Magnetohydrodynamics Boundary Layer Flows over a Stretching Surface with Radiation Effect and Embedded in Porous Medium

Siti Khuzaimah Soid, Zanariah Mohd Yusof, Ahmad Sukri Abd Aziz, Seripah Awang Kechil

***Abstract*—**A steady two-dimensional magnetohydrodynamics flow and heat transfer over a stretching vertical sheet influenced by radiation and porosity is studied. The governing boundary layer equations of partial differential equations are reduced to a system of ordinary differential equations using similarity transformation. The system is solved numerically by using a finite difference scheme known as the Keller-box method for some values of parameters, namely the radiation parameter *N*, magnetic parameter *M*, buoyancy parameter λ, Prandtl number *Pr* and permeability parameter *K*. The effects of the parameters on the heat transfer characteristics are analyzed and discussed. It is found that both the skin friction coefficient and the local Nusselt number decrease as the magnetic parameter M and permeability parameter *K* increase. Heat transfer rate at the surface decreases as the radiation parameter increases.

***Keywords*—**Keller-box, MHD boundary layer flow, permeability stretching.

I. Introduction

THE study of heat transfer over a stretching sheet has gained considerable attention due to its applications in industrial manufacturing processes, such as paper production, the aerodynamics extrusion of plastic sheet, glass blowing and metal spinning. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. The quality of the final product depends on the rate of heat transfer at the stretching surface.

Numerous researchers have investigated the heat transfer over stretching surface by considering the effect of magnetic field. The radiation effects are considered by Ghaly [1] for the magnetohydrodynamics free-convection flow and Raptis et al. [2] for the steady MHD asymmetric flow of an electrically conducting fluid past a semi-infinte stationary plate. Liu [3] analyzed the hydromagnetic fluid flow past a stretching sheet in the presence of a uniform transverse magnetic field. Chen [4] investigated the fluid flow and heat transfer on a stretching vertical sheet, and his work has been extended by Ishak et al. [5] to hydromagnetic flow and they found that as the magnetic field increases, the surface skin friction as well as the surface Nusselt number decrease.

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The radiative effects have important applications in physics and engineering particularly in space technology and high temperature processes [6]. Effects of radiation have been studied by Abdul Hakeem and Sathiyanathan [7], Seddeek and Abdelmeguid [8], Mamun Molla and Anwar Hossain [9], Chen [10], Cortell [11], Sajid and Hayat [12] and Bataller [13]. Chamkha [14] analyzed the steady hydromagnetic two-dimensional flow and heat transfer in a stationary electrically-conducting and heat-generating fluid driven by a continuously moving porous surface immersed in a fluid-saturated porous medium. It was shown the heat transfer characteristics can be enhanced by the porous medium. Ouaf [15] obtained an exact solution of thermal radiation on magnetohydrodynamics flow over a stretching porous sheet. Abbas and Hayat [16] investigated the magnetohydrodynamics boundary layer flow in a porous space. They found that the velocity and temperature increase for large value of radiaton parameter and the local porosity parameter.

Motivated by the above investigations and application, this study extends the work of Soid and Ishak [17] by considering the effect of radiation and porous media on the magnetohydrodynamics boundary layer flow and heat transfer over a stretching vertical sheet.

II. Mathematical Formulation

Consider a steady two-dimensional boundary layer flow over a vertical stretching surface in an incompressible, viscous and electrically conducting fluid in the presence of a transverse variable magnetic field B_0 and thermal radiation embedded in porous media. The stretching surface is assumed to have the velocity of the form $U(x)=ax$ where a is a constant. The ambient temperature T_∞ and the surface temperature T_w are assumed to be constant. Under these assumptions, the governing equations of partial differential equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma {B_0}^2}{\rho}u\pm g\beta(T-T_\infty)-\frac{\upsilon}{k}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_\rho}\frac{\partial q_r}{\partial y}, \tag{3}$$

along with the boundary conditions

$$u = U(x) = ax, \quad v = 0, \quad T = T_w, \qquad \text{at } y = 0,$$
$$u \to 0 \quad , \quad T \to T_\infty \qquad \text{as } y \to \infty, \tag{4}$$

where u and v are the velocity components along the x - and y -axes, respectively, g is the acceleration due to the gravity, α is the thermal diffusivity of the fluid, υ is the kinematic viscosity, σ is fluid electrical conductivity, β is the coefficient of thermal expansion, ρ is the fluid density, k is porous medium permeability, C_p is the specific heat at constant pressure, q_r is the radiative heat flux, T is the fluid temperature, B_0^2 is the magnetic induction and T_w is the surface temperature of the stretching surface.

