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Collective oscillations in a magnetized plasma subjected to a radiation field

Daniel Santos, Bruno Ribeiro, Marco Amato, Antonio Fonseca

Abstract—In this paper we discuss the behaviour of the longitudinal modes of a magnetized non collisional plasma subjected to an external electromagnetic field. We apply a semiclassical formalism, with the electrons being studied in a quantum mechanical viewpoint whereas the electromagnetic field in the classical context. We calculate the dielectric function in order to obtains the modes and found that, unlike the Bernstein modes, the presence of radiation induces oscillations around the cyclotron harmonics, which are smoothed as the energy stored in the radiation field becomes small compared to the thermal energy of the electrons. We analyze the influence of the number of photon involved in the electronic transitions between the Landau levels and how the parameters such as the external fields strength, plasma density and temperature affect the dispersion relation.

Keywords—Collective oscillations, External fields, Dispersion relation.

I. Introduction

THE quantum mechanical approach has been for a long time an important tool to describe classical plasmas in the presence of external fields. A broad and detailed study of this subject is provided in [1]. Although it may seem unusual to employ quantum mechanics and then discard the quantum corrections by taking the classical limit, this procedure has its advantages when more difficult problems are considered such as those in astrophysics. Nowadays phenomena involving laser-plasma interaction attract much attention, some of it due to the possibility of application in thermonuclear fusion. Another interesting aspect concerns the influence of external electromagnetic fields on plasma wave instabilities [2].

In this paper, we study the electrostatic oscillations in a magnetized plasma in the presence of an external radiation field. It is well known that, when the magnetic field is the only external field, the electrostatic oscillations supported by the plasma are the Bernstein modes [3], [4], whose frequencies lie between cyclotron harmonics. We shall investigate how the presence of the electromagnetic field affects these modes. For this, we assume an infinite and homogeneous plasma immersed in an axial magnetostatic field. The external electromagnetic field is considered to be a spatially independent classical plane waves (dipole approximation) polarized along the x-direction. The plasma electrons are described by the solution of the Schrödinger equation for an electron subjected to both an electromagnetic and a magnetostatic fields. To describe the system, we adopt the same theoretical framework used in [5] for a free plasma.

D. Santos and B. Ribeiro are Ph.D student at the Institute of Physics of University of Brasília, Brasília, DF, 70910-900 Brazil (e-mail: {das,brunovr}@fis.unb.br).

M. Amato and A. Fonseca are with International Centre for Condensed Matter Physics, Universidade de Brasília, Brasília, DF, 70910-900 Brazil (e-mail: {marco,alaf}@fis.unb.br).

II. Theoretical Formalism

We start from the problem of an electron subjected simultaneouly to a magnetostatic and radiation fields which is described by the Schrödinger equation

$$H_0\Psi^{(0)}(\mathbf{r},t) = i\hbar\frac{\partial}{\partial t}\Psi^{(0)}(\mathbf{r},t). \quad (1)$$

The hamiltonian H_0 is given by

$$H_0 = \frac{1}{2m_e}(\mathbf{p} - e\mathbf{A}(y,t))^2, \quad (2)$$

$\mathbf{p}$ being the electron momentum and $\mathbf{A}(y,t)$ the potential vector which is taken to be

$$\mathbf{A}(y,t) = -\frac{E}{\omega}\sin(\omega t)\ \hat{\mathbf{x}} - B_0 y\hat{\mathbf{x}}, \quad (3)$$

where ω and E are, respectively, the frequency and the amplitude of the electrical component of the electromagnetic field and B_0 represents the magnetostatic field strength.

