Prioritization Method in the Fuzzy Analytic Network Process by Fuzzy Preferences Programming Method

Tarifa S. Almulhim, Ludmil Mikhailov, and Dong-Ling Xu

***Abstract*—**In this paper, a method for deriving a group priority vector in the Fuzzy Analytic Network Process (FANP) is proposed. By introducing importance weights of multiple decision makers (DMs) based on their experiences, the Fuzzy Preferences Programming Method (FPP) is extended to a fuzzy group prioritization problem in the FANP. Additionally, fuzzy pair-wise comparison judgments are presented rather than exact numerical assessments in order to model the uncertainty and imprecision in the DMs' judgments and then transform the fuzzy group prioritization problem into a fuzzy non-linear programming optimization problem which maximize the group satisfaction. Unlike the known fuzzy prioritization techniques, the new method proposed in this paper can easily derive crisp weights from incomplete and inconsistency fuzzy set of comparison judgments and does not require additional aggregation producers. Detailed numerical examples are used to illustrate the implement of our approach and compare with the latest fuzzy prioritization method.

***Keywords*—**Fuzzy Analytic Network Process (FANP); Fuzzy Non-linear Programming; Fuzzy Preferences Programming Method (FPP); Multiple Criteria Decision-Making (MCDM); Triangular Fuzzy Number.

I. INTRODUCTION

THE Analytic Hierarchy Process AHP [1], is used widely in Multiple Criteria Decision-Making (MCDM) environment for dealing with complex decision making problems. It is especially appropriate for complex decision problems which involve the evaluation of decision elements (criteria, sub-criteria, or alternatives) which are difficult to measure. The most significant phase of the AHP is modeling problems as a hierarchy containing a decision goal and decision elements. It assumes that each element in the hierarchy is considered to be independent of all the others [1]. However, many decision making problems cannot be structured as a linear top-to-bottom form of hierarchy nor do they involve intersection and dependency of elements [2].

In order to overcome these limitations, the Analytic Network Process (ANP) was proposed by Saaty in 1996. The ANP deals with the decision making problems without assuming independencies among decision elements which might be useful in many real-world cases. To demonstrate this, consider a simple decision making problem regarding buying a car. Both AHP and ANP provide a helpful process for a decision maker (DM) for buying the car from different options. Assume that the DM wants to make the final decision based on three decision elements: comfort, purchase price, and fuel economy. The AHP assumes that the three elements are independent of one another, while the ANP allows consideration of the interdependence of comfort, purchase price, and fuel economy. So, when the DM desires to pay more for the car in order to obtain more comfort or fuel economy, or pay less to get less comfort or fuel economy, the ANP could allow that by taking into account intersections and dependencies among the decision elements.

The ANP uses a network without the need to specify levels as in the hierarchy. It is also called a super-matrix technique and it is a generalization of the AHP where the hierarchies are replaced by networks enabling the modeling of feedback loops. It uses a network without a need to specify levels and the levels are replaced by clusters. Paired comparison judgments in the ANP are similar to the AHP, and it uses some fixed preference scales which allows the DMs to identify how many times a decision element dominates another one. In order to construct Paired comparison judgments, the DM is asked to compare pairwisely any two decision elements and provide a numerical / linguistic judgment for their relative importance. Thus, the DM gives a set of ratio judgments to indicate the strength of his/her preferences by using some fixed preference scales. Then, the weights of criteria and the score of alternatives are derived by using DMs' assessments for decision elements.

However, in many practical cases the DMs' judgments might be uncertain, due to the subjective nature of DM's judgments, lack of data or incomplete information. The traditional ANP may not reflect human preferences properly when the DMs (experts, judges...) are unable to provide crisp values for comparison ratios. Suppose that the comparison ratios are expressed as fuzzy numbers in order to deal with the subjective uncertainty. There have been many attempts to modify the ANP in order to increase the capabilities of the ANP for deriving priority vectors (weights) from uncertain judgments by introducing fuzzy numbers in the pair-wise comparison of the ANP by converting linguistic judgments into fuzzy numbers [3]- [8].

