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Design of Nonlinear Observer by Using Chebyshev Interpolation based on Formal Linearization

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Abstract—This paper discusses a design of nonlinear observer by a formal linearization method using an application of Chebyshev Interpolation in order to facilitate processes for synthesizing a nonlinear observer and to improve the precision of linearization.

A dynamic nonlinear system is linearized with respect to a linearization function, and a measurement equation is transformed into an augmented linear one by the formal linearization method which is based on Chebyshev interpolation. To the linearized system, a linear estimation theory is applied and a nonlinear observer is derived. To show effectiveness of the observer design, numerical experiments are illustrated and they indicate that the design shows remarkable performances for nonlinear systems.

Keywords—nonlinear system, nonlinear observer, formal linearization, Chebyshev interpolation.

I. Introduction

NATURALLY, the estimation problem is more difficult and less understood when systems are nonlinear than linear. The most practical way to approach the nonlinear problem is to employ linearization in order to apply the linear system theories [1]–[6]. Formal linearization [7]–[10] is one of them to treat with these nonlinear problems.

In the previous work, a nonlinear observer design using the formal linearization method based on Chebyshev expansion was considered [10]. In this paper, we develop a nonlinear observer design by the formal linearization method based on Chebyshev interpolation in order to make processes of the design easier. Introducing a linearization function which consists of the Chebyshev polynomials of the state variables, and an augmented measurement vector which consists of polynomials of the measurement variables, a given nonlinear system is transformed into an augmented linear system by using Chebyshev interpolation. By this linearized system, a nonlinear observer is derived by applying linear system theories. Inversion is simple because of the original state variable involved in the linearization function.

An advantage in this method comparing with the previous work [10] is that coefficients of linearized system are simply obtained by carrying out summation due to the orthogonality for a finite sum, while they are executed by the integral calculus in the previous method.

Numerical experiments are illustrated to verify the effectiveness of this observer design in comparison with a conventional linearization based on Taylor expansion truncated at the first order.

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II. Statement of Problem

Consider a nonlinear dynamic system described by a state differential equation

$$\Sigma_1 : \dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t)), \tag{1}$$
$$\boldsymbol{x}(0) = \boldsymbol{x}_0 \in D$$

where t denotes time, $\cdot = d/dt$, $\boldsymbol{x}$ is an $n \times 1$ state vector, and $\boldsymbol{f}$ is a sufficiently smooth nonlinear function. D is a compact domain denoted by the Cartesian product:

$$D = \prod_{i=1}^{n} [l_i - p_i, l_i + p_i] \subset R^n$$

where l_i $(l_i \in R)$ is the middle of the domain of x_i and p_i $(p_i > 0)$ is half of the domain of x_i $(i = 1, \cdots, n)$.

Assumed that a measurement equation is given by

$$\boldsymbol{\eta}(t) = \boldsymbol{h}(\boldsymbol{x}(t)) \in R^\ell \tag{2}$$

where $\boldsymbol{\eta}$ is an $\ell \times 1$ output vector with $\ell < n$, and $\boldsymbol{h}(\boldsymbol{x})$ is a sufficiently smooth nonlinear function.

The problem is that the state of a nonlinear dynamic system (Eq. (1)) can be estimated by use of the given measurement output which is written by a nonlinear equation (Eq. (2)) .

III. Nonlinear Observer by Formal linearization

A. Formal Linearization for Dynamic System

In this method, Chebyshev interpolation is applied to linearize the given nonlinear system (Eq. (1)), and the state variable $\boldsymbol{x}$ is changed into $\boldsymbol{y}$ so that $\boldsymbol{y}$ has the basic domain of the Chebyshev polynomials

$$D_0 = \prod_{i=1}^{n} [-1, 1]$$

and $\boldsymbol{y}$ is rewritten by

$$\boldsymbol{y} = P^{-1}(\boldsymbol{x} - L) \in D_0 \tag{3}$$

where

$$L = \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}, P = \begin{pmatrix} p_1 & & 0 \\ & \ddots & \\ 0 & & p_n \end{pmatrix}, \boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}.$$

The given dynamic system (Eq. (1)) becomes

$$\dot{\boldsymbol{y}}(t) = P^{-1}\boldsymbol{f}(P\boldsymbol{y}(t) + L). \tag{4}$$