In the absence of radiation ($E \to 0$), the wave functions in Eq.(1) reduce to Landau wave functions [6]

$$\Phi_{n,k_x,k_z}(\mathbf{r},t) = \frac{1}{\sqrt{L_xL_z}}\exp(ik_xx)\exp(ik_zz) \cdot \chi_n(y)\exp\left(-\frac{i}{\hbar}E_{n,k_z}t\right), \quad (4)$$

where L_x and L_z are the length intervals at which the particle is confined, $\mathbf{k} = \mathbf{p}/\hbar$ is its wave vector and

$$\chi_n(y) = \frac{1}{\pi^{\frac{1}{4}}a_c{}^{\frac{1}{2}}\sqrt{2^n\ n!}}\exp\left[-\frac{(y-y_0)^2}{2\ a_c{}^2}\right]H_n\left(\frac{y-y_0}{a_c}\right). \quad (5)$$

The $H_n(y)$ are Hermite polynomials and $E_{n,k_z} = \hbar\omega_c(n + 1/2) + \hbar^2{k_z}^2/2m_e$ are the eigenenergies for an electron in an uniform and static magnetic field. The constants a_c and ω_c denote, respectively, the Larmor radius and the electron cyclotron frequency.

To solve Eq. (1), which has a time-dependent hamiltonian, we make use of an unitary transformation [7] in the Landau wave functions

$$\Psi^{(0)}_{n,k_x,k_z}(\mathbf{r},t) = U\Phi_{n,k_x,k_z}(\mathbf{r},t), \quad (6)$$

where

$$U = \exp\left(\frac{i}{\hbar}\boldsymbol{\alpha}(t)\cdot\mathbf{r}\right)\exp\left(\frac{i}{\hbar}\boldsymbol{\beta}(t)\cdot\mathbf{p}\right)\exp\left(\frac{i}{\hbar}\eta(t)\right). \quad (7)$$

The functions $\boldsymbol{\alpha}(t)$ and $\boldsymbol{\beta}(t)$ generate a spatial and momentum translation, respectively, while $\eta(t)$ is a phase factor.

Substituting equation (6) into (1) and then multiplying both members of the resulting equation by $U^\dagger$, we get a system of differencial equations for the functions that define the unitary operator (7). Solving this system, we obtain

$$U = \exp\left(\frac{i}{\hbar}F(\omega,t)\right)\exp\left(-\frac{i}{\hbar}m_e\omega_c\gamma_1\left(\cos(\omega t)-1\right)y\right) \cdot \exp\left(-i\gamma_1\left(\cos(\omega t)-1\right)k_x\right)\exp\left(i\frac{\omega_c}{\omega}\gamma_1\sin(\omega t)k_y\right), \quad (8)$$

where $\gamma_1 = eE/m_e({\omega_c}^2-\omega^2)$, $\gamma_2 = e^2E^2/8m_e\omega({\omega_c}^2-\omega^2)$ and $F(\omega,t) = 2\gamma_2\omega t - \gamma_2\sin(2\omega t)$.

The wave function given by (6) with the unitary operator derived in (8), just describes an electron subjected to both magnetostatic and radiation fields. However it does not take into account the presence of the self-consistent electrostatic field of the plasma. In order to take this into account on the electron dynamics, we introduce an additional term in the hamiltonian (2) and the equation of motion takes the form

$$i\hbar\frac{\partial}{\partial t}\Psi_{\mathbf{m}}(\mathbf{r},t) = (H_0 - e\varphi(\mathbf{r},t))\Psi_{\mathbf{m}}(\mathbf{r},t), \quad (9)$$

where the index $\mathbf{m}$ on the wave function represents the electron states $|n^{(m)}, k_x^{(m)}, k_z^{(m)}\rangle$ and the electrostatic potential can be written as a Fourier series expansion

$$\varphi(\mathbf{r},t) = \sum_{\mathbf{q}}\sum_{\Omega} \exp(i\mathbf{q}\cdot\mathbf{r})\exp(-i\Omega t)\varphi(\mathbf{q},\Omega). \quad (10)$$