Using the Rosseland approximation[17], the radiative heat flux

$$q_r = -\frac{4\sigma}{3K^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ is the Stefan-Boltzman constant and K^* is the absorption coefficient. The temperature differences within the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature, (T^4 may be expanded in a Taylor's series. Expanding T^4 about T_∞ and neglecting higher orders we get)

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

Substituting (5) and (6) into (3) yield

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha(1+N)\frac{\partial^2 T}{\partial y^2}, \tag{7}$$

where $\alpha = \frac{k_t}{\rho C_\rho}$ and $N = \frac{16\sigma T_\infty^3}{3k^* K^*}$ is the radiation parameter.

We introduce the similarity transformation as follows:

$$\eta = \left(\frac{a}{v}\right)^{\frac{1}{2}} y \;, \psi = (va)^{\frac{1}{2}} x f(\eta)\,, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{8}$$

where η is the similarity variable , ψ is the stream function θ and f are dimensionless temperature and stream function, respectively . The stream function ψ is defined by

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x}. \tag{9}$$

Substituting (8) into (1), (2) and (7) we obtain the transformed nonlinear ordinary differential equations as follows

$$f'''(\eta) + f(\eta)f''(\eta) - f'^2(\eta) - M^2 f'(\eta) + \lambda\theta(\eta) - Kf'(\eta) = 0 \tag{10}$$

$$\frac{1}{Pr}(1+N)\theta''(\eta) + f(\eta)\theta'(\eta) = 0 \tag{11}$$

where $P_r = \frac{\upsilon}{\alpha}$ is the Prandtl number and prime indicates differentiation with respect to η. $\lambda = \pm\frac{G_{rx}}{Re_x^{\,2}}$ is the buoyancy parameter, $Gr_x = \frac{g\beta(T_W - T_\infty)x^3}{\upsilon^2}$ is the Grashof number, $D = \frac{ka}{\upsilon}$ is the local Darcy number, M is the magnetic parameter and N is the radiation parameter [18]. To make the equation simpler, we take $\frac{1}{D}$ as K, represent permeability parameter.

The new transformed boundary conditions (4) become

$$f(0) = 0, \qquad f'(0) = 1, \qquad \theta(0) = 1,$$
$$f'(\eta) \to 0, \qquad \theta(\eta) \to 0 \qquad \text{as } \eta \to \infty. \tag{12}$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x defined by

$$C_f = \frac{\tau_w}{\rho U^2/2}, \qquad Nu_x = \frac{xq_w}{k_t(T_w - T_\infty)}, \tag{13}$$

respectively, where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=o}, \qquad q_w = -k_t\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{14}$$

with μ and k being the dynamic viscosity and the thermal conductivity, respectively.

Using non-dimensional variable in (8) we obtain

$$\frac{1}{2}C_f \operatorname{Re}_x^{1/2} = f''(0), \qquad Nu_x / \operatorname{Re}_x^{1/2} = -\theta'(0) \tag{15}$$

where $\operatorname{Re}_x = Ux/\upsilon$ is the local Reynolds number.

III. Result and Discussion

The transformed system (10) and (11) subjected to (12) are solved numerically using a finite-difference scheme known as the Keller-box method. To verify the validity and accuracy of the present analysis, the result for the values of heat transfer rate are compared with the exact solution given by Grubka and Bobba (1985) and the numerical of Soid and Ishak (2011) by using Runge Kutta Fehlberg with shooting technique shown in Table 1.It can be observed that the result by the Keller Box method are in excellent agreement with the result of [17] and [19].

Table I presents the values of the heat transfer rate at the surface for various values of *Pr* and *N*. It is seen that the heat transfer rate at the surface increases as the Prandtl number *Pr* increases. This is because increasing *Pr* is to increase the fluid viscosity but reduces the thermal conductivity, and in consequent increases the heat transfer rate at the surface. Opposite behaviors are observed for the effect of the radiation parameter *N*. As the radiation parameter increases, the fluid temperature inside the boundary layer increases, and this reduces the temperature gradient at the surface.

