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Estimation of Broadcast Probability in Wireless Adhoc Networks

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***Abstract*—** Most routing protocols (DSR, AODV etc.) that have been designed for wireless adhoc networks incorporate the broadcasting operation in their route discovery scheme. Probabilistic broadcasting techniques have been developed to optimize the broadcast operation which is otherwise very expensive in terms of the redundancy and the traffic it generates. In this paper we have explored percolation theory to gain a different perspective on probabilistic broadcasting schemes which have been actively researched in the recent years. This theory has helped us estimate the value of broadcast probability in a wireless adhoc network as a function of the size of the network. We also show that, operating at those optimal values of broadcast probability there is at least 25-30% reduction in packet regeneration during successful broadcasting.

Keywords- Crossover length, Percolation, Probabilistic broadcast, Wireless adhoc networks

I. INTRODUCTION

BROADCASTING is a very essential scheme to enable successful communication in wireless networks such as manets, sensor networks etc. essentially because these nodes don't really have information about the topology of the network. This approach like any other scheme has got its drawbacks. Problems that have been identified with broadcasting are categorized as broadcast storm problem [1].

A lot of research has also gone into optimizing broadcasting operation with neighbour based, area based etc techniques [2] [3]. Another such technique is the probability based technique first suggested by Yoav Sasson, David Cavin, Andre Schiper in their paper [4]. The authors in that paper have modelled the problem of optimizing broadcast operation using percolation theory [5]. We extend the idea suggested by that paper by exploring percolation theory further.

We have analytically derived and also verified, how good (and under what conditions) it is to use probability based techniques for optimizing broadcast operation. In the process, we have also identified the shortcomings and limitations of probabilistic techniques. All the results have been verified for ideal network conditions without loss of generality.

The structure of the paper is as follows, section II would be a brief introduction to percolation theory. Section III would elaborate on our model description followed by section IV which would illustrate results we have used from percolation theory and in section V we have verified them with our simulated results. Section VI would list out the major conclusions we have made through this paper and possible areas to work further.

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II. PERCOLATION THEORY

Percolation literally means movement of liquid inside a random medium. The theory of percolation deals with the study of percolation on lattices of different dimensions and structures. The random medium could be modelled, in two dimensions, as a square lattice. Two kinds of percolation, Bond and Site (refer to Fig. 1)have been studied very extensively on these lattices. Bond percolation involves uncertainty associated with the edges between the nodes of the lattice, i.e. each edge being either open (or closed) with a probability p (with $(1-p)$). Similarly the uncertainty in site percolation is associated with site rather than the edge i.e., each site is open with a probability p and closed with $(1-p)$. Movement within the lattice is restricted to paths which are connected by open edges (in the case of Bond percolation) or open sites (in the case of site percolation).

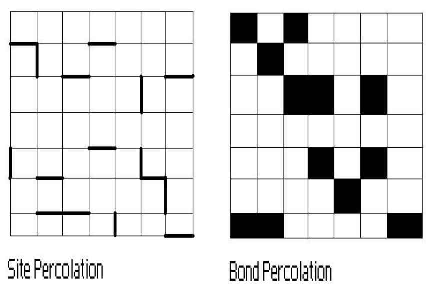

Fig. 1. Site Percolation and Bond Percolation

The existence of a critical probability p_c, beyond which there is an infinite cluster is one of the major results of percolation theory. The value of p_c has been determined analytically for very few cases, bond percolation on 2D square lattice being one of them. Thus the probability of existence of the infinite cluster undergoes a phase transition (as could be observed in Fig. 2) [6] when the connection probability is at p_c from $0 \rightarrow 1$.

This result has been of tremendous importance considering the applications of percolation theory. In a simple model of broadcasting in wireless adhoc networks using percolation theory, a critical probability would translate into the existence of critical broadcast probability beyond which a number of nodes would be connected. In the introductory paper [4] on probabilistic broadcasting, this has been the major argument.

This argument though does not take into account the number of nodes which become connected and the conditions under which they become connected. Exploring percolation theory further, one can find a lot of study conducted on the nature of these clusters/paths within the lattices, size of these clusters and the meaning of an infinite cluster. Infinite cluster in percolation theory is merely a path connecting the left end of the lattice to the right end, without necessarily connecting all nodes. We have explored these equations that estimate the sizes of these clusters.