The wave functions given in the left hand side of equation (6) form an orthonormal basis on which we can expand the new wave function $\Psi_{\mathbf{m}}(\mathbf{r},t)$ into $\left\{\Psi_{\mathbf{i}}^{(0)}(\mathbf{r},t)\right\}$

$$\Psi_{\mathbf{m}}(\mathbf{r},t) = \sum_{\mathbf{i}} a_{\mathbf{i},\mathbf{m}}(t)\Psi_{\mathbf{i}}^{(0)}(\mathbf{r},t). \quad (11)$$

Assuming a weak potential $(|e\varphi(\mathbf{r},t)| \ll H_0)$, so that we can expand the coeficients $a_{\mathbf{i},\mathbf{m}}(t)$ in a convergent series

$$a_{\mathbf{l},\mathbf{m}}(t) = a_{\mathbf{l},\mathbf{m}}^{(0)}(t) + a_{\mathbf{l},\mathbf{m}}^{(1)}(t) + a_{\mathbf{l},\mathbf{m}}^{(2)}(t) + \cdots, \quad (12)$$

where $a_{\mathbf{l},\mathbf{m}}^{(j)}(t) = O(|e\varphi(\mathbf{r},t)|^j)$, we can, by using perturbation theory, determine the coefficients of zero and first order

$$a_{\mathbf{l},\mathbf{m}}^{(0)}(t) = \delta_{\mathbf{m},\mathbf{l}}, \quad (13)$$

$$a_{\mathbf{l},\mathbf{m}}^{(1)}(t) = \frac{ie}{\hbar}\int_{-\infty}^{t} d\tau\ \varphi_{\mathbf{l},\mathbf{m}}(\mathbf{r},\tau), \quad (14)$$

where $\varphi_{\mathbf{l},\mathbf{m}}(\mathbf{r},\tau) = \langle\Psi_{\mathbf{l}}^{(0)}(\mathbf{r},\tau)|\varphi(\mathbf{r},\tau)|\Psi_{\mathbf{m}}^{(0)}(\mathbf{r},\tau)\rangle$.

Once obtained the expression for the coefficients of expansion (11), the wave function with first order correction is given by

$$\Psi_{\mathbf{m}}(\mathbf{r},t) = \Psi_{\mathbf{m}}^{(0)}(\mathbf{r},t) + \frac{ie}{\hbar}\sum_{\mathbf{l}}\int_{-\infty}^{t} d\tau\ \varphi_{\mathbf{l},\mathbf{m}}(\mathbf{r},\tau)\Psi_{\mathbf{l}}^{(0)}(\mathbf{r},t). \quad (15)$$

The second term in the right hand side of Eq.(15) represents the first correction in the wave function owing to the presence of the perturbative potential $\varphi(\mathbf{r},t)$. Using the Jacobi-Anger expansion

$$exp(ix\cos\theta) = \sum_{m=-\infty}^{+\infty} i^m J_m(x)e^{im\theta}, \quad (16)$$

$$exp(ix\sin\theta) = \sum_{m=-\infty}^{+\infty} J_m(x)e^{im\theta}, \quad (17)$$

we can carry out the integral and then rewrite the first correction as

$$\Psi_{\mathbf{m}}^{(1)}(\mathbf{r},t) = e\sum_{n^{(l)}=0}^{\infty}\sum_{\mathbf{q},\Omega}\sum_{m_1,m_2}\varphi(\mathbf{q},\Omega)\ \Theta_{m_1,m_2}(q_x,q_y) \exp(i\gamma_1 q_x)\frac{\exp\left(\frac{i}{\hbar}(\Delta E + (m_1+m_2)\hbar\omega - \hbar\Omega)t\right)}{\Delta E + (m_1+m_2)\hbar\omega - \hbar\Omega - i0^+} \langle n^{(l)}|\exp(iq_y y)|n^{(m)}\rangle\ \psi^{(0)}_{n^{(l)},k_x^{(m)}+q_x,k_z^{(m)}+q_z}(\mathbf{r},t), \quad (18)$$