The Chebyshev polynomials $\{T_r(\cdot)\}$ are defined by

$$T_r(y_i) = \cos(r \cdot \cos^{-1} y_i), \ (r = 0, 1, 2, \cdots) \tag{5}$$

or,

$$T_0(y_i) = 1,\ T_1(y_i) = y_i,\ T_2(y_i) = 2y_i^2 - 1,$$
$$T_3(y_i) = 4y_i^3 - 3y_i,\ T_4(y_i) = 8y_i^4 - 8y_i^2 + 1, \cdots .$$

Its recurrence formula is

$$T_{q+1}(y_i) = 2y_iT_q(y_i) - T_{q-1}(y_i),\ (q \geq 1) \tag{6}$$
$$T_0(y_i) = 1,\ T_1(y_i) = y_i\ .$$

Therefore, the derivative of the Chebyshev polynomials

$$S_q(y_i) \equiv \frac{dT_q(y_i)}{dy_i} \tag{7}$$

is given by

$$S_{q+1}(y_i) = 2T_q(y_i) + 2y_iS_q(y_i) - S_{q-1}(y_i),\ (q \geq 1) \tag{8}$$
$$S_0(y_i) = 0,\ S_1(y_i) = 1\ .$$

Using these Chebyshev polynomials, we define an N-th order linearization function $\boldsymbol{\phi}(\cdot) = \boldsymbol{\phi}(\boldsymbol{y}(\cdot))$ which consists of the Chebyshev polynomials by

$$\begin{aligned} \boldsymbol{\phi} &= [\phi_1, \phi_2, \cdots, \phi_i, \cdots, \phi_{(N+1)^n-1}]^T \\ &= [T_{(10\cdots0)}(\boldsymbol{y}), T_{(01\cdots0)}(\boldsymbol{y}), \cdots, T_{(0\cdots01)}(\boldsymbol{y}), \\ &\quad T_{(11\cdots0)}(\boldsymbol{y}), T_{(101\cdots0)}(\boldsymbol{y}), \cdots, T_{(10\cdots1)}(\boldsymbol{y}), \\ &\quad T_{(20\cdots0)}(\boldsymbol{y}), T_{(02\cdots0)}(\boldsymbol{y}), \cdots, T_{(r_1\cdots r_n)}(\boldsymbol{y}), \\ &\quad \cdots, T_{(N\cdots N)}(\boldsymbol{y})]^T \end{aligned} \tag{9}$$

where

$$T_{(r_1\cdots r_n)}(\boldsymbol{y}) = \prod_{i=1}^{n} T_{r_i}(y_i)\ .$$

The derivative of each element of $\boldsymbol{\phi}$ along with the solution of the given nonlinear system (Eq. (1)) becomes

$$\begin{aligned} \dot{\phi}_\alpha(\boldsymbol{y}) &= \dot{T}_{(r_1\cdots r_n)}(\boldsymbol{y}) = \frac{\partial T_{(r_1\cdots r_n)}(\boldsymbol{y})}{\partial \boldsymbol{y}^T}\dot{\boldsymbol{y}} \\ &= [S_{r_1}(y_1)T_{r_2}(y_2)\cdots T_{r_{n-1}}(y_{n-1})T_{r_n}(y_n), \cdots, \\ &\quad T_{r_1}(y_1)T_{r_2}(y_2)\cdots T_{r_{n-1}}(y_{n-1})S_{r_n}(y_n)]P^{-1}\boldsymbol{f}(P\boldsymbol{y}+L) \\ &\equiv G_{(r_1\cdots r_n)}(\boldsymbol{y}),\ \alpha = \alpha(r_1, \cdots, r_n). \end{aligned} \tag{10}$$

Applying Chebyshev interpolation up to the N-th order, this $G_{(r_1\cdots r_n)}(\boldsymbol{y})$ is approximated by

$$\hat{G}_{(r_1\cdots r_n)}(\boldsymbol{y}) = \sum_{q_1=0}^{N} \cdots \sum_{q_n=0}^{N} C_{(q_1\cdots q_n)}^{(r_1\cdots r_n)} T_{(q_1\cdots q_n)}(\boldsymbol{y}) \tag{11}$$

where

$$\begin{aligned} C_{(q_1\cdots q_n)}^{(r_1\cdots r_n)} &\equiv \frac{2^{n-\gamma}}{\prod_{i=1}^{n}(N+1)} \sum_{j_1=0}^{N}\sum_{j_2=0}^{N}\cdots \\ &\quad \sum_{j_n=0}^{N} G_{(r_1\cdots r_n)}(y_{1j_1}, y_{2j_2}, \cdots, y_{nj_n}) \\ &\quad \times T_{q_1}(y_{1j_1})T_{q_2}(y_{2j_2})\cdots T_{q_n}(y_{nj_n})\ , \end{aligned} \tag{12}$$
$$\gamma = \{\text{the number of } q_i = 0 : 1 \leq i \leq n\}.$$

