

# Application of Wavelet Neural Networks in Optimization of Skeletal Buildings under Frequency Constraints

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**Abstract**—The main goal of the present work is to decrease the computational burden for optimum design of steel frames with frequency constraints using a new type of neural networks called Wavelet Neural Network. It is contended to train a suitable neural network for frequency approximation work as the analysis program. The combination of wavelet theory and Neural Networks (NN) has lead to the development of wavelet neural networks. Wavelet neural networks are feed-forward networks using wavelet as activation function. Wavelets are mathematical functions within suitable inner parameters, which help them to approximate arbitrary functions. WNN was used to predict the frequency of the structures. In WNN a Rational function with Second order Poles (RASP) wavelet was used as a transfer function. It is shown that the convergence speed was faster than other neural networks. Also comparisons of WNN with the embedded Artificial Neural Network (ANN) and with approximate techniques and also with analytical solutions are available in the literature.

**Keywords**—Weight Minimization, Frequency Constraints, Steel Frames, ANN, WNN, RASP Function.

## I. INTRODUCTION

OPTIMIZATION techniques in civil engineering, nowadays are pursued for variety of reasons. One of the main reasons could be reduction in time and money, while aiming for an accurate design simultaneously. This article is mainly about optimization of steel frames under frequency constraints. However, during the optimization process, the analysis may be repeated for a considerable number of times for computing the frequency of the structure. For this reason the optimization procedure may require a significant amount of time as well as cost. Therefore, a new method is introduced to decrease the time of optimization. To achieve such aim, thus, we have suggested a new neural network called WNN. WNN estimates frequencies of the structures very fast. The reliability of using such a system will be investigated and results will be compared with those available in the literature. Through section II, optimization formulation is described. Section III to VII introduces neural networks and also a new approach to increase the efficiency of the neural networks. Finally, in section VIII and IX, numerical investigations are performed and conclusion will be presented.

## II. OPTIMIZATION PROBLEM FORMULATION

The constrained optimization problem, which is more of a practical one, has been formulated in terms of some parameters and restrictions. The parameters chosen to describe the design of a structure are known as design variables while the restrictions are known as constraint conditions. Optimum design of structures with frequency constraints is to minimize the weight of the structure while all the frequencies are kept in the specified limits. The objective function is the weight of the structure and the cross sectional areas of the members are the design variables. In this case the problem is to find a vector of design variables  $X = [x_1, x_2, \dots, x_n]$  that will minimize the weight of the structure  $W(x)$ , subjected to  $m$  natural frequency constraints and  $n$  design variables, as follow:

$$g_j(x) = \lambda_j - \bar{\lambda}_j \geq 0 \quad j = 1, m \quad (1)$$

$$x_i^L \leq x_i \leq x_i^U \quad i = 1, n \quad (2)$$

$\lambda_j$  represents the  $j$ th eigenvalue and  $\bar{\lambda}_j$  is the limit on the  $j$ th eigenvalue. The eigenvalue analysis of a structure is achieved by solving the homogenous set of equations:

$$(K - \lambda_j M)\phi = 0 \quad (3)$$

where  $\phi$  is the eigenvector corresponding to the  $j$ th eigenvalue.  $K$  and  $M$  are global stiffness and mass matrices, respectively.

In general, the optimum solution is obtained by mathematical programming methods [1]. In this work, the method employed to optimize the structure was Sequential Quadratic Programming (SQP). Now, the nature of the method is essentially based on an iterative process, as a result of which for large structures a great number of eigenvalue analyses should be performed. Thus, to reduce the computational burden, some approximation concepts have to be introduced. For this purpose, one can refer to the recent survey of the optimum design with frequency constraints presented by Grandhi [2].

However, in this paper, a known type of neural networks called Artificial Neural Network (ANN) was employed. Also, with the aim of increasing more the quality of approximation as well as reducing the computational time, a new type called Wavelet Neural Network (WNN) was introduced and applied.

### III. NEURAL NETWORK CONCEPTS

The first journal article on neural network application in civil/structural engineering was published in 1989[3]. The great majority of civil engineering applications of neural networks are based on the back propagation algorithm. The design of neural network has been inspired by the biological research on how the human's brain works. The brain is a network consisting of approximately 2.5 billion simple processors, called neurons, connected to one another through branchlike structures called axons and dendrites (see Figure (1)). Synapses connect the axons and dendrites of one neuron to those of another. The objective of NN is to mimic the neurons in the brain by linking together many simple processors, called Artificial Neurons or Nodes. Variable strength connections, called weights, implement the biological synapses [4].

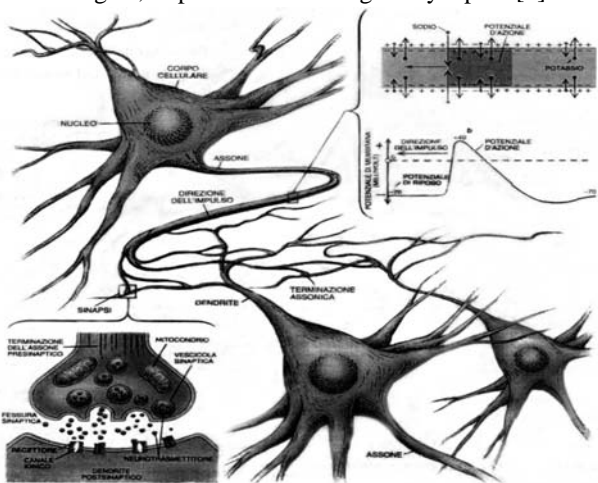


Fig. 1 the structure of a biologic neuron

Neural networks excel at recognition, identification, and classification types of problems. In an effort to model certain capabilities of the brain, Warren McCulloch and Walter Pitts established a simplified model of a biological neuron in 1943 called the McCulloch-Pitts model consisting of multiple inputs and one output with a Central Processing Unit (CPU) [5]. In the next section, the ANN, as a type of neural networks employed in this paper, will be briefly described.