where

$$\Delta E = E_{n^{(l)},k_z^{(l)}} - E_{n^{(m)},k_z^{(m)}}, \quad (19)$$

$$\langle n^{(l)}|\exp(iq_y y)|n^{(m)}\rangle = \int dy\ \chi_{n^{(l)}}(y)\exp(iq_y y)\chi_{n^{(m)}}(y), \quad (20)$$

and, for a matter of notation, we define the function

$$\Theta_{m_1,m_2}(q_x,q_y) = (-1)^{m_1+m_2}i^{m_1}J_{m_1}(\gamma_1 q_x)\cdot J_{m_2}\left(\gamma_1\frac{\omega_c}{\omega}q_y\right). \quad (21)$$

The knowledge of the wave function allows us to calculate the fluctuation on the charge density, for the $\mathbf{m}$-state electrons, due to the presence of the perturbative electrostatic potential

$$\delta\rho_{n^{(m)},k_z^{(m)}}(\mathbf{r},t) = -e\Psi_{\mathbf{m}}^*(\mathbf{r},t)\Psi_{\mathbf{m}}(\mathbf{r},t) - \rho_{n^{(m)}}^{(0)}(\mathbf{r}), \quad (22)$$

where $\rho_{n^{(m)}}^{(0)}$ is the unperturbed charge density

$$\rho_{n^{(m)}}^{(0)}(\mathbf{r}) = -e\Psi_{\mathbf{m}}^{(0)^*}(\mathbf{r},t)\Psi_{\mathbf{m}}^{(0)}(\mathbf{r},t). \quad (23)$$

Neglecting the second order term in $\varphi(\mathbf{q},\Omega)$, and noting that the sums in $\mathbf{q}$, Ω, m_1 and m_2 occur in a symmetrical interval, using the properties

$$J_{-m}(x) = (-1)^m J_m(x), \quad J_m(-x) = (-1)^m J_m(x), \quad (24)$$

$$\varphi^*(-\mathbf{q},-\Omega) = \varphi(\mathbf{q},\Omega), \quad (25)$$

and considering again the identities (16) and (17), we found the charge density fluctuation as

$$\delta\rho_{n^{(m)},k_z^{(m)}}(\mathbf{r},t) = -\frac{e^2}{L_xL_z}\chi_{n^{(m)}}(y)\sum_{l=-\infty}^{+\infty}\sum_{\mathbf{q},\Omega}\sum_{\substack{m_1,m_2\\m_3,m_4}} \cdot\varphi(\mathbf{q},\Omega)\exp(-\Omega t)\exp(iq_x)\exp(iq_z z)\chi_{n^{(m)}+l}(y) \cdot\Xi^{m_1,m_2}_{m_3,m_4}(q_x,q_y)\exp(i(m_1+m_2+m_3+m_4)\omega t) \cdot\langle n^{(m)}+l|\exp(iq_y y)|n^{(m)}\rangle\left\{① + ②\right\}, \quad (26)$$

where we define the integer $l = n^{(l)} - n^{(m)}$ related to the energy gap between two Landau levels and the function

$$\Xi^{m_1,m_2}_{m_3,m_4}(q_x,q_y) = (-1)^{m_1+m_2}\ i^{m_1+m_3}J_{m_1}(\gamma_1 q_x) \cdot J_{m_2}\left(\gamma_1\frac{\omega_c}{\omega}q_y\right)J_{m_3}(\gamma_1 q_x)J_{m_4}\left(\gamma_1\frac{\omega_c}{\omega}q_y\right). \quad (27)$$

