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Fuzzy Adjacency Matrix in Graphs

Mahdi Taheri and Mehrana Niroumand

Abstract**—**In this paper a new definition of adjacency matrix in the simple graphs is presented that is called fuzzy adjacency matrix, so that elements of it are in the form of 0 and $\frac{1}{n}, \; n \in N$ that are in the interval [0, 1], and then some charactristics of this matrix are presented with the related examples . This form matrix has complete of information of a graph.

Keywords**—**Graph, adjacency matrix, fuzzy numbers

I. Introduction

DEFINITION *1.* It is a classified tri-set (V(G),E(G),ψ (G)) which consist of an non empty collection V(G), Vertexes E(G) edges and ψ (G) incidence function that attributes.

Definition 2. A pair of G Vertexes which necessarily are not distinct to each G edge, If e is an edge and V_1, V_2 are vertexes that are connected by e therefore we will write ψ (G) (e)= υ V

*Definition 3***.** Two vertexes of a graph which are placed on the same edge are called adjacent and two edges placing on the same vertex are also called adjacent edge.

*Definition 4***.** An edge with two equal heads is called a loop and an edge with two distinct heads is called a linked loop.

*Definition 5***.** If the collection of vertexes and edges of a graph are finite that graph is called a finite graph.

*Definition 6***.** The graph in which there is not any loop, and also between its vertexes there is just one edge is called a simple graph otherwise it's called a multiple graph.

*Definition 7***.** Both G and H graph are called a like wherever V (G) = V (H) and E(G) =E (H) and ψ (G) = ψ (H), so we will write G=H.

*Definition 8***.** Both G and H graph are called Homomorphic if the two-way written θ: V (G) ➔ V (H) and ϕ E (G)➔ exist so as to we will have ψ G(e) =υ V if and if only $\psi_H(\phi_{(e)}) = \theta(\upsilon)\theta(V)$.

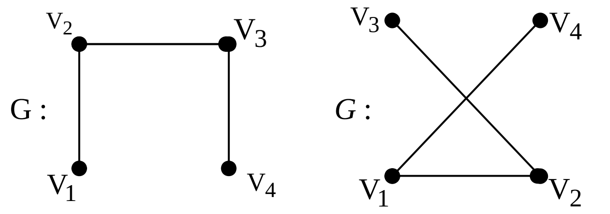
Definition 9. A simple graph in which two distinct vertexes are connected together by one edge is called a complete graph.

*Definition 10***.** G graph is called a connectivity graph in case otherwise it is called non-connectivity graph.

*Definition 11***.** The longest line between two vertexes of the same graph is called the graph consistency.

*Definition 12***.** G is used to show the supplementary graph of simple graph consisting of a collection of V vertexes in which both vertexes are adjacent if and if only the vertexes are not adjacent in G.

Example 1.



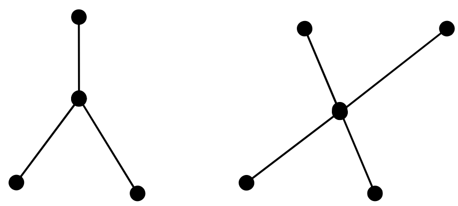
Definition 13. The edge of V vertexes in the G graph is shown by $D_G(V)$ and it is equal to the number of edge placed on V. in this definition each loop is counted as two edge.

Theorem 1. $\sum_{V \in V} d(V) = 2$ in which $\in$ shows the number of the graph edge.

Definition 14. The line of a graph that is equal that in both ends and its length is at least there called monocycle.

Definition 15. Graph is called three graphs when it lacks the loop.

Example 2.



Theorem 2. In each tree the both vertexes are connected by using a unique line.

Proving: by using contradiction theorem.

*Theorem 3***.** If G is a tree then we will have $\in$ =V-1 ($\in$ =the number of edge and V=number of Vertexes)

II. Incidence Matrix And Adjacency Matrix

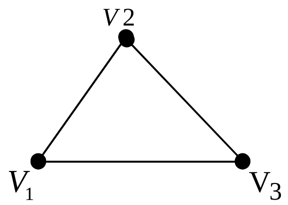
Parallel to each G graph there is an m×n Matrix.(m=number of vertexes and n=number of edge) that is called G incidence matrix. If we show vertexes of G using V_1---------V_m and the edge by ; . . . en then the G incidence matrix will be a matrix like M(G)=[m_{ij}] in which m_{ij} is equal to the times that V_1 is placed on ei(i.e. (0 , 1 , 2). Also parallel to each G graph thee is one n×n matrix like: A(G)=[a_{ij}] in which a_{ij} equal the number of edge between two vertexes V_j and V_i . it is obvious that if graph G is a simple graph, we will have :