TABLE I
VALUES OF HEAT TRANSFER RATE FOR VARIOUS VALUE OF *Pr* AND *N*

N	Exact solution			Numerical solution					
	Grubka and Bobba [19] (1985)			Soid and Ishak [17] (2011)			Present		
	Pr=0.7	Pr=1	Pr=3	Pr=0.7	Pr=1	Pr=3	Pr=0.7	Pr=1	Pr=3
0	0.4539	0.5820	1.1652	0.4539	0.5820	1.1652	0.4539	0.5820	1.1652
0.5	0.3364	0.4383	0.9114	0.3364	0.4383	0.9114	0.3364	0.4383	0.9114
1	0.2688	0.3544	0.7603	0.2688	0.3544	0.7603	0.2688	0.3544	0.7603
5	0.1052	0.1444	0.3544	0.1052	0.1444	0.3544	0.1052	0.1444	0.3544
10	0.0599	0.0836	0.2195	0.0599	0.0836	0.2195	0.0599	0.0836	0.2195

TABLE II
VALUES OF $f''(0)$ AND $-\theta'(0)$ FOR VARIOUS VALUES OF PRANDTL NUMBER P

Pr	$f''(0)$	$-\theta'(0)$
0.7	-1.2280	0.2919
1	-1.2501	0.3604
3	-1.3402	0.7023
7	-1.4251	1.1780
10	-1.4607	1.4581

Table II shows the numerical skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ for various values of Prandtl number. It is seen that the values of $f''(0)$ are always negative. Physically, negative sign of $f''(0)$ implies that the stretching sheet exerts a drag force on the fluid that cause the movement of the fluid on the surface. The absolute values of skin friction coefficient and the local Nuselt number increase with increasing Prandtl number .

Fig. 1 presents the temperature and velocity profiles, respectively, for various values of radiation parameter when all the other parameters values are fixed, .

It can be seen that the momentum and thermal boundary layer thickness increase as N increases. These induce the decrease in the absolute value of the velocity and temperature gradient at the surface. Thus, the heat transfer rate at the surface decreases with increasing of N .

Fluid flow and heat transfer towards porous stretching sheet have an important bearing on several technological processes. Fig. 2 shows the effects of permeability parameter on the velocity and temperature profiles. For temperature profiles, it is obvious that the presence of porous medium reduced the temperature distribution. The thermal boundary layer thickness increases as permeability parameter K increases.

The rate of heat transfer at the surface decreases with an increase of K. Conversely, the absolute value of the velocity gradient at the surface increases with an increase in K. Thus, the fluid velocity increases.

Figs. 3 depicts the temperature and velocity profiles for different values of magnetic parameter M. It is seen that the thermal boundary layer thickness increases as M increases. The heat transfer rate at the surface decreases with an increase of M. The transverse magnetic field opposes the transport phenomena [4]. This is related to the fact that the variation of magnetic parameter leads to the variation of the Lorentz force due to magnetic field and the Lorentz force produces more resistance to the transport phenomena.

IV. Conclusions

The steady laminar two-dimensional magnetohydrodynamics (MHD) boundary layer flow over stretching surface with presence of radiation and porous media have been investigated. The system of coupled boundary layer equations was solved using a finite difference method known as Keller-box method. The influences of physical parameters such as magnetic parameter, radiation parameter, buoyancy, permeability parameter and Prandtl number were examined. It can be concluded that both the skin friction coefficient and the local Nusselt number increses as Prandtl number increases. The temperature profiles for various values of magnetic parameter, radiation parameter and permeability parameter showed the same behaviour where the heat transfer rate at th e surface decreases as the parameters increases. In the presence of magnetic field and porous media, the fluid velocity increases as magnetic parameter and permeability parameter increases.