### IV. THE THEORY OF ANN

Figure (2), shows the model for an artificial neuron, which is described by:

$$y = f\left(\sum_{i=1}^N w_i p_i - b_t\right) \quad (4)$$

where  $f(X) = \frac{1}{1 + e^{-cX}}$  (sigmoid function)  $i = 1, \dots, N$

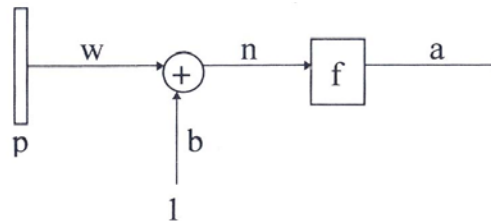


Fig. 2 the McCulloch-Pitts model of a neuron

Where  $w_i$  and  $p$  are synaptic weights and input signals.  $N$  represents the number of design variables.  $b$  is threshold or bias which is a constant term.  $f(.)$  is the activation function sometimes called squashing function or processing element. Output signal of the neuron is shown as  $y$ . The use of threshold,  $b$ , is to provide a bias to the activation function  $f(.)$ . In ANN the most common activation function is sigmoid function. A neural network has at least two physical components, namely, the neurons and the connections between them. The neurons are called activation functions, and the connections between the neurons are known as links or weights. Every link has a weight parameter associated with it. Each neuron receives stimulus from the neighboring neurons connected to it, process the information, and produces an output. Neurons that receive stimuli from outside the network are called input neurons. Neurons whose outputs are used externally are called output neurons. Neurons that receive stimuli from other neurons and whose output is a stimulus for other neurons in the network are known as hidden neurons. There are different ways in which information can be processed by a neuron, and different ways connecting neurons to others. The main objective in neural model development is to find an optimal set of weight parameters  $w$  such that  $y = y(x, w)$  closely approximates the original problem behavior. This is achieved through a process called training. During training, the neural network performance is evaluated by computing the difference between actual NN outputs and desired outputs for all the training samples. The difference is also known as the error. The weight parameters  $w$  are adjusted during training, such that this error is minimized. The technique by which the training phase is processed is called back propagation, a detail of which is given in section V.

### V. FUNDAMENTALS OF WAVELETS THEORY

The term wavelet as it implies means a little wave. This little wave must have at least a minimum oscillation and a fast decay to zero, in both the positive and negative directions, of its amplitude. This property is analogous to an admissibility condition of a function that is required for the wavelet transform. During last 10 years, the wavelet theory has been developed in mathematics. It is a mathematical function with suitable inner parameters, which help them to approximate arbitrary functions. In fact they cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over

traditional Fourier methods in analyzing physical states where the signal contains discontinuities and sharp spikes. Unlike the Fourier transform, the wavelet transform has dual localization, both in frequency and in time. These characteristics make wavelets an active subject with many exciting applications, not only in pure mathematics, but also in acoustics, image compression, turbulence, human vision, radar, and earthquake prediction, fluid mechanics and chemical analysis. Figure 3.a is an example of a wavelet called Morlet wavelet after the name of the inventor Jean Morlet in 1984 [6]. Sets of "wavelets" are employed to approximate a signal. The goal is to find a set of constructed wavelets which are referred to as daughter wavelets by a dilated and translated original wavelets namely mother wavelets. So by traveling from a large scale to a fine scale, one can zoom in and arrive at more and more exact representations of the given signal. Figures 3b to 3d, display various daughter wavelets where  $a$  is a dilation and  $b$  is a translation corresponding to the Morlet mother wavelet (Figure 3a). Note the number of oscillations which are the same for all the daughter wavelets.

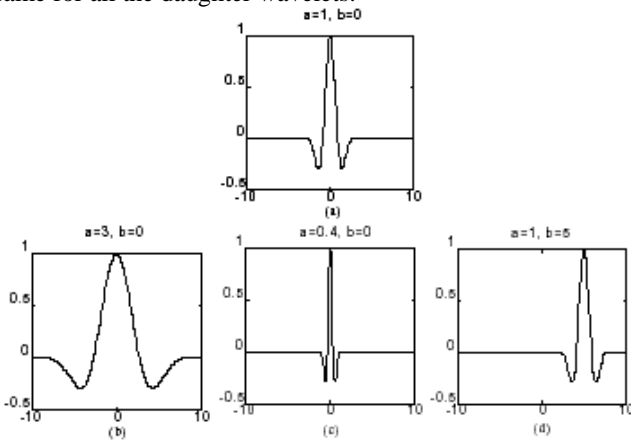


Fig. 3 Dilated and translated Morlet mother wavelets

#### A. Wavelet Transform

The wavelet transform is an operation that transforms a function by integrating it with modified versions of some kernel functions. The kernel function is called the mother wavelet, and the modified version is its daughter wavelet. For a function to be a mother wavelet it must be admissible. Recall from the beginning of this section that for a function to be a wavelet it must be oscillatory and have fast decay towards zero. If these conditions are combined with the condition that the wavelet must also integrate to zero, then these three conditions are said to satisfy the Grossmann-Morlet admissibility condition. More rigorously, a function  $h \in L^2(R)$ , where  $L^2$  being a set of all squared integral functions or finite energy functions, is admissible if [6]:

$$C_h = \int_{-\infty}^{\infty} \frac{|H(\omega)|^2}{|\omega|} d\omega < \infty \quad (5)$$