The interpolating points $\{y_{ij_i}\}$ are set to be

$$y_{ij_i} = \cos\frac{2j_i+1}{2N+2}\pi,\ (i = 1, \cdots, n,\ j_i = 0, \cdots, N)\ . \tag{13}$$

Substituting this $\hat{G}_{(r_1\cdots r_n)}(\boldsymbol{y})$ into Eq. (10) yields

$$\dot{\boldsymbol{\phi}}(\boldsymbol{y}) \approx A\boldsymbol{\phi}(\boldsymbol{y}) + b \tag{14}$$

where

$$[A_{\alpha\ \beta}] = [C_{(q_1\cdots q_n)}^{(r_1\cdots r_n)}] \in R^{((N+1)^n-1)\times((N+1)^n-1)}\ ,$$
$$[b_\alpha] = [C_{(0\cdots0)}^{(r_1\cdots r_n)}] \in R^{(N+1)^n-1},\ \beta = \beta(q_1, \cdots, q_n).$$

Thus a formal linear state differential equation is derived by

$$\Sigma_2 : \dot{\boldsymbol{z}}(t) = A\boldsymbol{z}(t) + b\ , \tag{15}$$
$$\boldsymbol{z}(0) = \boldsymbol{\phi}(\boldsymbol{y}(0)) = \boldsymbol{\phi}(P^{-1}(\boldsymbol{x}(0) - L))\ .$$

From Eqs. (3) and (9), the inversion is carried out as $\hat{\boldsymbol{x}}(t)$ by

$$\begin{aligned} \hat{\boldsymbol{x}}(t) &= P[I\ 0\ \cdots\ 0]\boldsymbol{\phi}(\boldsymbol{y}(t)) + L \\ &= P[I\ 0\ \cdots\ 0]\boldsymbol{z}(t) + L \end{aligned} \tag{16}$$

where I is an $n \times n$ unit matrix.

B. Formal Linearization for Measurement Equation

An augmented M-th order measurement vector $\boldsymbol{Y}(\cdot) = \boldsymbol{Y}(\boldsymbol{\eta}(\cdot))$ which consists of polynomials of the measurement variables are defined as

$$\begin{aligned} \boldsymbol{Y} &= [Y_1, Y_2, \cdots, Y_i, \cdots, Y_{(M+1)^\ell-1}]^T \\ &= [T'_{(10\cdots0)}(\boldsymbol{\eta}), T'_{(01\cdots0)}(\boldsymbol{\eta}), \cdots, T'_{(0\cdots01)}(\boldsymbol{\eta}), \\ &\quad T'_{(11\cdots0)}(\boldsymbol{\eta}), T'_{(101\cdots0)}(\boldsymbol{\eta}), \cdots, T'_{(10\cdots1)}(\boldsymbol{\eta}), \\ &\quad T'_{(20\cdots0)}(\boldsymbol{\eta}), T'_{(02\cdots0)}(\boldsymbol{\eta}), \cdots, T'_{(r_1\cdots r_\ell)}(\boldsymbol{\eta}), \\ &\quad \cdots, T'_{(M\cdots M)}(\boldsymbol{\eta})]^T \end{aligned} \tag{17}$$

where

$$T'_{(r_1\cdots r_\ell)}(\boldsymbol{\eta}) = \prod_{i=1}^{\ell} \eta_i^{r_i}\ .$$

From the given measurement equation (Eq. (2)), each element function of Eq. (17) is written as

$$\begin{aligned} Y_{\alpha'}(\boldsymbol{\eta}) &= T'_{(r_1\cdots r_\ell)}(\boldsymbol{\eta}) = \prod_{i=1}^{\ell} \eta_i^{r_i} \\ &= h_1^{r_1}(P\boldsymbol{y}+L)h_2^{r_2}(P\boldsymbol{y}+L)\cdots h_\ell^{r_\ell}(P\boldsymbol{y}+L) \\ &\equiv G'_{(r_1\cdots r_\ell)}(\boldsymbol{y}),\quad \alpha' = \alpha'(r_1, \cdots, r_\ell). \end{aligned} \tag{18}$$

