{22}$$

Considering positive definitness and upper boundness of T_2 , it can be inferred that, generalized velocities are also bounded. So if the condition (1) of Theorem holds, the system will be stable and all states remains in certain region, in particular the displacement of rotor is confined in Γ_S . For further discussion of this type of stability, the following definition is given:

Definition of δ_s -region stable: A system is δ_s -region stable if $\Gamma_S \subseteq C(\delta_s)$ where, $C(\delta_s)$ is square region in X-Y plane with width $2\delta_s$ and its center is at origin.

The term $U - T_0$ can be written as:

$$U - T_0 = \Delta - \frac{1}{2} I_p \Omega^2 \tag{23}$$

where the term Δ for roller bearing and ball bearing can be written as:

$$\Delta_R = \sum_{j=1}^{N} 18 \frac{K_R}{19} \delta_R^{\frac{19}{9}} \left(\psi_j\right) - \frac{1}{2} m A^2 \Omega^2 (X^2 + Y^2) \tag{24}$$

$$\Delta_B = \sum_{j=1}^{N} 4\frac{K_B}{5}\delta_B^{\frac{5}{2}}\left(\psi_j\right) - \frac{1}{2}mA^2\Omega^2(X^2+Y^2) \tag{25}$$

where Δ_R and Δ_B are related to the rotor system which is supported by roller bearings and ball bearings respectively.
In the similar way, the term H_0 can be rewritten as:

$$H_0 = -\frac{1}{2}I_p\Omega^2 + \Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) \tag{26}$$

where for roller bearing, $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0)$ is defined as:

$$\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) = \sum_{j=1}^{N} 18\frac{K_R}{19}\left(\delta_R^{\frac{19}{9}}\left(\psi_j\right)\Big|_{\substack{X=X_0\\Y=Y_0}}\right) + \frac{1}{2}m(\dot{X}_0^2+\dot{Y}_0^2) - \frac{1}{2}mA^2\Omega^2(X_0^2+Y_0^2) \tag{27}$$

and for ball bearing, we have:

$$\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) = \sum_{j=1}^{N} 4\frac{K_B}{5}\left(\delta_B^{\frac{5}{2}}\left(\psi_j\right)\Big|_{\substack{X=X_0\\Y=Y_0}}\right) + \frac{1}{2}m(\dot{X}_0^2+\dot{Y}_0^2) - \frac{1}{2}mA^2\Omega^2(X_0^2+Y_0^2) \tag{28}$$

Hence, without any loss of generality, the expressed condition in Eq.(21) can be expressed as:

$$\Delta = \Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) \tag{29}$$

which means the boundary of Γ_S is dependent on the initial value of $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0)$, which itself dependent on initial displacement and velocities of dynamical system.
Depending on initial conditions, $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0)$ could take positive or negative values. But for analysis in the rest of article the value of Λ equals zero is taken as a reference. In this case H_0 becomes $-\frac{1}{2}I_p\Omega^2$.

IV. Results And Discusions

In this section, regional stability of rotor-roller bearing system and rotor-ball bearing system are studied numerically. Also the effect of load-stiffness parameter and rotation speed on the regional stability of system for both types of bearings will be inspected. The specifications of roller and ball bearing are described in Table 1.
Due to restricted deflection capacity of each rolling element, in this study the generalized coordinate are considered to have the maximum value of $n\times\delta_c$ (n is real positive number which is chosen considering the desired region of stability). So, by considering this assumption, the functions Δ_R and Δ_B are plotted with respect to the displacement (x , y) within this domain. It is notable that in this system the switching phenomenon takes place frequently, as a result in each displacement the number of active rolling elements alters. For instance when the generalized coordinates are in the clearance area, $X^2+Y^2 \le \delta_c^2$, there are no rolling elements under compression and the terms Δ_R and Δ_B has negative value in this region, so the generalized coordinates x and y are inclined to increase. But out of this clearance area the rolling elements comes in contact gradually and the values of Δ_R and Δ_B increase accordingly. It is assumed that the radial clearance value and inner and outer race radius are the same in both types of bearing which support the same rotor.

TABLE I
Design Parameters For Typical Ball And Roller Bearings

Parameters	Descriptions	Values
r_i	Bearing's inner race radius	39(mm)
r_0	Bearing's outer race radius	51(mm)
δ_c	Radial clearance	20 μm
m	Rotor mass	10Kg

For roller bearing, the term Δ_R is plotted in Fig.3 for *n=3*. The result also clearly emphasizes that due to higher power of generalized coordinates in the term *U* with respect to T_0, the system is stable.