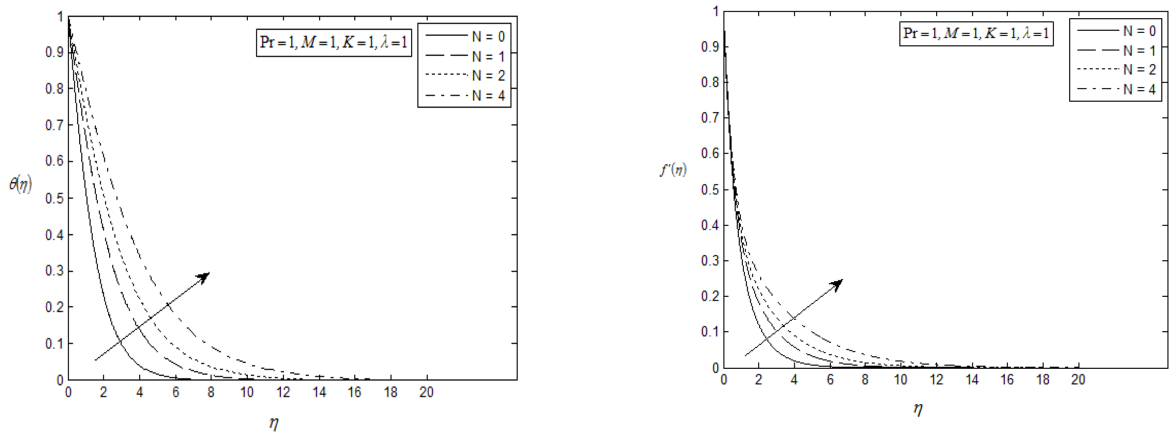

Fig. 1 Temperature and velocity profiles for various values of radiation parameter N

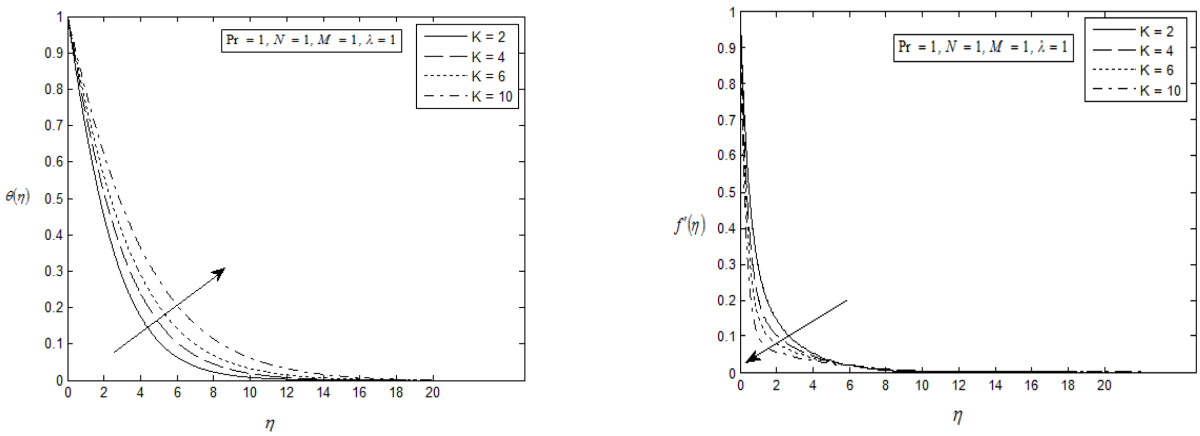

Fig. 2 Temperature and velocity profiles for various values of permeability parameter K