To this new augmented measurement equation, apply Chebyshev interpolation up to the N-th order, and this $G'_{(r_1\cdots r_\ell)}(\boldsymbol{y})$ is approximated by

$$\hat{G'}_{(r_1\cdots r_\ell)}(\boldsymbol{y}) = \sum_{q_1=0}^{N} \cdots \sum_{q_n=0}^{N} C'^{(r_1\cdots r_\ell)}_{(q_1\cdots q_n)} T_{(q_1\cdots q_n)}(\boldsymbol{y}) \tag{19}$$

where

$$C'^{(r_1\cdots r_\ell)}_{(q_1\cdots q_n)} \equiv \frac{2^{n-\gamma'}}{\prod_{i=1}^{n}(N+1)} \sum_{j_1=0}^{N}\sum_{j_2=0}^{N}\cdots$$

$$\sum_{j_n=0}^{N} G'_{(r_1\cdots r_\ell)}(y_{1j_1}, y_{2j_2}, \cdots, y_{nj_n})$$

$$\times T_{q_1}(y_{1j_1})T_{q_2}(y_{2j_2})\cdots T_{q_n}(y_{nj_n})\ , \qquad (20)$$

$$\gamma' = \{\text{the number of } q_i = 0 : 1 \le i \le n\}.$$

Substituting this $\hat{G}'_{(r_1\cdots r_\ell)}(\boldsymbol{y})$ into Eq.(18), the augmented measurement equation becomes

$$\boldsymbol{Y} \approx D\boldsymbol{\phi}(\boldsymbol{y}) + e \qquad (21)$$

where

$$[D_{\alpha'\ \beta'}] = [C'^{(r_1\cdots r_\ell)}_{(q_1\cdots q_n)}] \in R^{((M+1)^\ell-1)\times((N+1)^n-1)}\ ,$$

$$[e_{\alpha'}] = [C'^{(r_1\cdots r_\ell)}_{(0\cdots 0)}] \in R^{((M+1)^\ell-1)},\ \beta' = \beta'(q_1, \cdots, q_n).$$

Thus a formal linear measurement equation is derived by

$$\boldsymbol{Y}(t) = D\boldsymbol{z}(t) + e\ . \qquad (22)$$

C. Design of Nonlinear Observer

To the above linearized system (Eqs. (15) and (22)), a linear estimation theory is applied so that the identity observer [11] is synthesized as

$$\dot{\hat{\boldsymbol{z}}}(t) = A\hat{\boldsymbol{z}}(t) + b + K(t)\big(\boldsymbol{Y}(t) - (D\hat{\boldsymbol{z}}(t) + e)\big), \qquad (23)$$

$$\hat{\boldsymbol{z}}(0) = \boldsymbol{\phi}(\hat{\boldsymbol{y}}(0)) = \boldsymbol{\phi}(P^{-1}(\hat{\boldsymbol{x}}(0) - L))$$

where $\hat{\boldsymbol{x}}(0)$ is an initial value of the observer, $K(t)$ is an observer gain as

$$K(t) = \frac{1}{2}R(t)D^T W(t) \in R^{\left((N+1)^n-1\right)\times\left((M+1)^\ell-1\right)}.$$

$R(t)$ satisfies the matrix Riccati differential equation as

$$\dot{R}(t) = AR(t) + R(t)A^T + Q(t) - R(t)D^T W(t)DR(t) \quad (24)$$

where $Q(t), W(t)$ and $R(0)$ are chosen to be arbitrary real, symmetric, and positive definite. With the reference to the exponential estimator [11], the error in the state estimate $\boldsymbol{e} = \boldsymbol{z} - \hat{\boldsymbol{z}}$ is uniformly asymptotically stable in the sense of Lyapunov.

From Eq.(16), the estimate of the nonlinear observer $\hat{\boldsymbol{x}}(t)$ becomes

$$\hat{\boldsymbol{x}}(t) = P[I\ 0\ \cdots\ 0]\boldsymbol{\phi}(\hat{\boldsymbol{y}}(t)) + L$$

$$= P[I\ 0\ \cdots\ 0]\hat{\boldsymbol{z}}(t) + L. \qquad (25)$$

IV. Numerical Experiments

Numerical experiments for nonlinear observers of scalar and multidimensional systems are illustrated. For comparison, a conventional linearization based on Taylor expansion truncated at the first order is also depicted.