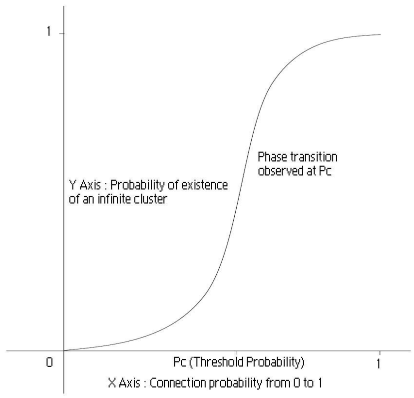

Fig. 2. Phase-transition phenomena observed in percolation

We will begin our arguments by first explaining our model in the next section along side getting into deeper understanding of percolation theory.

III. Model Description

Let us consider a wireless adhoc network which has been deployed on an area A and with node density ηper Sq. units. The total number of nodes is equal to $A\eta$. Without loss of generality, we can align each node to the closest intersection on the grid, with no more than one node at every intersection and none of the intersections points in the grid are left empty. We thus have a square grid of $L\ X\ L$ in 2 dimensions, with L nodes in one dimension.

Therefore, $L^2 =$Aη

Since we plan to model the broadcast operation in wireless adhoc network, its easier to apply site percolation rather than bond percolation on our 2D square lattice. Connection probability in site percolation is equivalent to the broadcast probability in wireless adhoc network (2D square) which is to say that a site is open with probability a p is equivalent to a node rebroadcasting the packet with probability p(refer to Fig. 3).

With this basic model of a wireless adhoc network as a 2D

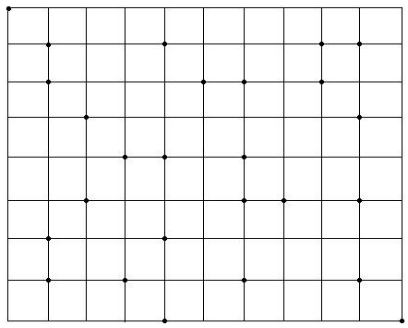

Fig. 3. All the intersection points are nodes of the wireless adhoc network. The darkened ones indicate the nodes which would rebroadcast

square lattice, we can now apply results from percolation theory. Some of the important assumptions made here are,

1) The entire grid is occupied.
2) All the immediate neighbours of a node in the horizontal and vertical direction can listen to that particular node and the diagonal neighbours cannot hear that particular node.
3) There is no loss of information between two nodes.
4) Nodes are immobile.
5) Collision and other traffic conditions do not apply here.

IV. Results from percolation theory

The quantity of interest so far has been the connection probability p, which determined the existence of infinite cluster and in the case that there exists an infinite cluster for a particular p, we could say all nodes with p as their broadcast probability would be connected.

Some of these quantities become very confusing while considering finite systems, which is precisely what a wireless adhoc network is (e.g. What would an infinite cluster mean in a finite lattice?). A wireless adhoc network consists of a limited number of nodes and is far from being considered an infinite system and so far in the literature it has been modelled as an infinite system.

The moment we shift our focus from the infinite cluster, to the largest cluster of the lattice, some of these questions could be answered. An infinite cluster as stated in percolation theory is a cluster which connects the left end of the lattice to the right. This as we notice is not the requirement for successful broadcast, we not just need the left end to be connected to the right but also all the nodes within the lattice to be connected. This shift of concern from the infinite cluster to the largest cluster of the lattice surprisingly shifts the focus from critical probability p_c, to another term known as the crossover length denoted as ξ. Percolation theory offers us lot of mathematical equations which govern the behaviour of the size/mass(M) of the largest cluster.

Our major focus has been to understand how and when this mass of the largest cluster in a lattice varies as the total size of the lattice (L^2 is the total number of nodes in the lattice) i.e., $M \propto L^2$.

$M \propto L^D$ for $L < \xi$

$M \propto L^d$ for $L > \xi$

$$\xi \propto (p - p_c)^{-\nu};\ P \propto (p - p_c)^{\beta};$$

$\beta \cong 0.14; \nu \cong 1.33$

D is the $fractal\ dimension$,d is the $dimension$ of the lattice, L is the $length$ of the cluster in one dimension, P $strength$ of the infinite cluster, M $mass$ of the largest cluster.