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The ANP can be used for both individuals and group decision making problem [9]. Furthermore, there have been attempts to integrate the fuzzy set theory and the group ANP for expressing the uncertain preferences in fuzzy group decision making problem [10] - [12].

Several fuzzy prioritization methods in the Fuzzy Analytic Network Process (FANP) have been developed for deriving the priority vectors (weights): Fuzzy Preference Programming (FPP) [3], the logarithmic least square method [4], the Chang's extent analysis method [5], [6], and a fuzzy Eigen-value method [7], [8]. However, the previous prioritization methods provide a fuzzy priority vector or multiple crisp priority vectors, so they cannot directly be used in the FANP. Therefore, they require an additional aggregation method or a fuzzy ranking method. Using different ranking methods, for converting fuzzy numbers into crisp numbers, might lead to different ranking results [13]. Additionally, in the previously mentioned methods for solving fuzzy group prioritization problems in the FANP, the methods are assumed that all the DMs have the same weight of importance and have equal experience to assess all decision elements. Nevertheless, in the real group decision making problems, sometimes there are important experts, such as the executive managers of the organization. Also, some experts are more experienced than others; therefore the final decision should be influenced by the degree of importance of each expert.

Reference [3] shows that the FPP can be applied for fuzzy group prioritization problem in order to derive priorities from fuzzy comparison judgments. In this method all individuals' fuzzy judgments for group decision makers (DMs) combine into a linear programming method model by using different values of an alpha-cuts concept. However, the model assumes that all DMs have the same weight of importance. Besides, the FPP needs an additional aggregation procedure to obtain the final crisp vector at the different values of alpha-cuts.

In order to overcome some of the drawbacks of the existing fuzzy prioritization methods, a modification of the non-linear Fuzzy Preference Programming (FPP) method is proposed by introducing importance weights of decision makers (DMs). The proposed method has some attractive features. It does not require any fuzzy ranking procedure or any aggregation procedure. Moreover, it provides a priority vector from an incomplete and inconsistent set of fuzzy judgments and can easily be modified for handling the fuzzy group prioritization problem.

The organization of this paper is as follows. The next section summaries steps of the FANP. Section III discusses the version of the FPP method. Section IV presents the new modified FPP method, which is further transformed into a single non-linear optimization problem. Numerical examples are in section V which illustrates the proposed method applicability. Finally, the conclusion is in section VI.

II. Fuzzy Analytic Network Process (FANP)

In this section the algorithm of FANP, that combines the ANP and the FPP, is summarized in steps as follows:

Step 1: Structuring a network model which includes decision elements (alternatives, criteria, sub criteria, clusters, and actors).

Step 2: Identifying dependences among all elements of the prior network model.

Step 3: Establishing pair-wise comparison matrices with fuzzy individual/group ratio judgments.

Step 4: Determining the consistency index.

Step 5: Applying the FPP method to obtain relative importance weights (individual/group priority vectors) from each matrix, which should model the uncertain judgments.

Step 6: Checking the consistency index. If it is acceptable (less than 10%), continue to Step 7, otherwise, return to Step 3.

Step 7: Forming an un-weighted super-matrix by filling a super-matrix with the obtained relative importance weights (the priority vector). For more details see Appendix A.

Step 8: Producing a weighted super-matrix by adjusting the un-weighted super-matrix to column stochastic so that the sum of the elements in each column is equal to one.

Step 9: Limiting the weighted super-matrix by raising itself to power $C+1$, where C is an arbitrarily large number, until the row elements converge to the same value for each column of the matrix. The resulting matrix is called a limiting super-matrix

Step 10: Aggregating the weights of criteria and the score of alternatives into a final priority vector.

The estimation of the priority vectors from pair-wise comparison judgments matrices is the major phase of the FANP. Thus, this paper focuses on the problem of deriving the group priority vector (crisp weights of decision elements) from fuzzy group comparison judgments.