A. Nonlinear Observer for Scalar System

As a simple example, consider a dynamic scalar system

$$\dot{x} = x^2 \qquad (26)$$

$$x(0) = 0.9,\ D = [-1, 0] \subset R$$

and a measurement equation

$$\eta = \sqrt{1 + x}. \qquad (27)$$

To apply the above formal linearization in Sec. III, the values for changing state variable in Eq. (3) are set as

$$L = -0.5, P = 0.5,$$

the linearization function and the augmented measurement vector are

$$\boldsymbol{\phi} = \begin{pmatrix} y \\ 2y^2 - 1 \\ 4y^3 - 3y \end{pmatrix}, \boldsymbol{Y} = \begin{pmatrix} \eta \\ \eta^2 \\ \eta^3 \end{pmatrix}$$

respectively, when the order of the linearization function N and the measurement vector M are $N = M = 3$. In this case, the formal linear system (Eq. (15)) becomes

$$\dot{\boldsymbol{z}}(t) = \begin{pmatrix} -1 & 0.25 & 0 \\ 3.5 & -2 & 0.5 \\ -6 & 5.25 & -3 \end{pmatrix} \boldsymbol{z}(t) + \begin{pmatrix} 0.75 \\ -2 \\ 3 \end{pmatrix},$$

and the augmented measurement equation (Eq. (22)) is

$$\boldsymbol{Y}(t) = \begin{pmatrix} 0.416 & -0.075 & 0.023 \\ 0.5 & 0 & 0 \\ 0.51 & 0.072 & -0.007221 \end{pmatrix} \boldsymbol{z}(t) + \begin{pmatrix} 0.641 \\ 0.5 \\ 0.424 \end{pmatrix}.$$

Fig. 1 shows the true value x of Eq. (26) and the approximated values $\hat{x}$ of Eq.(16) when N is varied from $N = 1$ to 3. $\hat{x}$(Taylor) refers to a result obtained by Taylor expansion truncated at the first order :

$$\dot{\hat{x}} = -\hat{x} - \frac{1}{4}$$

when the operating point is -0.5 for comparison.

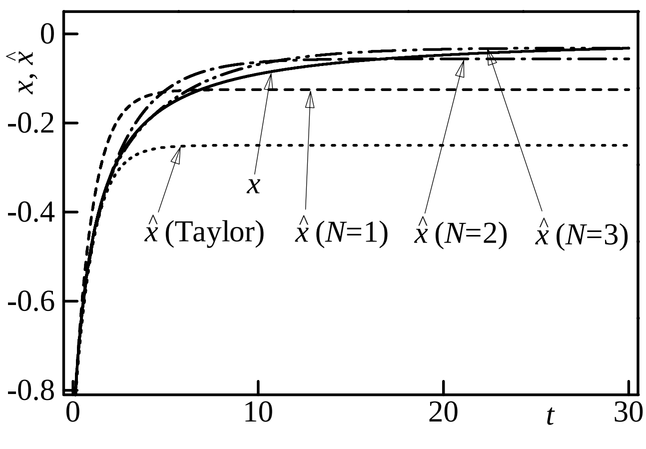

Fig. 1. True value $x(t)$ and approximated values $\hat{x}(t)$ of linearization

Fig. 2 depicts the true value η of Eq. (27) and the approximated values Y of Eq.(21) when N is varied from $N = 1$ to

3 and M is fixed at 1. $\hat{\eta}$(Taylor) is a result obtained by Taylor expansion truncated at the first order :

$$\hat{\eta} = \frac{1}{\sqrt{2}}x + \frac{3\sqrt{2}}{4}$$

when the operating point is -0.5.

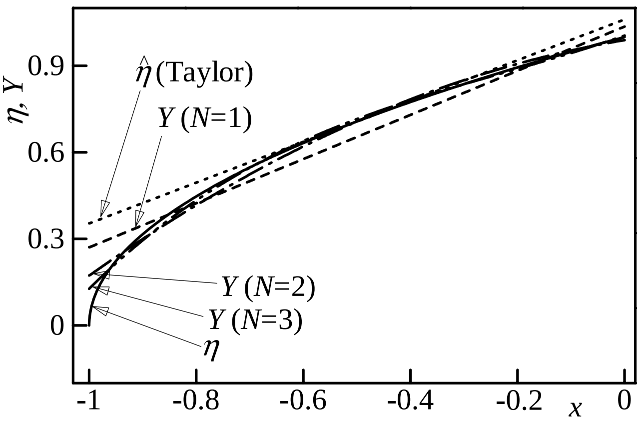

Fig. 2. True value η and approximated values of measurement equation

To synthesize the nonlinear observer for the given system (Eqs. (26) and (27)), parameters for the nonlinear observer (Eq. (23)) are set by the unknown value $x(0) = 0.9$, an initial value of the observer $\hat{\hat{x}}(0) = -0.01$ in Eq. (23),

$$R(0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = diag(1, 1, 1),$$

$$W(t) = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} = diag(5, 5, 5)$$

when $N = 3$,

$$R(0) = diag(1, 1),\ W(t) = diag(5, 5)$$

when $N = 2$, and

$$R(0) = 1,\ W(t) = 5$$

when $N = 1$.