$D = d - (\beta/\xi)$ for $d < 6$

Theoretical value of $D = 91/48 = 1.9$

Also the number of nodes which receive a broadcasted message is greater than the actual mass of the largest cluster. The reason for this is, in site percolation a path is possible only when both the neighbours are open whereas in broadcasting even if one node does not rebroadcast the message it can definitely receive a broadcasted message from a broadcasting neighbour.

To represent this relation in mathematical form, we say

Total number of nodes receiving broadcasted message (T_{br}) =

Mass of the Cluster (M) +

Perimeter of the cluster (t)

Perimeter of a cluster (t) denotes the number of nodes which are the immediate neighbours for the cluster.

In order to execute a successful broadcast operation one must operate at a value of p which makes sure the number of nodes receiving broadcasted message is proportional to L^2 and hence its not just sufficient to be operating at $p > p_c$

An interesting point to note here is that selection of p is also a function of the lattice size in one dimension L i.e., for a given lattice of size $L_1\ X\ L_1$, we need to check what the closest value of crossover length is and then determine the corresponding p with the above mentioned equation. Although these equations represent proportionalities a more accurate value can be estimated using simulations.

Another interesting point to note here is, when operating at $p = p_c$, the value of crossover length is infinite, hence no matter what the size of the lattice is, the mass of the largest cluster would vary as L^D and not L^d. So it is definitely advisable to operate above p_c.

In order to verify our conclusions, we present our simulated results in the next section.

V. Simulation

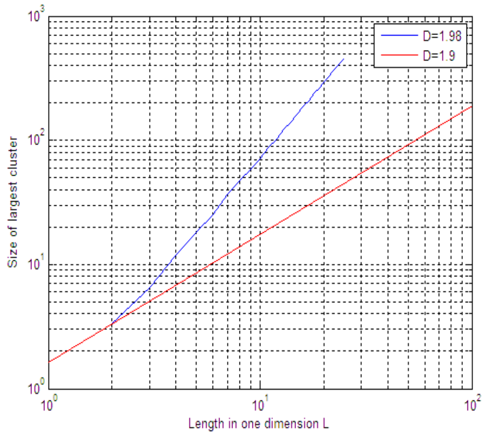

Fig. 4. Largest Cluster vs. Length in one dimension, for number of sites varying between 4 and 625, for p = 0.75

We have verified the results used in the above section by simulation of site percolation(Fig. 4,5,6) on a 2D square lattice on MATLAB. We have varied the sizes of lattice from 4 sites to 625 sites. The simulation involved the following:

1) Generating random numbers and depending on the value of connection probability either place an open or closed site.
2) Using the Hoshmen Kopelman [7] algorithm, we percolated labels through the lattice.
3) Estimated the size of the largest cluster in the lattice for varying sizes of lattice for a particular value of connection probability p.

The value of the fractal dimension D, has been estimated from the log log plot of size of largest cluster vs. length of lattice in one dimension. It turned out be around 1.98(refer to Fig. 4), just as it has been theoretically derived.

We also simulated an ideal wireless adhoc network(Fig. 7,8,9) on MATLAB using the model described in section III. The number nodes again varied between 4 nodes to 625 nodes. The simulation involved the following:

1) Decided with a probability p if each node rebroadcasts or not to its neighbours in the vertical and horizontal direction.
2) Broadcasting a message from a different node every time, we calculated the number of nodes which are actually receiving it.

Roughly around when $p = 0.7$ we can observe(refer to Fig. 8, 9, 10) the number of nodes receiving broadcasted message varying as the number of nodes in the wireless adhoc network.

p	$\xi \propto$
0.59	∞
0.6	457
0.65	42
0.7	18
0.75	12
0.8	8
0.85	6
0.9	5
0.95	4

TABLE I

p AND CORRESPONDING CROSSOVER LENGTH ξ VALUE

The major conclusion of the paper is as follows, given a wireless adhoc network of size $L\ X\ L$, we need to be able to predict the broadcast probability with which nodes have to broadcast so as to ensure its successful (complete connectivity).

Ideally to optimize power consumption, one has to operate at the lowest possible value of p and also simultaneously ensure connectivity. This can be done by selecting the value of crossover length that is close to L. As noted in the equations above, when the value of crossover length is close to or less than L, the size of the largest cluster is proportional to the size of the lattice.

What is also to be noticed are the values of crossover length at a particular value of p from the Table I.