III. Deriving Group Priorities from Uncertain Judgments

A. Fuzzy Group Pair-Wise Comparison Judgments

Consider a group of K decision makers ($DM_k, k = 1,2,...,K$) evaluate n elements (clusters, criteria, sub-criteria, or alternatives).

Suppose that each DM provides a set $A^k = \{\tilde{a}_{ijk}\}$ of $m_k \le n(n-1)/2$, incomplete fuzzy comparison judgments, $i = 1,2,..,n-1,\ j = 2,3,..n,\ k = 1,2,.,k$ and $j \succ i$ represented as Triangular Fuzzy Numbers (TFNs) $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$, where l_{ijk}, m_{ijk} and u_{ijk} are the lower, mode and upper bounds, respectively.

Fig. 1 shows the TFN $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$:

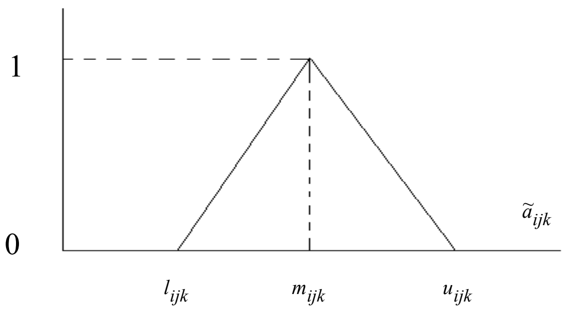

Fig. 1 Triangular Fuzzy Number $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$

And A^k can be represented by:

$$A^k = \begin{bmatrix} (1,1,1) & (l_{12k}, m_{12k}, u_{12k}) & \cdots & (l_{1jk}, m_{1jk}, u_{1jk}) \\ (l_{21k}, m_{21k}, u_{21k}) & (1,1,1) & \cdots & (l_{2jk}, m_{2jk}, u_{2jk}) \\ \cdots & \cdots & \cdots & \cdots \\ (l_{i1k}, m_{i1k}, u_{i1k}) & (l_{i2k}, m_{i2k}, u_{i2k}) & \cdots & (1,1,1) \end{bmatrix} \quad (1)$$

Then, a group prioritization problem is to determine a priority vector (weights) $w = (w_1, w_2, ..., w_n)^T$ from A^k, such that w_i represents the relative importance weight of n elements.

B. Fuzzy Preferences Programming Method (FPP)

The FPP method [13] proposed to derive crisp priority vectors from fuzzy numbers by applying alpha-cuts (or α-level sets) before comparisons and hence avoiding the final fuzzy scores that other methods obtain.

The judgments are represented by TFN numbers with applying the alpha-cuts which expresses the degree of confidence of the DMs in the judgments they make. The alpha-cut is used in this method in order to transform the initial fuzzy judgments into an interval judgments series. Then the fuzzy prioritization problem is transformed into an optimization problem that maximizes the DMs' overall satisfaction with the final optimal solution where there are missing or inconsistent judgments in the fuzzy pair-wise comparison matrices.

For a given α-level, each fuzzy judgment $\tilde{a}_{ijk}$ can be decomposed into an interval sets $\tilde{a}_{ijk} = [l_{ijk}(\alpha), u_{ijk}(\alpha)]$, where:

$l_{ijk}(\alpha) = (m_{ijk} - l_{ijk}) * \alpha + l_{ijk}$,

$u_{ijk}(\alpha) = -(u_{ijk} - m_{ijk}) * \alpha + u_{ijk}$.

If the interval pair-wise comparison judgments are consistent, the FPP derives a priority vector $w = (w_1, w_2, ..., w_n)^T$, which satisfies:

$$l_{ijk}(\alpha) \le \frac{w_i}{w_j} \le u_{ijk}(\alpha) \quad (2)$$

For inconsistence judgments, the FPP method tries to find the crisp priority vector which satisfies:

$$l_{ijk}(\alpha) \tilde{\le} \frac{w_i}{w_j} \tilde{\le} u_{ijk}(\alpha)$$

$$s.t. \quad (3)$$

$$-w_i + w_j l_{ijk}(\alpha) \tilde{\le} 0$$

$$w_i - w_j u_{ijk}(\alpha) \tilde{\le} 0$$

where $\tilde{\le}$ implies 'fuzzy less or equal to'. If m denotes the overall number of fuzzy group comparison judgments. Thus, a set of $2m$ fuzzy constraints of the type (3) are obtained.