The terms within braces in Eq.(26) are given by

$$① = \left(\hbar\omega_c l + \frac{\hbar^2}{2m_e}(k_z^{(m)}+q_z)^2 - \frac{\hbar^2}{2m_e}k_z^{(m)^2} + (m_1+m_2)\hbar\omega - \hbar\Omega\right)^{-1}, \quad (28)$$

$$② = \left(\hbar\omega_c l + \frac{\hbar^2}{2m_e}(k_z^{(m)}+q_z)^2 - \frac{\hbar^2}{2m_e}k_z^{(m)^2} - (m_1+m_2)\hbar\omega + \hbar\Omega\right)^{-1}. \quad (29)$$

For a maxwellian electron distribution function f_{n,k_z}, the total density fluctuation can be written as

$$\delta\rho(\mathbf{r},t) = \sum_{n,k_z} f_{n,k_z}\ \delta\rho_{n,k_z}(\mathbf{r},t). \quad (30)$$

This fluctuation induces a potential in the medium which can be calculated via Poisson equation

$$\sum_{\mathbf{q},\Omega} q^2\exp(-i\Omega t)exp(i\mathbf{q}\cdot\mathbf{r})\varphi_{ind}(\mathbf{q},\Omega) = \frac{\delta\rho(\mathbf{r},t)}{\epsilon_0}, \quad (31)$$

where we express the induced potential as a Fourier series along the same lines as those discussed to obtain Eq.(10).

From equations (30) and (31) we find an expression for the induced potential. Taking the time average of the induced potential over the period of the radiation field, we notice from the factor $\exp(i(m_1+m_2+m_3+m_4)t)$, that only the terms in the sums that satisfy $m_1+m_2+m_3+m_4 = 0$ contribute. Then, we obtain

$$\varphi_{ind}(\mathbf{q},\Omega) = -\frac{e^2}{\epsilon_0 L_xL_z}\frac{\varphi(\mathbf{q},\Omega)}{q^2}\sum_l\sum_{n,k_z}\chi_n(y) \cdot\exp(-iq_y y)\chi_{n+l}(y)\langle n+l|\exp(iq_y y)|n\rangle \cdot\sum_{\substack{m_1+m_2\\+m_3+m_4=0}}\Xi^{m_1,m_2}_{m_3,m_4}(q_x,q_y)\ f_{n,k_z}\left\{① + ②\right\}. \quad (32)$$

The full potential $\varphi(\mathbf{q},\Omega)$ can be written as a sum of two terms [8]

$$\varphi(\mathbf{q},\Omega) = \varphi_{ext}(\mathbf{q},\Omega) + \varphi_{ind}(\mathbf{q},\Omega) = \frac{\varphi_{ext}(\mathbf{q},\Omega)}{\varepsilon(\mathbf{q},\Omega)}, \quad (33)$$

where $\varepsilon(\mathbf{q},\Omega)$ is the dielectric function which can be written as

$$\varepsilon(\mathbf{q},\Omega) = 1 - \frac{\varphi_{ind}(\mathbf{q},\Omega)}{\varphi(\mathbf{q},\Omega)} \quad (34)$$

Proceeding further, the spatial average of the dielectric function $\epsilon(\mathbf{q},\Omega) = \langle\varepsilon(\mathbf{q},\Omega)\rangle$ is found to be

$$\epsilon(\mathbf{q},\Omega) = 1 + \frac{e^2}{\epsilon_0 Vq^2}\sum_l\sum_{n,k_z}|\langle n+l|\exp(iq_y y)|n\rangle|^2 \sum_{\substack{m_1,m_2\\m_3,m_4}}\Xi^{m_1,m_2}_{m_3,m_4}(q_x,q_y)\ f_{n,k_z}\left\{① + ②\right\}. \quad (35)$$