At $p = p_c$,

crossover length$(\xi) \to \infty$.

This value falls sharply as we move away from p_c, but still remains well over usual wireless adhoc network dimensions. Hence it is interesting to see that the size of wireless adhoc network in one dimension would always be less than crossover length if we use p close to p_c and in which case the size of largest cluster would vary as L^D and not L^d. This picture though changes drastically when p is roughly around 0.7 when, crossover length becomes close to the size of a realistic wireless adhoc network in one dimension.

By fixing the value of crossover length for a lattice of size $L\ X\ L$, one could fix p from Table I(the left column is the connection probability p and the right gives the proportional value of crossover length ξ). To be noted is the value of p_c for site percolation on 2D square lattice which is roughly around 0.59 [5]. For example, if the size of the wireless adhoc network $\approx 10^6$, one can assume the length of crossover length ξ to be around 400 and the corresponding value of broadcast probability (p) is 0.6(from Table I). Therefore for a wireless adhoc network of such huge(unrealistic) size, $p > 0.6$ could be an appropriate value. Similarly as the size of the wireless adhoc network decreases, the corresponding value of the ξ also reduces and the value of p increases.

Hence, for more realistic wireless adhoc network sizes $\approx 10^2$, the value of $\xi \approx 10$ to 40. Thus it is more sensible to use $0.65 < p < 0.8$. This has been verified in our simulations (Fig. 8,9,10).

Hence we draw a major conclusion here, which is to say that, it is not sufficient to operate at $p > p_c$ for successful broadcast, but also the size of the wireless adhoc network in one dimension is to be taken into account to determine the broadcast probability. Hence as stated in the introduction of the paper we have shifted the focus from connection probability(p) to the crossover length(ξ).

VI. Conclusion and Future work

Broadcasting, as mentioned also in the abstract, is very fundamental to a network with nodes which have no information about the topology of the network. Broadcasting operation is definitely optimized using probabilistic schemes but only under a lot of constraints on the network size, shape, density etc. A more effective technique for general networks would be to use an intelligent probabilistic scheme, where the value of rebroadcast probability could change with the density of nodes around. This could be particularly effective when the nodes are mobile and neighbourhood of a node keeps constantly changing. This idea has been explored in [8].

It is plausible to say that rebroadcast probability should be a function of the neighbour density, i.e., if the number of nodes in the neighbourhood is high, we could use a smaller broadcast probability and vice versa. This argument leads to another interesting point which is, the neighbour density of a node is a function of the transmission range, i.e. a node with larger transmission range is likely to have more number of neighbours purely by the virtue of its reachibility. What we can hence conclude is that, there is an inherent relation between the rebroadcast probability and transmission range of the node. It would thus be very interesting to work on dynamically changing rebroadcast probability(p) and transmission range(r)[9] simultaneously with a logic implemented with it. The logic could be as simple as, a node with large transmission range has lower rebroadcast probability and vice versa with,

$$rp = k(constant).$$

The other interesting result that percolation theory gives us is the value of $p_c \cong 0.246$ [5] for a BCC lattice. This is to say that the broadcast operation in a three dimensional wireless adhoc network can be optimized even further. To be able to arrive at the exact values of broadcast probability for 3D wireless adhoc networks, one has to do a similar research as we have done for 2D wireless adhoc networks.

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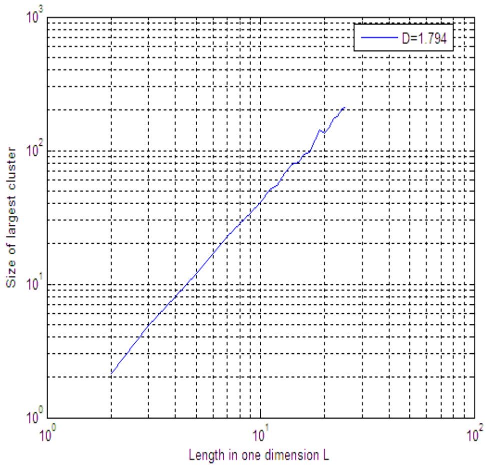

Fig. 5. Largest Cluster vs. Length in one dimension, for number of sites varying between 4 and 625, for $p = 0.6$