M. Taheri, Department of mathematics, Islamic Azad university, Malayer branch, Malaer, Iran. Email: taheri@iau-malayer.ac.ir
M. Niroumand, Department of Mathematics, Islamic Azad university, Malayer branch, Malaer, Iran. Email: mehdimehrana@yahoo.com

$$a_{ij} = \begin{cases} 1 & \text{If } V_i \text{ and } V_j \text{ are connected by the same edge (they are adjacent)} \\ 0 & \text{Otherwise it is O} \end{cases}$$

Two example of adjacency matrix (Complete simple matrix):

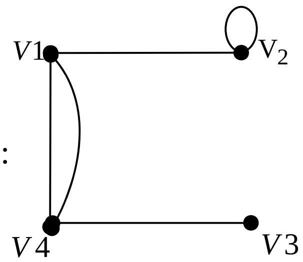
G_1:



(Multiple Matrix)

$$A_{G_1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

G_2:



$$A_{G_2} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

A. Properties of adjacency matrix

In adjacency Matrix of a simple graph all elements placed on the diameter are Zero. Adjacency Matrix of a graph is an isomorphic square matrix. The sum of each line in adjacency matrix of a graph shows the relevant degree of the line. In the fuzzy adjacency matrix of a complete graph all elements are one except the main diameter.

III. Fuzzy Sets

Definition 16. If x is a set of elements which is shown by x, then the fuzzy set of $\tilde{A}$ in x is defined as fallow:

$$\tilde{A} = \left\{ (x, M_{\tilde{A}}\ (x) \mid x \in X \right\}$$

Here $M_{\tilde{A}}\ (x)$ is x membership function or the membership degree in $\tilde{A}$ membership function illusion X set in M area. If the area of M membership function only consists of Zero (0) and one then this set will be a classic set. And, if the set of M consist of real numbers between Zero and one the set of $\tilde{A}$ will be a fuzzy set.

Example 3.

Suppose that fuzzy set of $\tilde{A}$ is defined on a set of real numbers around 10 its membership function then is defined as follow: (fig. 1)

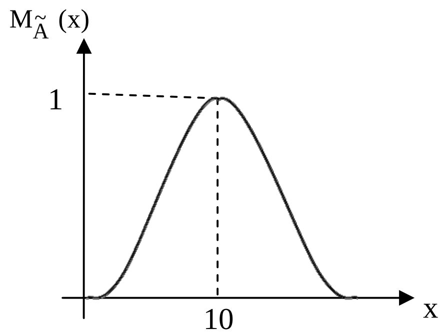


Fig. 1 membership function

Definition 17. The highest elements membership degree of a fuzzy set is called its height.

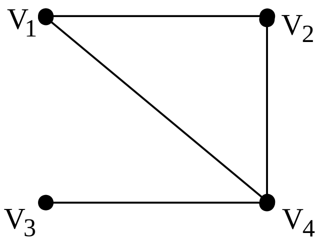
$$h(\tilde{A}) = \sup_{x \in x} \mu_{\tilde{A}}(x)$$

IV. Fuzzy Adjacency Matrix In Graph

If G is a graph and $V_1, \ldots, V_n$ are its vertexes then $A_f=[bji]_{n \times n}$ is called fuzzy adjacency matrix of the graph G so that:

$$b_{ij} = \begin{cases} \dfrac{1}{n} & \text{n is the number of degree in the shortest line of i , j} \\ 0 & \text{If } i = j \text{ or is not relevant to } V_i \text{ and } V_j \end{cases}$$

Example 4.



$$A_F = \begin{bmatrix} 0 & 1 & 1 & \frac{1}{2} \\ 1 & 0 & 1 & \frac{1}{2} \\ 1 & 1 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 & 0 \end{bmatrix}$$

A. Fuzzy adjacency matrix

Theorem 4.

i) Fuzzy adjacency matrix ia an isomorphic graph.

ii) If $A=[a_{ij}]$ is adjacency matrix and $A_F=[b_{ij}]=a_{ij}$

iii) If G is a complete graph $A_F=A$

iv) If G is a connectivity simple graph then all elements except the main diameter are non-Zero.

Theorem 5.

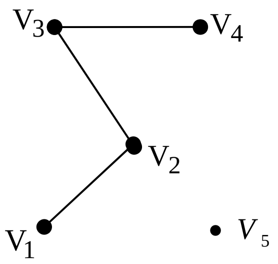
Function of proximity degree of graph

If G is a simple graph and AF= [bij] is its fuzzy adjacency

Then Mf is called the function of proximity degree of V_i , V_j Vertexes so as to

Example 5.

$$\mu_F : V \times V \to [o,1]$$
$$\mu_F : V_i \times V_j \to a_{ij} \ (i \neq j)$$



$$A_F = \begin{bmatrix} 0 & 1 & \frac{1}{2} & \frac{1}{3} & 0 \\ 1 & 0 & 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mu_f(V_1, V_2) = 1 , \quad \mu_f(V_1, V_2) = \tfrac{1}{2} \quad , \ . . .$$

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