The $2m$ fuzzy constraints in (3) can be represented in a matrix form as:

$$RW \tilde{\le} 0 \quad (4)$$

The q-th fuzzy inequality in (4) can be indicated by $RW_q \tilde{\le} 0$, $q = 1,2,...2m$.

For a given priority vector W, the degree of satisfaction of this fuzzy inequality is measured by a linear membership function as below:

$$\mu_q(R_q W) = \begin{cases} 1 - \dfrac{R_q W}{d_q} & , R_q W \le d_q \\ 0 & , R_q W \ge d_q \end{cases} \quad (5)$$

where d_q is a deviation parameter, specified by the k-th DM, denoting the allowed range of approximate satisfaction of the soft inequality constraints (3).

The solution to the prioritization problem by the FPP method is based on two assumptions [14]. The first on requires the existence of a *non-empty fuzzy feasible area* $\tilde{P}$ which is defined as an intersection of all fuzzy constraints, described by the following membership function:

$$\mu_{\tilde{P}}(W) = Min\{\mu_1(R_1 W), \mu_2(R_2 W), ...\mu_{2m}(R_{2m} W)\} \quad (6)$$

where the normalization condition, $\sum_{i=1}^{n} w_i = 1$, is satisfied.

According to [14], the second assumption identifies a selection rule, which determines a priority vector, having the highest degree of membership in the aggregated membership function as described in (5). Thus, there is *a maximizing solution* W^* (a crisp priority vector) that has a maximum degree of membership λ^* (the consistency index), in $\tilde{P}$, such that :

$$\lambda^* = \mu_{\widetilde{P}}(W^*) = Max[Min\{\mu_1(R_1W),...,\mu_{2m}(R_{2m}W)\}]$$
$$s.t. \tag{7}$$
$$\sum_{i=1}^{n} w_i = 1$$

A new decision variable λ is introduced which measures the maximum degree of membership in the fuzzy feasible area $\widetilde{P}$. When the interval judgments are consistent, λ is equal to one. For inconsistent judgments the consistency index λ takes a value between one (meaning complete consistent) and zero (meaning complete inconsistent) that depends on the degree of inconsistency. The above max-min optimization problem is transformed into the following:

$$\begin{aligned} & Max \quad \lambda \\ & s.t. \\ & \lambda \le \mu_q(R_qW) \\ & \sum_{i=1}^{n} w_i = 1, \quad w_i \succ 0, \\ & i = 1,2,...,n, \quad q = 1,2,..2m \end{aligned} \tag{8}$$

Since the membership functions $\mu_q(R_qW)$ are linear. The prioritization problem (8) is a linear program and can be represented as:

$$\begin{aligned} & Max \quad \lambda \\ & s.t. \\ & d_q\lambda + R_qW \le d_q \\ & \sum_{i=1}^{n} w_i = 1, \quad w_i \succ 0, \\ & i = 1,2,...,n, \quad q = 1,2,..2m \end{aligned} \tag{9}$$

IV. Proposed Method for Fuzzy Group Prioritization Problem

A. The Fuzzy Group Pair-Wise Comparison Judgments

The FPP method presented in the section III has some drawbacks. In the first place, it needs a number of α -levels, which transfers the fuzzy judgments into interval series, for solving the linear programming (9). Thus, the traditional FPP method requires an additional aggregation technique to obtain the priority vector at different α -levels. Consequently, this process is time consuming due to several computation steps needed for applying the α -cuts concept. A further limitation is that the FPP method does not consider the DMs' importance weights and ignores the DMs' expertise. In order to overcome these limitations, a non-linear model for fuzzy group prioritization problem is proposed, which can derives crisp priorities from a fuzzy set of judgments, expressed as TFNs.