Fig. 3 shows true value x and estimates $\hat{\hat{x}}$ of Eq. (25) when the orders of N and M are varied from 1 to 3. $\hat{\hat{x}}$ (Taylor) is a result by the conventional first order Taylor expansion for comparison when the parameters for the observer are the same $R(0) = 1$ and $W(t) = 5$ as our method of the order $N = M = 1$.

Fig. 4 shows the integral square errors of estimation

$$J(t) = \int_0^t \left(x(\tau) - \hat{\hat{x}}(\tau)\right)^2 d\tau$$

for the various orders from $N = M = 1$ to $N = M = 3$ and the conventional Taylor method($Taylor$).

Next experimentats are the results when the order of linearization function is fixed at $N = 3$ and the order of the augmented measurement vector is varied from $M = 1$ to 3. Fig. 5 shows x and $\hat{\hat{x}}$. Fig. 6 shows the integral square errors of estimation in this case.

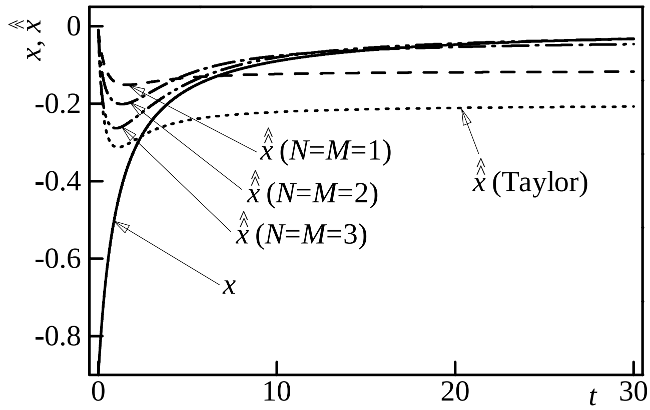

Fig. 3. Estimates $\hat{\hat{x}}(t)$ of the scalar system by various orders

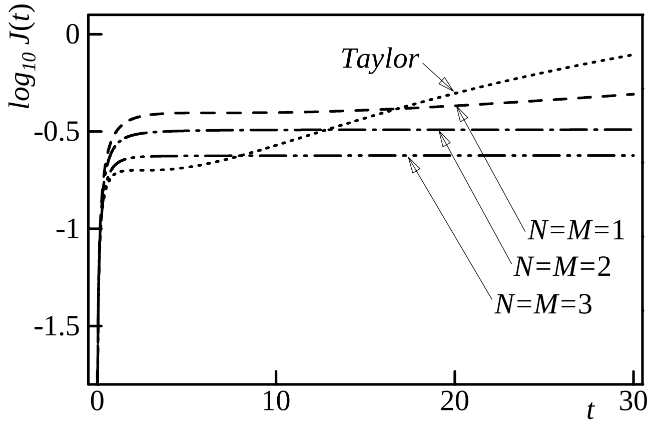

Fig. 4. Integral square errors of estimation of the scalar system by various orders

B. Nonlinear Observer for Multidimensional System

As a multidimensional system, consider the simple pendulum in which the bob is connected to the rod which has zero mass. Let θ denote the angle subtended by the rod and the vertical axis through the pivot point. The dynamic equation of this system is written as

$$\frac{d^2}{dt^2}\theta + a_1 \frac{d}{dt}\theta + a_2 \sin(\theta) = 0. \tag{28}$$

Assume that the position of the bob is measured from above and the measurement equation is

$$\eta = a_3 \sin(\theta). \tag{29}$$

Taking the state variables as $x_1 = \theta$ and $x_2 = \dot{\theta}$, the dynamic system (Eq. (28)) is described as

$$\begin{cases} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_1 x_2 - a_2 \sin(x_1) \end{cases} \tag{30}$$

and the measurement equation is

$$\eta = a_3 \sin(x_1) \tag{31}$$

Applying the above formal linearization in Sec. III, a formal linear system is obtained by