$H(\omega)$  is the Fourier transform of  $h(t)$  and the constant  $C_h$  is the admissibility constant of the function  $h(t)$ . The requirement that  $C_h$  is finite allows for inversion of the

wavelet transform. Now since any admissible function can be a mother wavelet, therefore for a given  $h(t)$ , the condition  $C_h < \infty$  holds only if  $H(\omega) = 0$ . The wavelets are inherently band-pass filters in the Fourier domain or equivalently so, if  $\int_{-\infty}^{\infty} h(t) dt = 0$  (zero-mean value in time domain). The wavelet transform of a function  $f \in L^2(R)$  with respect to a given admissible mother wavelet  $h(t)$  is defined as [7]:

$$w_f(a,b) = \int_{-\infty}^{\infty} f(t) h_{a,b}^*(t) dt \quad (6a)$$

where

$$h_{a,b}(t) = \frac{1}{\sqrt{a}} h\left(\frac{t-b}{a}\right) \quad (6b)$$

where  $*$  denotes the complex conjugate. However, most wavelets are real valued. The constant term  $a^{-1/2}$  is for energy normalization that keeps the energy of the daughter wavelet equal to the energy of the original mother wavelet. The inversion formula of the wavelet transform is given by [7]:

$$f(t) = \frac{1}{c_h} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w_f(a,b) h_{a,b}(t) \frac{dadb}{a^2} \quad (7)$$

#### B. RASP Wavelets

Rational functions with Second-order Poles (RASP) arise from the residue theorem of complex variables [7], the theorem of which is beyond the scope of this paper. Figure 4 presents three examples of the RASP mother wavelets with the normalized gains  $k_i$ ,  $i = 1, 2, 3$  equal to 3.0788, 2.7435, and 0.6111, respectively. These wavelets are real, odd functions with zero mean. The distinction among these mother wavelets is their rational form of functions being strictly proper and having simple/double poles of second order. According to the admissibility condition, equation (5), it must be shown that the families of RASP mother wavelets have zero mean value. This can be done easily using the residue theorem for evaluation of integrals [7].

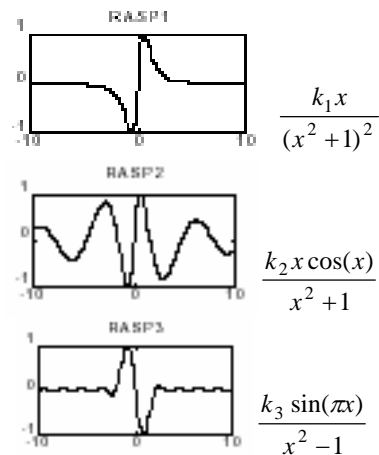


Fig. 4 Examples of Rational functions with Second-order Poles (RASP) wavelets

## VI. THE THEORY OF WNN

Neural networks are combined with other computing paradigms such as wavelet, genetic algorithm and fuzzy logic to enhance the performance of neural networks models. As a part of this work, eigenvalues are approximated also by WNN. It is a feed-forward neural network in which wavelets are used as activation functions. They are used to decompose the input signals under analysis. This introduces therefore an additional layer in the NN system inside which the dilation and translation parameters ( $a$  and  $b$ ) are modified as well the weight parameters during the process of training. This flexibility facilitates more reliability towards optimum learning solution. Based on wavelet theory, WNN has been proposed as a novel universal tool for functional approximation, which may create a surprising effectiveness in solving the conventional problem of poor convergence or even divergence encountered in other kinds of neural networks. Despite of the WNN great potential applicability, there are only a few papers on its applications in civil engineering [3]. As the eigenvalues are highly nonlinear functions of the design variables, in this work a WNN system is employed to approximate the frequencies.

The topological structure of the WNN is shown in Figure (5). The WNN consists of three layers; input, hidden and output layers. The connections between input and hidden units, and also between hidden and output units are called weights  $w_{it}$  and  $W_{ij}$  respectively. The topological structure of the ANN is shown in Figure (6). As clear in the figure, only the activation function in hidden layer is different from that in WNN. The back propagation algorithm as the basis of WNN process, for each sample of  $N$  training samples in figure (7) has been shown and is described as follows:

- (1) Initializing the dilation and translation parameters  $a_t$  and  $b_t$  and also node connection weights  $w_{it}$ ,  $W_{ij}$  using some random values. All these random values are limited in the interval (0,1).
- (2) Inputting the first sample data  $x_n(i)$  and their corresponding output values  $V_{nj}^G$  where  $i$  and  $j$  vary from 1 to  $S$  and 1 to  $J$  respectively.  $S$  and  $J$  represent the number of the input and output nodes respectively. The suffix  $n$  represents the  $n$ th data sample of training set, and  $G$  represents the desired output state.
- (3) Using the feed-forward step of the back propagation algorithm, the output value of the sample  $V_{nj}$  is computed using (8):

$$V_{nj} = \sum_{t=1}^T W_{tj} f(X) = \sum_{t=1}^T W_{tj} f\left(\frac{\sum_{i=1}^S w_{it} x_n(i) - b_t}{a_t}\right) \quad \text{where}$$

$$f(X) = \frac{X \cos(X)}{X^2 + 1} \quad (\text{RASP2 wavelet}) \quad (8)$$

where  $T$  represents the number of neurons in hidden layer and  $f$  could be any wavelet such as those listed in Table I [7]. Here is shown a sample wavelet from the list called RASP2.