As in section III, we can define K fuzzy feasible areas, $\widetilde{P} = \bigcap \widetilde{P}_k$, where $\widetilde{P}_k$ is the intersection of the membership functions, corresponding to the k -th DMs' fuzzy judgments .

By introducing a new decision variable λ_k, which measures the maximum degree of membership of a given priority vector in the fuzzy feasible area $\widetilde{P}_k$, we can introduce membership functions that represent the DMs' satisfaction with different crisp solution ratios $\frac{w_i}{w_j}$. Each crisp priority vector $w = (w_1, w_2,..., w_n)^T$ satisfies:

$$l_{ijk} \lessapprox \frac{w_i}{w_j} \lessapprox u_{ijk} \tag{10}$$

And can be measured by a membership function [13]:

$$\mu_q^{\ k}(R_q^{\ k}W) = \begin{cases} \dfrac{\left(w_i^k / w_j^k\right) - l_{ijk}}{m_{ijk} - l_{ijk}}, & w_i^k / w_j^k \le m_{ijk} \\[2ex] \dfrac{u_{ijk} - \left(w_i^k / w_j^k\right)}{u_{ijk} - m_{ijk}}, & w_i^k / w_j^k \ge m_{ijk} \end{cases} \tag{11}$$

The max-min optimization problem (8) is transformed into the following model:

$$\begin{aligned} & Max \quad \lambda_k \\ & s.t. \\ & \lambda_k \le \mu_q^{\ k}(R_q^{\ k}W) \\ & \sum_{i=1}^{n} w_i = 1, \quad w_i \succ 0, \\ & i = 1,2,...,n, \quad q = 1,2,..2m_k \end{aligned} \tag{12}$$

where $\mu_q^{\ k}(R_q^{\ k}W)$ are the membership functions of the type (11) corresponding to the soft constraints of the k -th DMs.

For introducing the DMs' importance weights, let us define I_k as the importance weight of the $DM_k; k = 1,2,..., K$, where I_k are weights related to the fuzzy k -th set of judgments (1). For aggregating all individual models of type (12) into a single group model a weighted additive goal-programming (WAGP) model [15] is applied.

B. Aggregation Individual Models into a Single Model

The WAGP model transforms the multi-objective decision-making problem to a single objective using fuzzy set theory. Therefore, it is used in order to combine all individual models (12) into a new single model by taking into account the DMs' importance weights.

The WAGP model considers the different importance weights of gaols and constraints. The WAGP model is:

$$\mu_D(x) = \sum_{s=1}^{p} \alpha_s \mu_{z_s}(x) + \sum_{r=1}^{h} \beta_r \mu_{g_r}(x)$$
$$\sum_{s=1}^{p} \alpha_s + \sum_{r=1}^{h} \beta_r = 1 \qquad (13)$$

where:

μ_{z_S} : are membership functions for the p–th fuzzy goal $z_S, s = 1,2,...p$.

μ_{g_r} : are membership functions the h-th fuzzy constraints $g_r, r = 1,2,...h$.

x : is the vector of decision variables.

α_s : are weighting coefficients that show the relative important of the fuzzy goals.

β_r : are weighting coefficients that show the relative important of the fuzzy constraints.

A single objective model in WAMP is the maximization of the weighted sum of the membership functions μ_{z_S} and μ_{g_r} . By identifying new decision variables λ_s and γ_r , the model (13) can be transferred into a crisp single objective model as follows:

$$\begin{aligned}
&Max \ \ \sum_{s=1}^{p} \alpha_s \lambda_s + \sum_{r=1}^{h} \beta_r \gamma_r \\
&s.t. \\
&\lambda_s \le \mu_{z_s}(x), \qquad s = 1,2,...p \\
&\gamma_r \le \mu_{g_r}(x), \qquad r = 1,2,...h \\
&\sum_{s=1}^{p} \alpha_s + \sum_{r=1}^{h} \beta_r = 1 \\
&\lambda_s, \gamma_r \in [0,1]; \qquad \alpha_s, \beta_r \ge 0
\end{aligned} \qquad (14)$$