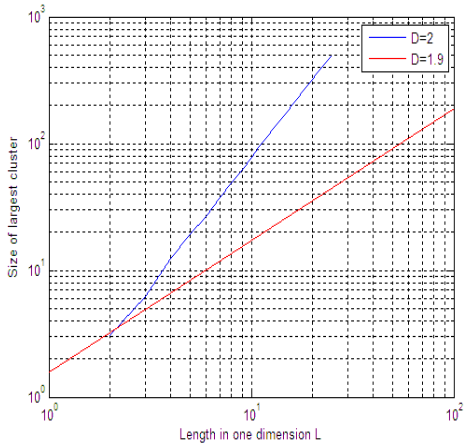

Fig. 6. Largest Cluster vs. Length in one dimension, for number of sites varying between 4 and 625, for $p = 0.8$

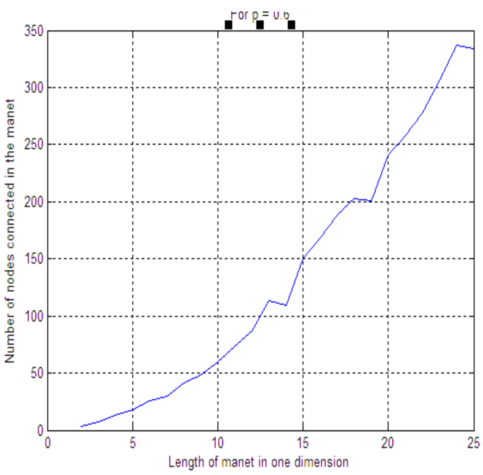

Fig. 7. Number of nodes reachable by broadcasting(T_{br}) vs. size of wireless adhoc network in one dimension, for $p = 0.6$

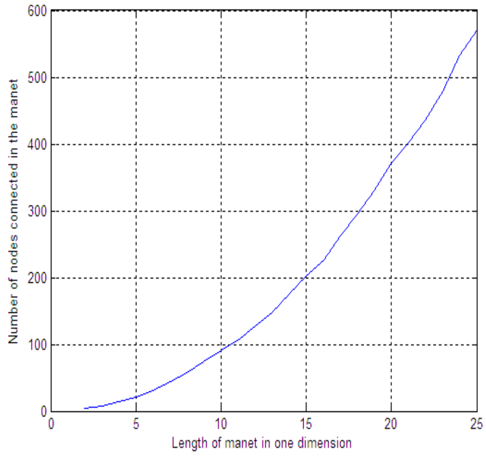

Fig. 8. Number of nodes reachable by broadcasting(T_{br}) vs. size of wireless adhoc network in one dimension, for $p = 0.7$

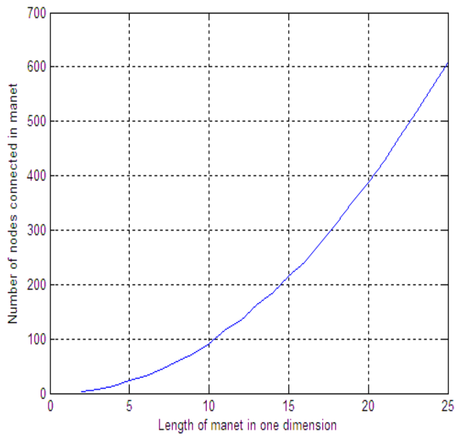

Fig. 9. Number of nodes reachable by broadcasting(T_{br}) vs. size of wireless adhoc network in one dimension, for $p = 0.75$

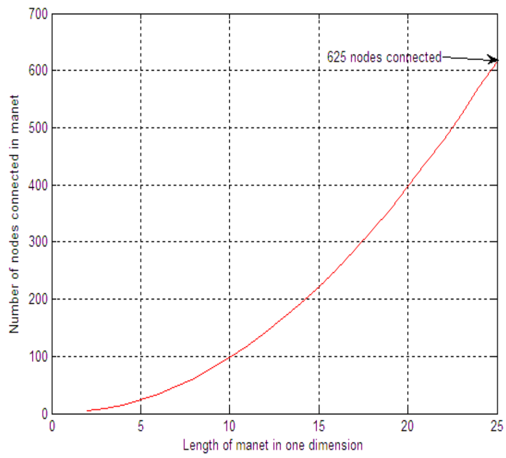

Fig. 10. Number of nodes reachable by broadcasting(T_{br}) vs. size of wireless adhoc network in one dimension, for $p = 0.8$

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