(4) Using the backward step of the algorithm, the error  $E_n$  described in (8), is reduced by adjusting the parameters  $W_{ij}$ ,  $w_{it}$ ,  $a_t$  and  $b_t$ . For this purpose, one needs to compute  $\Delta W_{ij}$ ,  $\Delta w_{it}$ ,  $\Delta a_t$  and  $\Delta b_t$  as indicated in (10) and (11), using the gradient descend algorithm. This leads to the determination of equation (12) and equation (13) as follow:

$$E_n = \frac{1}{2} \sum_{j=1}^J (V_{nj}^G - V_{nj})^2 \quad (9)$$

$$\Delta W_{ij}(k+1) = -\eta \frac{\partial E}{\partial W_{ij}(k)} + \alpha \Delta W_{ij}(k) \quad (10)$$

$$\Delta w_{it}(k+1) = -\eta \frac{\partial E}{\partial w_{it}(k)} + \alpha \Delta w_{it}(k)$$

$$\Delta a_t(k+1) = -\eta \frac{\partial E}{\partial a_t(k)} + \alpha \Delta a_t(k) \quad (11)$$

$$\Delta b_t(k+1) = -\eta \frac{\partial E}{\partial b_t(k)} + \alpha \Delta b_t(k)$$

$$W_{ij}(k+1) = W_{ij}(k) + \Delta W_{ij}(k+1) \quad (12)$$

$$w_{it}(k+1) = w_{it}(k) + \Delta w_{it}(k+1)$$

$$a_t(k+1) = a_t(k) + \Delta a_t(k+1) \quad (13)$$

$$b_t(k+1) = b_t(k) + \Delta b_t(k+1)$$

Where  $k$  represents the backward step number and  $\eta$  and  $\alpha$  being the learning and the momentum constants, differing in the ranges 0.01 to 0.1 and 0.1 to 0.9, respectively.

(5) Returning to step (3), the process is continued until  $E_n$  satisfies the given error criterion, after which the training of

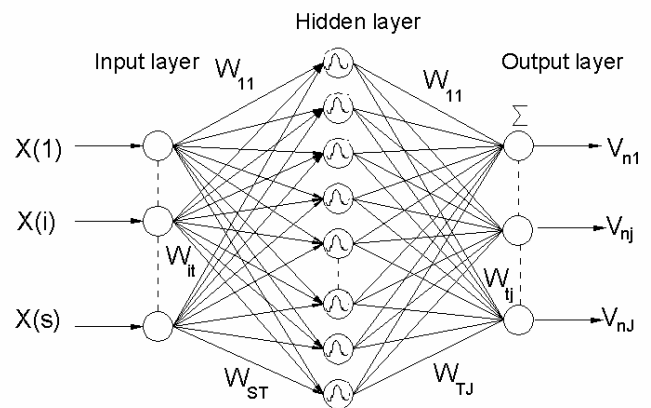


Fig. 5 the WNN topology structure

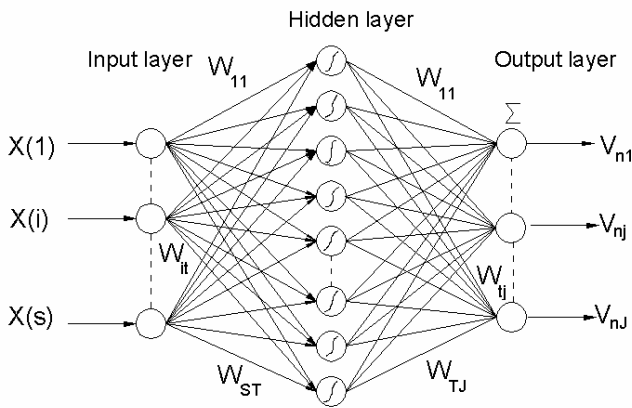


Fig. 6 the ANN topology structure

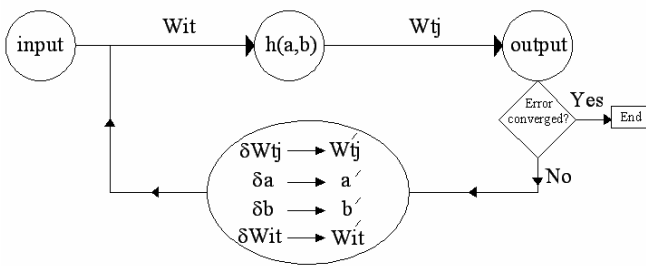


Fig. 7 the WNN training cycle

the next sample is started. This will eventually lead to the training of the network for the whole samples. Now the number of epochs is the number of times where the set of the samples are trained once. It is also called number of iterations. This will eventually lead to the training of the network for the whole samples.

### VII. MAIN STEPS

The procedure of structural optimization with frequency constraints using WNN is summarized as follows:

- (1) Assume some initial values for the design variables.
- (2) Solve (3) to evaluate the eigenvalues. Returning to step (1) depending on the number of design samples, a set of cross sections and their relative frequencies are obtained. These are the input data for training the WNN system.
- (3) Train WNN in the manner that was explained in section (4). The parameters in WNN will be fixed after training.
- (4) Solve the approximated design problem by the numerical optimization methods. A Sequential Quadratic Programming (SQP) method is used for solving the problem [8]. Obviously, during the process of optimization, the eigenvalue analysis of the structure is not needed and the WNN approximates the eigenvalues.
- (5) Check for optimization convergence; if not converged, go to step (4). Otherwise, analyze the optimum solution and make sure of constraints satisfaction. Print the results and stop the process.

### VIII. NUMERICAL RESULTS

The described method has been embedded as a computer program and a comparative example between ANN and WNN and three frame optimization examples were attempted, the results of which are followed.