By comparing the model (12) with (14), one can observe the similarity between them. However, the proposed FPP method (13) does not deal with fuzzy goals; it just represents the non-linear fuzzy constraints. Thus, by taking into the account the specify form of $R_q{}^k W \lesssim 0$, and introducing the important weights of the DMs, the problem can be further presented into a non-linear program by utilizing WAGP model (14) as:

$$\begin{aligned}
&Max \ \ Z = \sum_{k=1}^{K} I_k \lambda_k \\
&s.t. \\
&(m_{ijk} - l_{ijk})\lambda_k w_j - w_i + l_{ijk} w_j \le 0 \\
&(u_{ijk} - m_{ijk})\lambda_k w_j + w_i - u_{ijk} w_j \le 0 \\
&i = 1,2,...n-1; \quad j = 2,3,..n \quad ; \ j \succ i; \\
&\sum_{i=1}^{n} w_i = 1 \ ; \quad w_i \succ 0; \\
&i = 1,2,...,n \ ; \quad k = 1,2,...K
\end{aligned} \qquad (15)$$

where the decision variable λ_k measures the degree of membership of a given priority vector in the fuzzy feasible area $\widetilde{P}_k$, I_k denotes the importance weight of the k-th decision makers, $k = 1,2,...K$. In the model (15) the value of Z call a consistency index, this measures the overall consistency of the initial set of fuzzy judgments. When the set of fuzzy judgments is consistent, the optimal value of Z is grater or equal to one. For the inconsistent fuzzy judgments, the maximum value of Z takes a value less than one.

V. Numerical Examples

The first example illustrates the solution to the fuzzy group prioritization problem for obtaining a priority vector and a final group ranking. The second example demonstrates how the importance weights of DMs influence the final group ranking.

A. Example 1

We consider the example in [16], where three DMs ($K = 3$) rank three elements ($n = 3$), and the importance weights of DMs are $I_1 = 0.3$, $I_2 = 0.2$, $I_3 = 0.5$. The DMs provide an incomplete set of five fuzzy judgments ($m = 5$) presented as triangular fuzzy numbers:

DM 1: $a_{121} = (1,2,3)$; $a_{131} = (2,3,4)$.

DM 2: $a_{122} = (1.5,2.5,3.5)$; $a_{132} = (3,4,5)$.

DM 3: $a_{123} = (2,3,4)$.

The prioritization problem is to derive a crisp priority vector $W = (w_1, w_2, w_3)^T$ that approximately satisfies the following fuzzy constraints:

For DM 1: $1 \lesssim \dfrac{w_1}{w_2} \lesssim 3$; $2 \lesssim \dfrac{w_1}{w_3} \lesssim 4$

For DM 2: $1.5 \lesssim \dfrac{w_1}{w_2} \lesssim 3.5$; $3 \lesssim \dfrac{w_1}{w_3} \lesssim 5$

For DM 3: $2 \lesssim \dfrac{w_1}{w_2} \lesssim 4$

Using the above data and the non-linear optimization model (15), the following formulation is obtained:

$$
\begin{aligned}
& Max\ Z = 0.3\lambda_1 + 0.2\lambda_2 + 0.5\lambda_3 \\
& s.t. \\
& \lambda_1 w_2 - w_1 + w_2 \leq 0 \\
& \lambda_1 w_2 + w_1 - 3w_2 \leq 0 \\
& \lambda_1 w_3 - w_1 + 2w_3 \leq 0 \\
& \lambda_1 w_3 + w_1 - 4w_3 \leq 0 \\
& \lambda_2 w_2 - w_1 + 1.5w_2 \leq 0 \\
& \lambda_2 w_2 + w_1 - 3.5w_2 \leq 0 \\
& \lambda_2 w_3 - w_1 + 3w_3 \leq 0 \\
& \lambda_2 w_3 + w_1 - 5w_3 \leq 0 \\
& \lambda_3 w_2 - w_1 + 2w_2 \leq 0 \\
& \lambda_3 w_2 + w_1 - 4w_2 \leq 0 \\
& w_1 + w_2 + w_3 = 1 \\
& w_1 \geq 0, \quad w_2 \geq 0, \quad w_3 \geq 0
\end{aligned}
\tag{16}
$$