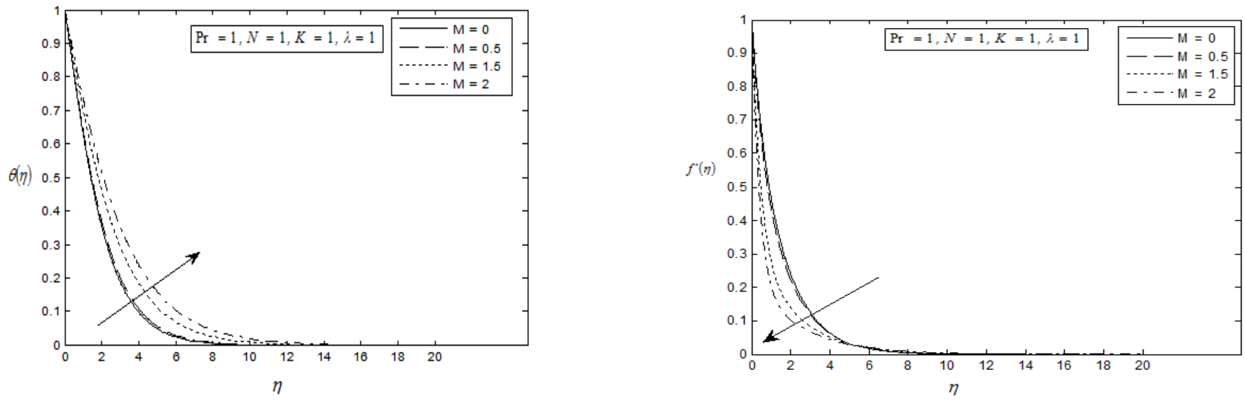

Fig. 3 Temperature and velocity profiles for various values of Magnetic parameter M

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REFERENCES

[1] Ghaly, A.Y. (2002). Radiation effects on a certain MHD free-convection flow. Chaos, Solitons and Fractals, 13, pp. 1843-1850.
[2] Raptis, A. (2004). Effect of thermal radiation on MHD flow. Applied Mathematics and Computation, 153, pp. 645-649.
[3] Liu, I.C. (2005). A note on heat and mass tranfer for a hydromagnetic flow over a stretching sheet. *International Communications in Heat and Mass Transfer, 32*, pp. 1075-1084.
[4] Chen, C.H. (1998). Laminar mixed convection adjacent to vertical, continuously stretching sheets. Heat and Mass Transfer, 33, pp. 471-476.
[5] Ishak, A., Nazar, R., & Pop, I. (2008). Hydromagnetic flow and heat transfer adjacent to a stretching vertical sheet. Heat Mass Transfer, 44, pp. 921-927.
[6] Mukhopadhyay, S., & Layek, G.C. (2008). Effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface. International Journal of Heat and Mass Transfer, 51, pp. 2167-2178.
[7] Abdul Hakeem, a.k., & Sathiyanathan, K. (2009). An analytic solution of an oscillatory flow through a porous medium with radiation effect. Nonlinear Analysis:Hybrid Systems, 3, pp. 288-295.
[8] Seddeek, M.A., & Abdelmeguid, M.S. (2006). Effects of radiation and thermal diffusivity on heat transfer over a stretching surface with variable heat flux. Physics Letter A, 348, pp. 172-179.
[9] Mamun Molla, M., & Anwar Hossain, M. (2007). Radiation effect on mixed convection laminar flow along a vertical wavy surface. International Journal of Thermal Sciences, 46, pp. 926-935.
[10] Chen, C.H. (2010). On the analytic solution of MHD flow and heat transfer for two types of viscoelastic fluid over a stretching sheet wuth energy dissipation, internal heat source and thermal radiation. International Journal of Heat and Mass Transfer, 53, pp. 4264-4273.
[11] Cortell, R. (2008). Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. Physics Letters A, 372, pp. 631-636.
[12] Sajid, M., & Hayat, T. (2008). Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. International Communications in Heat and Mass Transfer, 35, pp. 347-356.
[13] Bataller, R.C. (2008). Radiation effects for the Blasius and Sakiadis flows with a convective surface boundary condition. Applied Mathematic and Computation, 206, pp. 832-840.
[14] Chamkha, A.J. (1997). Hydromagnetic flow and heat transfer of a heat-generating fluid over a surface embedded in a porous medium. International Communications Heat Mass Transfer, 24(6), pp. 815-825.
[15] Ouaf, M.E.M. (2005). Exact solution of thermal radiation on MHD flow over a stretching porous sheet. Applied Mathematics and Computations, 170, pp. 1117-1125.
[16] Abbas, Z., & Hayat, T. (2008). Radiation effects on MHD flow in a porous space. International Journal of Heat and Mass Transfer, 51, pp. 1024-1033.
[17] Soid, S.K & Ishak, A. (2011). Thermal boundary layer flow along stretching sheet, IEEE, ICBEIA, pg. 120-122.
[18] Mukhopadhyay, S. (2009). Effect of thermal radiation on unsteady mixed convection flow and heat trabsfer over a porous stretching surface in porous medium. International Journal of Heat and Mass Transfer, 52, pp. 3261-3265.
[19] Grubka, L.J., & Bobba, K.M. (1985). Heat transfer characteristics of a continuous, stretching surface with variable temperature. ASME J. Heat Transfer, 107, pp. 248-250.