#### A. Problem 1: A comparison between ANN and WNN

Neural networks are compared with a standard problem called XOR. In this example NN is trained to estimate the XOR function. XOR function is described as (table I):

TABLE I  
 INPUT AND OUTPUTS OF XOR FUNCTION

$(x_1)$ input(1)	$(x_2)$ input(2)	$(y)$ output
0	0	0
0	1	1
1	0	1
1	1	0

Each row of the table makes one sample of training sample set. A 2-5-1 structure was used for both ANN and WNN that is the networks are consist of 2 nodes as input layer, 5 neurons as hidden layer and 1 neuron as output layer.

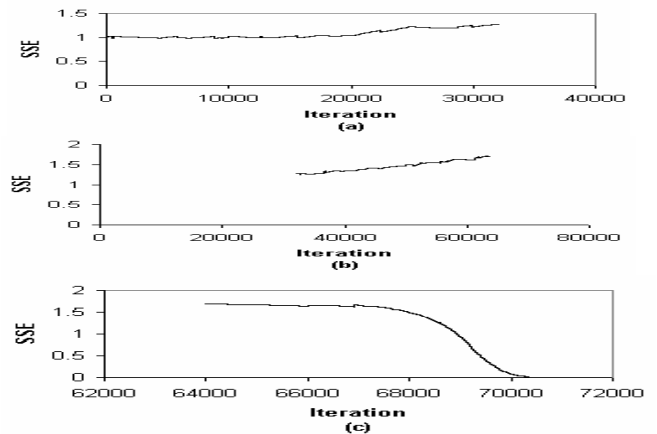


Fig. 8 Variation of SSE during ANN training process in XOR problem

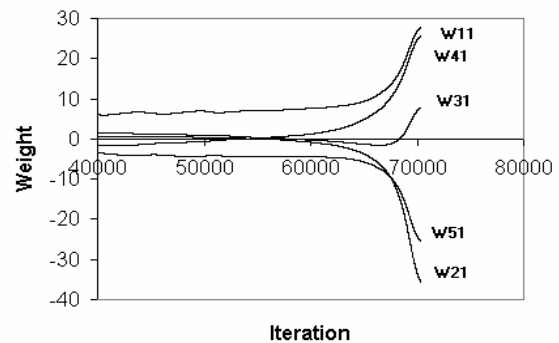


Fig. 9 Variation of weights between hidden and output layer during ANN training process in XOR problem

ANN was trained after 70375 iterations. An SSE (Sum Squared Error) variation is indicated in Figure (8). A very slow convergence was seen during ANN training. The weight variation between hidden and output layer is shown in Figure (9). After iteration 67000, a fast convergence to minimization of the error happens.

The same structure was used for WNN. In this case just in 33468 the network was trained. This faster convergence refers to existence of adaptable factors;  $a$  and  $b$  which makes NN to fit the function. In Figures 10, 11, 12 and 13, variation of SSE, parameters;  $a$ ,  $b$  and weight is shown.

TABLE II  
 RESULTS OF XOR PROBLEM

$(x_1)$ input (1)	$(x_2)$ input (2)	(y) output	ANN method	WNN method
0	0	0	0.020	-0.041
0	1	1	0.911	0.969
1	0	1	0.990	0.945
1	1	1	0.010	-0.006
			No. of epochs 70375	No. of epochs 33468

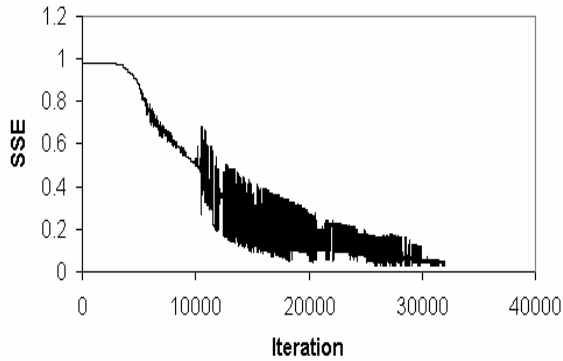


Fig. 10 Variation of SSE during WNN training process in XOR problem

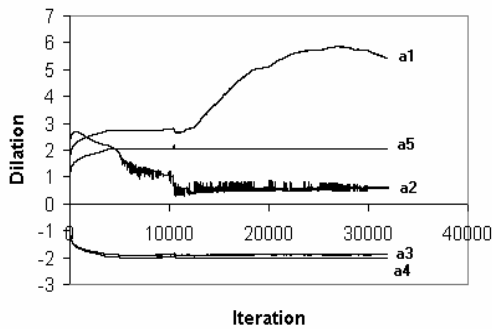


Fig. 11 Variation of parameter a during WNN training process in XOR problem

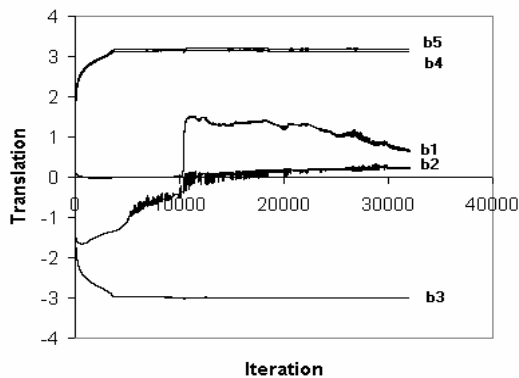


Fig. 12 Variation of parameter b during WNN training process in XOR problem

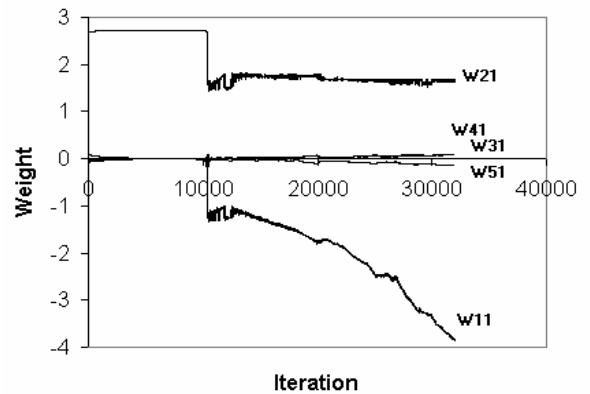


Fig. 13 Variation of weights between hidden and output layer during WNN training process in XOR problem