$$\dot{\boldsymbol{z}}(t) = A\boldsymbol{z}(t) + b\ , \tag{32}$$

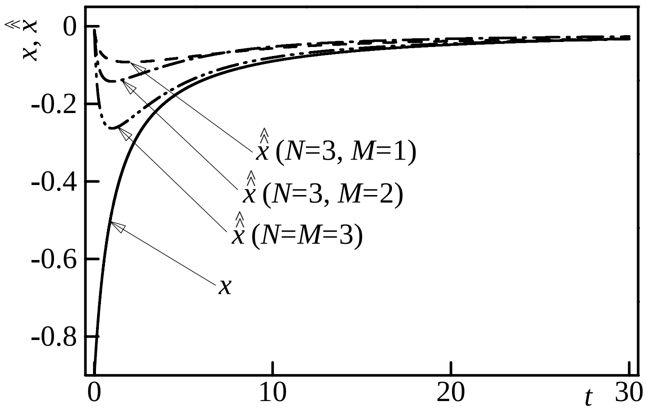

Fig. 5. Estimates $\hat{\hat{x}}(t)$ of the scalar system by various orders of M when N is fixed at 3

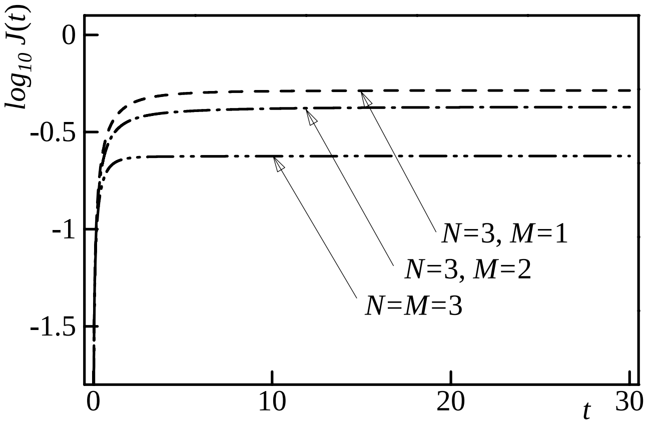

Fig. 6. Integral square errors of estimation of the scalar system by various orders of M when N is fixed at 3

$$\boldsymbol{Y}(t) = C\boldsymbol{z}(t) + d\ , \tag{33}$$

and the observer of this pendulum system is

$$\dot{\hat{\boldsymbol{z}}}(t) = A\hat{\boldsymbol{z}}(t) + b + K(t)\big(\boldsymbol{Y}(t) - C\hat{\boldsymbol{z}}(t) - d\big), \tag{34}$$
$$\hat{\boldsymbol{z}}(0) = \boldsymbol{\phi}(\hat{\boldsymbol{x}}(0)).$$

Throughout this experiments, the system parameters are set as

$$a_1 = 0.5,\ a_2 = \frac{980.7}{400},\ a_3 = 1.$$

For the formal linearization, the values for changing state variable in Eq. (3) are set as

$$L = \begin{pmatrix} 0.4 \\ -0.4 \end{pmatrix},\ P = \begin{pmatrix} 1.3 & 0 \\ 0 & 1.5 \end{pmatrix}.$$

To synthesize the observer, parameters for the nonlinear observer (Eq. (34)) are set by

$$\boldsymbol{x}(0) = \begin{pmatrix} 1.5 \\ 1 \end{pmatrix},\ \hat{\boldsymbol{x}}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$R(0) = I$,$W(t) = diag(10, 5, 2)$, $N = 3$, and $M = 3$.

Figs. 7 and 8 show the true values $x_i(t)$ and the estimated values $\hat{\hat{x}}_i(t)$ for $i = 1$ and 2, respectively. $\hat{\hat{x}}_i$(Taylor) are the

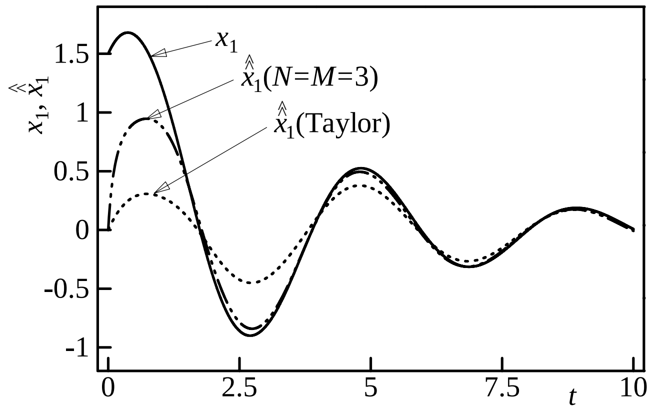

Fig. 7. Estimates $\hat{\hat{x}}_1(t)$ of the pendulum system by new method and the conventional method