Using LINGO V13.0 software, the solution to the non-linear problem (16) is found as:

$$
\begin{aligned}
& w_1 = 0.623\ ,\ w_2 = 0.216, \\
& w_3 = 0.161\ ,\ \lambda_1 = 0.123, \\
& \lambda_2 = 0.623\ ,\ \lambda_3 = 0.876
\end{aligned}
$$

And the maximum value of the objective function is $Z = 0.600$. Thus, $W = (0.623, 0.216, 0.161)$ is a crisp priority vector generated from the group fuzzy judgments set by selecting the solution that has the highest degree of membership of the fuzzy judgments set. Also, it can be seen that the consistency index value is $Z = 0.600$, which means that the fuzzy judgments are slightly inconsistent, since the consistency index is non-negative (Z is less than one).

TABLE I
RESULTS FROM THE TWO PRIORITIZATION METHODS

Method	w_1	w_2	w_3
Weighted FPP method [a]	0.615	0.205	0.179
Non-linear FPP method [b]	0.623	0.216	0.161

[a] The method proposed in [16] with applying α - cut
[b] The method proposed in this paper without applying α - cut

This result can be compared with the crisp results from the example in [16], as shown in Table I. We may observe that we have the same final ranking $w_1 \succ w_2 \succ w_3$, from applying the two different prioritization methods. However, the Weighted FPP method applies an aggregation procedure for obtaining the crisp vector from different values of priorities at different α - levels, while, the proposed FPP method in this paper does not require an additional aggregation procedure.

Nevertheless, if the third DM who has the highest important weight provides a new fuzzy comparison judgment $a_{213} = (2,3,4)$ which means that the second element is about three times more important than the first one. Then, the obtained final weights for the three elements are $w_1 = 0.310$, $w_2 = 0.620,$ $w_3 = 0.069$ and the final ranking is $w_2 \succ w_1 \succ w_3$. Thus, it can be observed that the third DM's judgments strongly influence the final ranking. However, if the importance weight of the third DM is lower of equal to the first two DMs' weights, then the new fuzzy comparison judgment $a_{213} = (2,3,4)$ does not change the final ranking.

B. Example 2

This example shows that the importance weights of the DMs influence the final group ranking.

Consider two decision makers DMs ($K = 2$) assess three criteria ($n = 3$). The DMs provide an incomplete set of four fuzzy judgments ($m = 4$) presented as triangular fuzzy numbers:

DM 1: $a_{121} = (1,2,3); a_{131} = (2,3,4).$

DM 2: $a_{212} = (3,4,5); a_{312} = (2,3,4).$

Two situations are investigated when both DMs have the following different weights:

1. $I_1 = 0.2$, $I_2 = 0.8$
2. $I_1 = 0.8$, $I_2 = 0.2$

For both situations, the final rankings for both individual DMs are shown in Tables II and III respectively. The final group rankings are shown in Tables II and III (the third row for each table). The results are obtained by using LINGO V13. Each final group ranking is obtained by solving a non-linear program of type (15), which includes eight non-linear inequality constraints corresponding to the given DMs' fuzzy comparison judgements.

It can be observed from Tables II and III that the final group ranking tends to be the individual ranking of the DM who has the highest importance weights. In more details, it can be seen from Table II that the judgements of the second DM with the highest importance weight ($I_2 = 0.8$) influence more strongly the final group ranking. On the other hand, the final group ranking in Table III depends on the first DM, who has the highest importance weight ($I_1 = 0.8$).

From examples 1 and 2, we can notice the significance of introducing importance weights of the DMs to the fuzzy group prioritization problem. It is seen that the final group ranking depends on the DMs' importance weights.