**B. Problem 2: A One story plane frame structure**

This problem is designed with two frequency constraints. The lowest allowable natural frequency is 78.8 rad/s. The principal moments of inertia  $I_z$  are considered in terms of CSA (A) as follow [9]:

$$I_z = 4.6248 A^2 \quad \text{if} \quad 0 \text{ in}^2 < A \leq 44.2 \text{ in}^2 \quad (14)$$

$$I_z = 256 A - 2300 \quad \text{if} \quad 44.2 \text{ in}^2 < A < 88.28 \text{ in}^2 \quad (15)$$

$$\text{where } A_L \leq A \leq A_U \Rightarrow 3 \text{ in}^2 \leq A \leq 88.28 \text{ in}^2 \quad (16)$$

The frequency constraints are considered as follows ( $\omega = \lambda^{1/2}$ ):

$$\omega_1 \geq 5 \text{ Hz} \quad \omega_2 \geq 18 \text{ Hz} \quad (17)$$

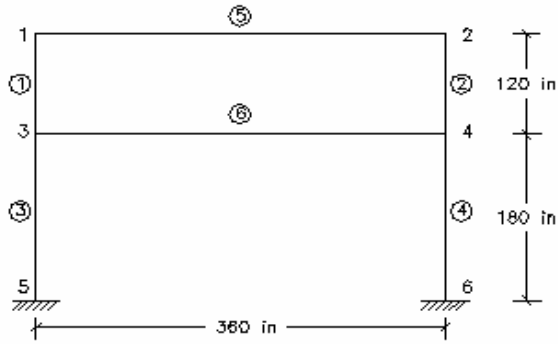


Fig. 14 plane frame structure

In ANN method, a sigmoid function was employed as a transfer function. The best network architecture was determined as 4-5-2, meaning 4 nodes as the number of design variables in the input layer, 5 neurons in the hidden layer and 2 neurons as the number of constrains in the output layer. The best learning and momentum rates were found as 0.01, 0.9 respectively. 20 samples were used for training ANN. After 9338 epochs, the network was trained. Results show that the optimum weight determined is not reliable because of the violation of constraints by 20%. This could well be due to low number of training samples. It is worth to note that a least number of 100 training samples are used generally in different literatures [4].

TABLE IV  
 OPTIMUM RESULTS FOR PLANE FRAME STRUCTURE

Element no.	CSA using WNN method( $in^2$ )	CSA using Three point method( $in^2$ )
1	8.98	5.9
2	8.98	5.9
3	14.36	19.24
4	14.36	19.24
5	3.00	5.8
6	7.63	3.00
Weight (lb)	3122.63	3222.91

To further improve the prediction accuracy and investigate the capability of WNN, a similar study was carried out using a WNN. The same network architecture as ANN, that is 4-5-2 was used. Learning and momentum rates of 0.01 and 0.9 were set respectively. Similarly WNN was trained with 20 samples. Among different number of wavelet functions studied, as shown in Table III, It was found that RASP2 function is the best choice for approximating the frequencies. Only in 6295 epochs, the system was trained with an optimum weight of 3122.63 lb with no constraints violation. A 3% reduction in

optimum weight design was observed compared with the two-point approximation [10] (see Table IV).

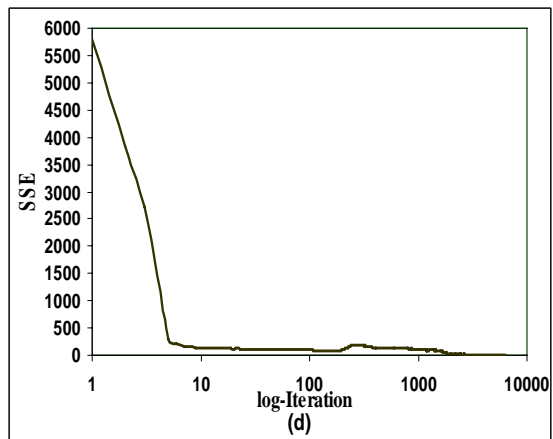
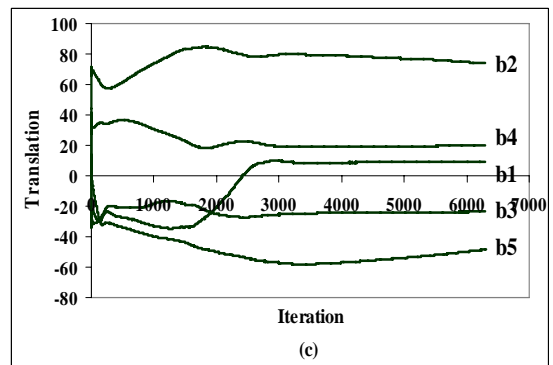
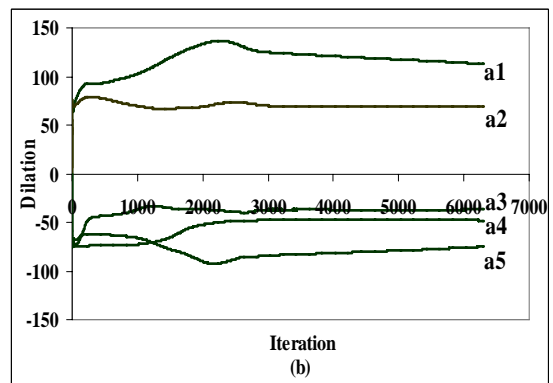
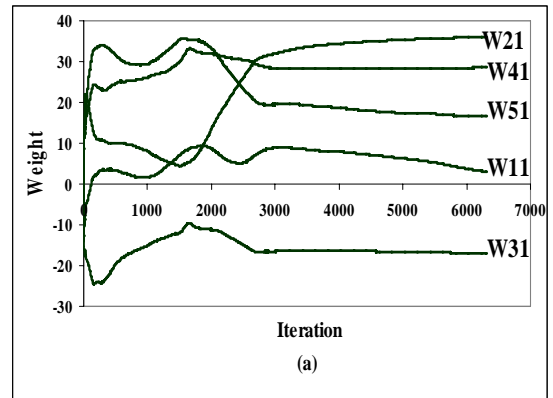