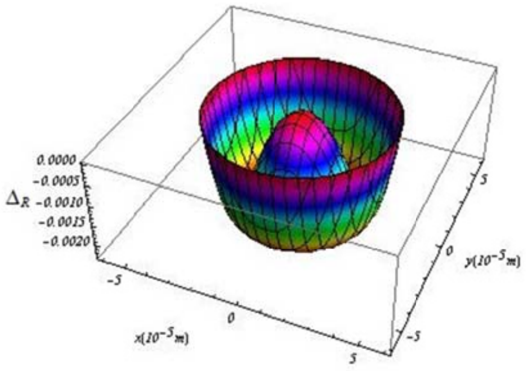
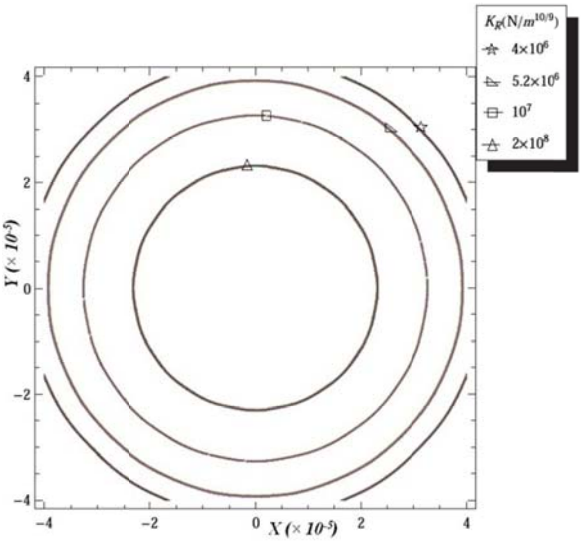


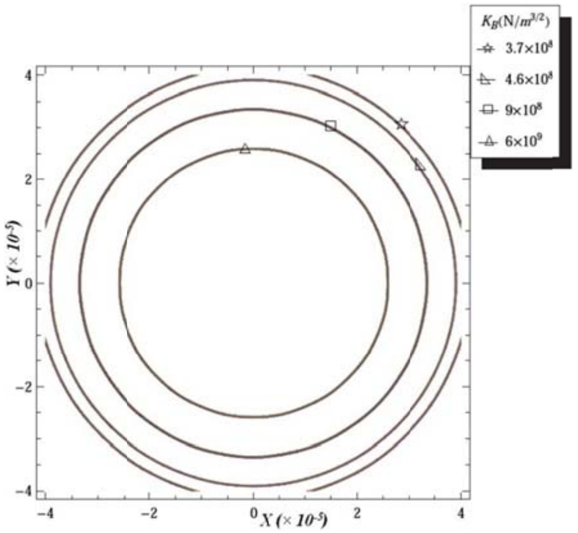
Fig.3 Surface of Δ_R for rotor- roller bearing system in the case of $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) = 0$; $\Omega = 2000\,rad/s$, $N = 14$, $K_R = 2.16\times10^7\ N/m^{10/9}$

Taking into account "δ_s-region stable" definition which is highlighted before, $2\delta_c$-region stability of the system is investigated for both of the rotor-roller bearing and the rotor-ball bearing systems which is illustrated in Figs.4. These figures show the effect of load-stiffness parameters (K_R, K_B) on desired regional stability ($2\delta_c$) of the system. As it can be seen in Figs.4 (a) and Figs.4(b), increase in the value of K_R, K_B, makes system to reach smaller stability region Γ_s (which means lower amplitudes in oscillations of rotor). For instance with $K_R = 2\times10^8\ N/m^{10/9}$ for different initial conditions which satisfy $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) = 0$ or $H_0 = -1/2\,I_p\Omega^2$, the system would remain in its corresponding region, Γ_s.

But as K_R decrease the region enlarges where in $K_R = 4\times10^6\ N/m^{10/9}$ the system in no more $2\delta_c$-region stable.



(a)



(b)

Figs. 4 Effect of load- stiffness parameter of rolling element contact on stability region, Γ_S, for ; a) rotor-roller bearing system, b) rotor-ball bearing system. For both cases

$N = 16, \Omega = 1000\, rad\,/\,s$

As it is obvious from Eq. (24) and (25) the increase in rotation speed will enlarge the region that system remains in. Considering the definition of $2.5\delta_c$ stability, the effect of rotation speed on regional stability of both rotor-roller bearing and rotor-ball bearing systems are inspected and shown in Fig.5. Accordingly, the results show by increase in the value of rotation speed of the rotor, the region Γ_s; would grow up which means higher amplitude in oscillation of rotor.

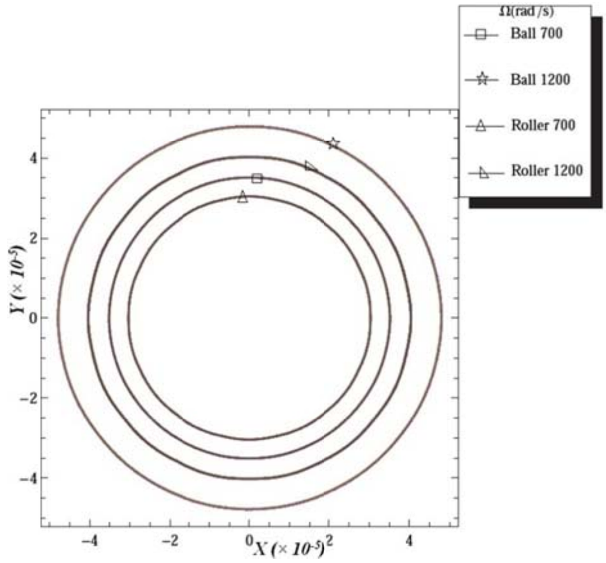


Fig. 5 Effect of rotation speed of rotor on stability region, Γ_S, for both roller bearing and ball bearing cases,