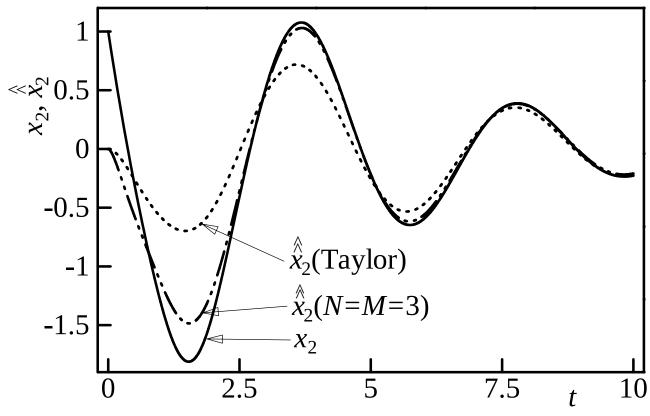

Fig. 8. Estimates $\hat{\hat{x}}_2(t)$ of the pendulum system by new method and the conventional method

results synthesized by the conventional observer based on the linearization of Taylor expansion truncated at the first order whose system is written by

$$\begin{cases} \dot{\hat{x}}_1 &= \hat{x}_2 \\ \dot{\hat{x}}_2 &= -a_1\hat{x}_2 - a_2\hat{x}_1 \end{cases},$$
$$\hat{\eta} = a_3\hat{x}_1$$

when the operating point is the origin for comparison.

Fig. 9 shows the integral square errors of estimation

$$J(t) = \int_0^t \big(\boldsymbol{x}(\tau) - \hat{\boldsymbol{x}}(\tau)\big)^T \big(\boldsymbol{x}(\tau) - \hat{\boldsymbol{x}}(\tau)\big) d\tau$$

for the various orders of the linearization function N and the augmented measurement vector M from 1 to 3. $Taylor$ is the error by the conventional method based on Taylor expansion. Fig. 10 shows the errors of estimation for the various orders of M from 1 to 3 when the order of N is fixed at 3.

V. Conclusions

We have developed an observer design for a nonlinear system by a formal linearization method exploiting Chebyshev interpolation. By this method, a nonlinear observer design is

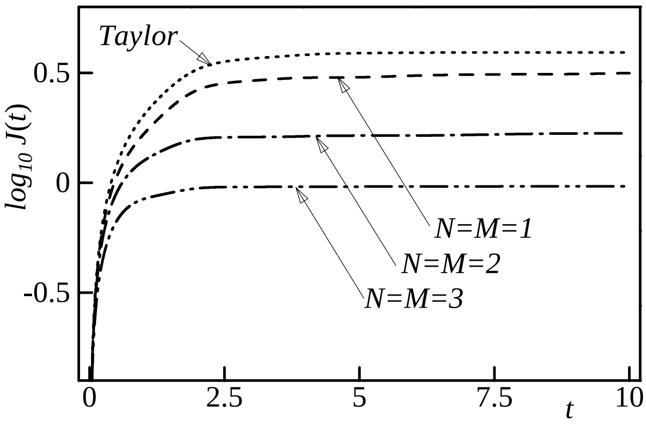

Fig. 9. Integral square errors of estimation of the pendulum system by various orders

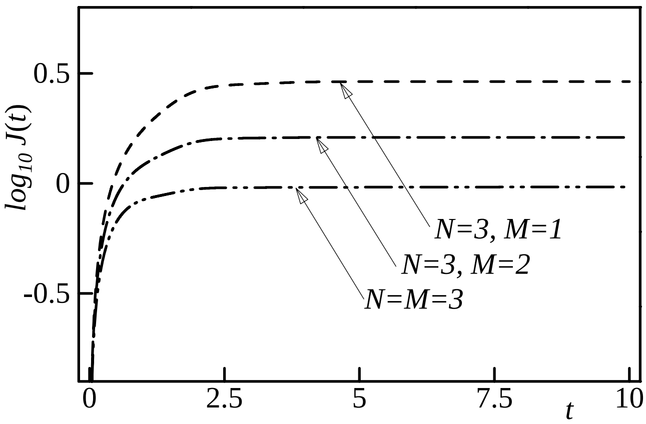

Fig. 10. Integral square errors of estimation of the pendulum system by various orders of M when N is fixed at 3

synthesized easier than the previous work because linearization processes are mechanically computed with a computer.

Numerical experiments show that our method is better than the previous works and the accuracy is improved as both the orders of the linearization function and the augmented measurement vector increase.

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