Fig. 15 WNN parameters variation

TABLE III  
WAVELET TRANSFER FUNCTIONS

Name	$F(x)$	Name	$F(x)$
Morlet	$\cos(\omega_0 x)e^{(-0.5x^2)}$	Polywog1	$xe^{(-x^2/2)}$
RASP1	$x/(x^2 + 1)^2$	Polywog2	$(x^3 - 3x)e^{(-x^2/2)}$
RASP2	$x \cos x/(x^2 + 1)$	Polywog3	$(x^4 - 6x^2 + 3)e^{(-x^2/2)}$
RASP3	$\sin \pi x/(x^2 - 1)$	Polywog4	$(1 - x^2)e^{(-x^2/2)}$
Polywog5	$(3x^2 - x^4)e^{(-x^2/2)}$	Slog2	$1/(1 + e^{-x+1}) - 1/(1 + e^{-x+3}) - 1$
Slog1	$1/(1 + e^{-x+1}) - 1/(1 + e^{-x+3})$	Shannon	$(\sin 2\pi x - \sin \pi x) / \pi x$

Figure (15a) indicates weight variation between hidden and output layers belonging to the first output. Dilation and translation variations for wavelet functions were also shown in Figures (15b) and (15c) respectively. Figure (15d) shows a quick convergence of the SSE which within the first 6 iteration has been reduced from nearly 6000 to less than 100. In WNN and ANN, only 20 samples were used, that is just 20 analyses were required, whereas in the approach presented by McGee and Phan a total number of up to 70 analyses were required [9].

### C. Problem 3: One story space frame structure

The eight-member space frame shown in Figure (16) is chosen from McGee and Phan [11]. The structure is designed for minimum weight subjected to multiple frequency constraints. A non-structural mass of  $3.0 \text{ lb}\cdot\text{s}^2 / \text{in}$  is stipulated at the free nodes of the space frame. The material properties are the weight density  $\rho = 0.28 \text{ lb}/\text{in}^3$  and the modulus of elasticity  $E = 30 \times 10^6 \text{ psi}$ . The cross-sectional areas of the structure are taken as the design variables and the moments of inertia of each member are expressed in terms of the area of the member  $A$  as:

$$I_z = aA_i^p \quad I_y = bA_i^q \quad J = cA_i^r \quad (18)$$

where  $a, b$  and  $c$  are constants governed by the dimensions of the section and  $p, q$  and  $r$  are some positive numbers. Thus [11]:

$$I_z = 4.6248 A^2 \quad \text{if } 0 \text{ in}^2 < A \leq 44.2 \text{ in}^2 \quad (19)$$

$$I_z = 256 A - 2300 \quad \text{if } 44.2 \text{ in}^2 < A \leq 88.28 \text{ in}^2$$

$$I_y = 0.1248 A^2 \quad \text{if } 0 \text{ in}^2 < A \leq 61.8 \text{ in}^2 \quad (20)$$

$$I_y = 91.207 A - 5225.59 \quad \text{if } 61.8 \text{ in}^2 < A \leq 67.6 \text{ in}^2$$

$$J = 0.0072 A^2 \quad \text{if } 0 \text{ in}^2 < A < 88.3 \text{ in}^2 \quad (21)$$

$$A_L \leq A \leq A_U \Rightarrow 0.1 \text{ in}^2 \leq A \leq 88.3 \text{ in}^2 \quad (22)$$

$$\omega_1 \geq 5 \text{ Hz} \quad \omega_2 \geq 9 \text{ Hz} \quad \omega_3 \geq 15 \text{ Hz} \quad (23)$$

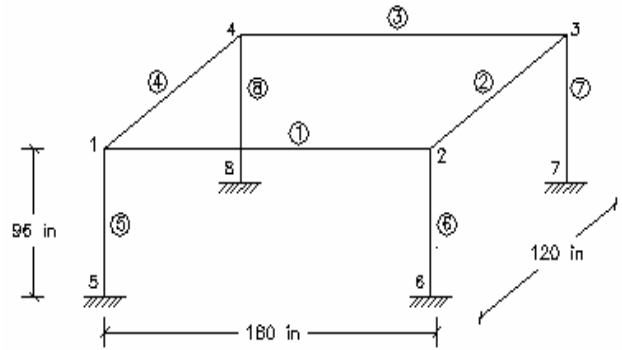


Fig. 16 One story space frame structure

TABLE V  
SPACE FRAME STRUCTURE RESULTS

	Three point method	ANN method	WNN method
No. of analyses	6	9	9
No. of epochs	-	41442	36449
Weight (lb)	1772.1	2437.21	1403.81

In ANN and WNN methods, the best network architecture was determined as 3-10-3. In ANN the best learning and momentum rates were found as 0.06, 0.5 respectively. The corresponding rates in WNN were found as 0.05, 0.3 respectively. 9 samples were used for training the ANN and WNN, results of which are available in Table 5. The same problem solved by McGee and Phan [11], with various options of optimality criteria methods, was reported a requirement of 25 to 65 analyses to be determined.