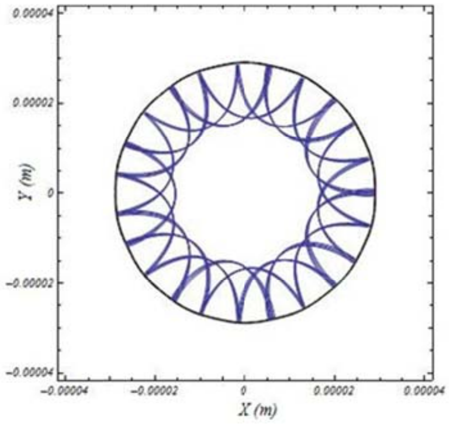
$K_R = 8\times10^6\, N\,/\,m^{10/9}$, $K_B = 4\times10^8\, N\,/\,m^{3/2}, N = 14$

In order to validate the stability results, the nonlinear dynamic equations of motion for system are solved, numerically. By utilizing Lagrange method the equations of motion for system can be written as:

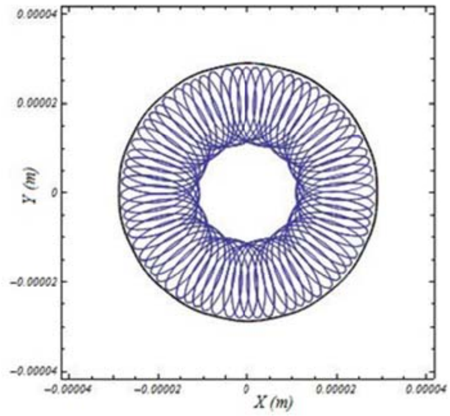
$$
\begin{aligned}
&m\ddot{X} + Am\Omega\dot{Y} - mA^2\Omega^2 X + 2\sum_{j=1}^{N} K_R Sin\varphi_j \delta_R^{10/9}\left(\psi_j\right) = 0 \\
&m\ddot{Y} - Am\Omega\dot{X} - mA^2\Omega^2 Y + 2\sum_{j=1}^{N} K_R Cos\varphi_j \delta_R^{10/9}\left(\psi_j\right) = 0
\end{aligned}
\tag{30}
$$

Above equations are solved for two different initial conditions which satisfy $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) = 0$ and an initial condition where $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) < 0$. Results are shown in X-Y plane, illustrating the center of mass movement (see Figs.6). Also the stability region of system; Γ_s which is the result of regional stability analysis of system has given in Fig.6(a)-6(c) for comparison.

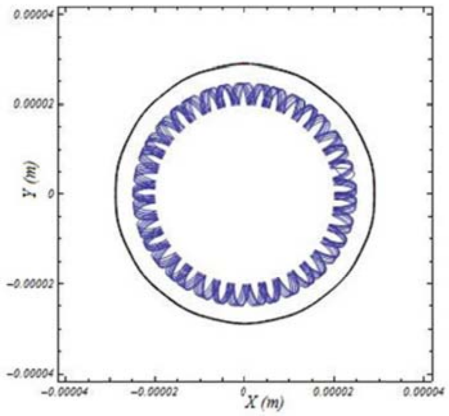
As it can be seen in Figs. 6(a) and 6(b) for $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) = 0$, the trajectory of center of mass remains in its corresponding stability region, Γ_s, where at some location becomes tangent with the border of Γ_s. As it is expected, for initial conditions where $\Lambda(X_0, Y_0, \dot{X}_0, \dot{Y}_0) < 0$, the trajectory of system is completely inside the obtained stability region, admittedly Fig. 6(c) depicts this prediction as well.



(a)



(b)



(c)

Figs. 6 Displacement of rotor-roller bearing system($N = 14$, $K_R = 2.16 \times 10^7 \, N / m^{10/9}$,$\Omega = 1000 \, rad / s$) in X-Y plane under different initial condition for; a) $\Lambda(X_0 ,Y_0 ,\dot{X}_0 ,\dot{Y}_0) = 0$ where $X_0 = Y_0 = 2 \times 10^{-5} m$, $\dot{X}_0 = \dot{Y}_0 = 2.47 \times 10^{-3} m / s$; b) $\Lambda(X_0 ,Y_0 ,\dot{X}_0 ,\dot{Y}_0) = 0$ where $X_0 = Y_0 = \sqrt{2} \times 10^{-5} \, m$ $\dot{X}_0 = \dot{Y}_0 = 6.13 \times 10^{-3} m / s$; c) $\Lambda(X_0 ,Y_0 ,\dot{X}_0 ,\dot{Y}_0) =$ -0.00038 where $X_0 = \sqrt{2} \times 10^{-5}$,$Y_0 = \sqrt{2} \times 10^{-5} \, m$, $\dot{X}_0 = \dot{Y}_0 = 0 \, m / s$

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