TABLE II

INDIVIDUAL AND GROUP RESULTS ($I_1 = 0.2$, $I_2 = 0.8$)

DMs	w_1	w_2	w_3	Final ranking
DM 1	0.545	0.273	0.182	$w_1 \succ w_2 \succ w_3$
DM 2	0.117	0.530	0.353	$w_2 \succ w_3 \succ w_1$
Group	0.117	0.529	0.354	$w_2 \succ w_3 \succ w_1$

TABLE III

INDIVIDUAL AND GROUP RESULTS ($I_1 = 0.8$, $I_2 = 0.2$)

DMs	w_1	w_2	w_3	Final ranking
DM 1	0.545	0.272	0.181	$w_1 \succ w_2 \succ w_3$
DM 2	0.117	0.530	0.353	$w_2 \succ w_3 \succ w_1$
Group	0.402	0.397	0.201	$w_1 \succ w_2 \succ w_3$

VI. CONCLUSION

This paper deals with the fuzzy group prioritization problem in the FANP for deriving a crisp priority vector. An extension of the non-linear FPP method for group decision making under uncertainty is proposed. The main advantage of the proposed method for the fuzzy group prioritization problem in the FANP is that it considers the DMs' importance weights based on their expertise. Furthermore, it does not require an additional aggregation technique because it does not apply the α - cut concept. An additional advantage is that it derives priorities from an incomplete and inconsistence set of fuzzy judgments. Moreover, it is suitable for group decision making problem. All these characteristics make the proposed method an appropriate alternative to existing fuzzy group prioritization methods in the FANP.

Future work includes considering the different importance weights for the DMs by applying fuzzy weights. Then, employ the proposed method in the FANP for solving a complex network structure.

APPENDIX

In the ANP, any decision problem is decomposed into a network of decision elements, where all the elements correspond to clusters and the relevant elements combine into the same clusters.

For a system of N clusters, [17] described the process of establishing the super-matrix as the following:

$$
W = \begin{array}{cc} & \begin{array}{ccccc} C_1 & \cdots & C_k & \cdots & C_N \\ e_{11}\cdots e_{1n_1} & \cdots & e_{k1}\cdots e_{kn_k} & \cdots & e_{N1}\cdots e_{Nn_N} \end{array} \\ \begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ \vdots \\ e_{1n_1} \end{array} \\ \vdots & \vdots \\ C_k & \begin{array}{c} e_{k1} \\ \vdots \\ e_{kn_k} \end{array} \\ \vdots & \vdots \\ C_N & \begin{array}{c} e_{N1} \\ \vdots \\ e_{Nn_N} \end{array} \end{array} & \begin{pmatrix} W_{11} & \cdots & W_{1k} & \cdots & W_{1N} \\ \vdots & & \vdots & \ddots & \cdots \\ W_{k1} & \cdots & W_{kk} & \cdots & W_{kN} \\ \vdots & & \vdots & \ddots & \cdots \\ W_{N1} & \cdots & W_{Nk} & \cdots & W_{NN} \end{pmatrix} \end{array}
$$

Where C_k is the k -th cluster ($k = 1,2,..., N$) which has n_k elements denoted as $e_{k1}, e_{k2},...., e_{kn_k}$ .The influence of an element in a cluster on another element inside the same cluster is judged by ratio scale and places in a pair-wise comparison matrix. The typical entry W_{ij} in the super-matrix is called a block of the super-matrix, which represents a relationship, between the i -th cluster and the j -th cluster, and is illustrated below:

$$
W_{ij} = \begin{bmatrix} W_{i1}^{(j_1)} & W_{i1}^{(j_2)} & .... & W_{i1}^{(j_{n_j})} \\ W_{i2}^{(j_1)} & W_{i2}^{(j_2)} & .... & W_{i2}^{(j_{n_j})} \\ ... & ... & .... & ... \\ W_{in_i}^{(j_1)} & W_{in_i}^{(j_2)} & .... & W_{in_i}^{(j_{n_j})} \end{bmatrix}
$$

Each column of W_{ij} is a local vector of each element that is derived from paired comparisons by applying a proper prioritization method. Some of entries may be zero corresponding to those elements that have no influence (Saaty and Vargas, 2006).

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