### D. Problem 4: Ten story space frame structure

The structure is shown in figure (17) with 88 nodes. The design variables are the 180 cross-sectional areas of the members. The material properties and relations between the areas and second moments of inertia of the members are those indicated in problem 3. A non-structural mass of  $1.0 \text{ lb}\cdot\text{s}^2 / \text{in}$  is placed at the free nodes of the frame. The frequency constraints are considered as:

$$\omega_1 \geq 0.5 \text{ Hz} \quad \omega_2 \geq 0.8 \text{ Hz} \quad \omega_3 \geq 1.2 \text{ Hz} \quad (24)$$

In ANN and WNN methods, the best network architecture was determined as 20-8-3. In ANN the best learning and momentum rates were found as 0.01, 0.9 respectively. The corresponding rates in WNN were found as 0.01, 0.9 respectively. 20 samples were used for training the ANN and WNN, results of which are available in Table VI. The same problem solved by McGee and Phan [11], with six options of optimality criteria methods, was reported a requirement of 23



to 95 analyses to be determined.

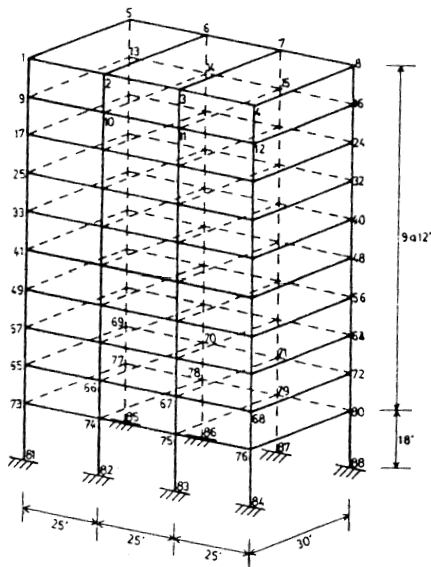


Fig. 17 Ten story space frame structure

TABLE VI  
SPACE FRAME STRUCTURE RESULTS

	Three point method	ANN method	WNN method
No. of analyses	9	20	20
No. of epochs	-	65536	16334
Weight (lb)	13704.8	16126.6	12729.3

## IX. CONCLUSION

A new method for estimating the frequency of two and three dimensional frame structures during optimization process was introduced. This method is on the basis of the wavelet theory. We used wavelet functions as the activation functions in the skeleton of the traditional neural network. This substitution forms a robust neural network called WNN. Results show that WNN is not only faster than ANN but also more accurate than the other neural networks. Main reason for this capability is the existence of two parameters in wavelet function called dilation and translation. Wavelet functions adjust these parameters automatically to detect the vicinity of the original signal. We used WNN instead of the numerical analysis to predict the frequency of the structure. A number of structures are attempted for optimization based of WNN. Having investigated different wavelet functions for the optimization study, the determined results show that RASP2 wavelet families are the best choice for estimating the frequency of this type of structures.

## REFERENCES

- [1] TH. Haftka and Z. Gurdal, Elements of structural optimization. 3rd ed. Netherlands: Kluwer Academic Publishers, 1992. W.-K. Chen, Linear Networks and Systems (Book style). Belmont, CA: Wadsworth, 1993, pp. 123-135.
- [2] R.V. Grandhi, Structural optimization with frequency constraints-a review. AIAA J., 1993, 31:2296-22303.

- [3] H. Adeli, Neural networks in civil engineering:1989-2000. Comp-Aided Civil and Infrastructures Eng. J., 2001, 6:126-142.
- [4] B. Muller and J.Reinhardt, Neural Networks. Springer Verlag,1990, Berlin.
- [5] R. Lippmann, An Introduction to Computing with Neural Networks. IEEE ASSP Mag., April 1987, pp 4-22.
- [6] A. Grossmann and J. Morlet, Decomposition of Hardy Functions into Square Integrable Wavelets of Constant Shape. SIAM J., Math. Anal., v15, n4, 1984, pp. 723-736.
- [7] G. Lekutai, Adaptive self-tuning neuro wavelet network controllers. PhD thesis, Elec Eng, 1997, Virginia, 24061-01115.
- [8] A. D. Belegundu and T. R. Chandrupatla, Optimization Concepts and Applications in Engineering, 1999, Prentice Hall, Upper Saddle River, New Jersey 07451.
- [9] O. G. McGee and K. F. Phan, A robust optimality criteria procedure for cross-sectional optimization of frame structures with multiple frequency limits. Comp Struct J., 1991, 38:485-500.
- [10] E. Salajegheh, Optimum design of structures with high-quality approximation of frequency constraints. Advances in Engineering Software J., 2000, 31:381-384.
- [11] O. G. McGee and K. F. Phan, On the convergence quality of minimum weight design of large space frames under multiple dynamic constraints. Struct Opt J., 1992, 4:156-164.



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