The fact that the sums over n and k_z are infinite along with the conditions $l \ll n$ and $q_z \ll k_z$ allow us to make the transformations $n \to n-l$ and $k_z + q_z \to k_z$ into term ① in Eq.(35) and then rewrite the dielectric function as

$$\epsilon(\mathbf{q},\Omega) = 1 + \frac{e^2}{\epsilon_0 Vq^2}\sum_{\substack{m_1,m_2\\m_3,m_4}}\Xi^{m_1,m_2}_{m_3,m_4}(q_x,q_y)\sum_l\sum_{n,k_z} \frac{f_{n-l,k_z-q_z} - f_{n,k_z}}{\frac{\hbar^2}{2m_e}k_z^2 - \frac{\hbar^2}{2m_e}(k_z-q_z)^2 + \hbar\omega_c l + (m_1+m_2)\hbar\omega - \hbar\Omega} \cdot|\langle n+l|\exp(iq_y y)|n\rangle|^2. \quad (36)$$

In the classical limit the summations over n, k_z and the Landau energies become

$$\sum_{n,k_z}(...)\ \underset{V\to\infty}{\longrightarrow}\ V\int(...)d^3v, \quad (37)$$

$$\hbar\omega_c\left(n+\frac{1}{2}\right) \to \frac{m}{2}v_\perp^2, \quad (38)$$

and, according to [9]

$$|\langle n+l|exp(iq_y y)|n\rangle|^2 \to J_l^2\left(\frac{q_\perp v_\perp}{\omega_c}\right). \quad (39)$$

Under these assumptions, the dielectric function may be written as

$$\epsilon(\mathbf{q},\Omega) = 1 + \frac{e^2}{m_e\epsilon_0 q^2}\sum_{\substack{m_1,m_2\\m_3,m_4}}\Xi^{m_1,m_2}_{m_3,m_4}(q_x,q_y) \sum_l\int d^3v\frac{J_l^2\left(\frac{q_\perp v_\perp}{\omega_c}\right)}{\Omega - \omega_c l - (m_1+m_2)\omega - q_z v_z} \left[\frac{l\omega_c}{v_\perp}\frac{\partial F}{\partial v_\perp} + q_z\frac{\partial F}{\partial v_z}\right], \quad (40)$$

where we expand f_{n-l,k_z-q_z} to first order around (n, k_z).

In the absence of external fields, the argument of the Bessel functions vanish, and it may be expressed as $J_m(0) = \delta_{m,0}$.

Thus, in this limit we can immediately check that our results are in agreement with those derived in [1] for the dielectric function of a magnetized plasma.

To investigate the collective oscillations perpendicular to the magnetostatic field we take the axial component of the plasmon wave number equal to zero, which yields

$$\epsilon(\mathbf{q},\Omega) = 1 + \frac{e^2}{m_e\epsilon_0 q^2}\sum_{\substack{m_1,m_2\\m_3,m_4}} \Xi^{m_1,m_2}_{m_3,m_4}(q_x,q_y) \sum_l \int d^3v \frac{J_l^2\left(\frac{q_\perp v_\perp}{\omega_c}\right)}{\Omega-\omega_c l-(m_1+m_2)\omega}\frac{l\omega_c}{v_\perp}\frac{\partial f}{\partial v_\perp}. \quad (41)$$

The integral in Eq.(41) can be performed in cylindrical coordinates by using Weber's second exponential integral formula [10]. The result, in terms of the more convenient adimensional variables $\xi = (k_B T_e/\omega_c^2 m_e)^{1/2}\ \mathbf{q}$ and $\bar{\Omega} = \Omega/\omega_c$, is given by

$$\epsilon(\xi,\bar{\Omega}) = 1 - \frac{\omega_p^2}{\omega_c^2}\frac{\exp(-\xi_\perp^2)}{\xi_\perp^2}\sum_{\substack{m_1,m_2\\m_3,m_4}} \Xi^{m_1,m_2}_{m_3,m_4}(q_x,q_y) \sum_{l=-\infty}^{+\infty}\frac{l}{\bar{\Omega}-l-(m_1+m_2)\bar{\omega}} I_l(\xi_\perp^2), \quad (42)$$

where $I_l(x)$ is the modified Bessel function, $\omega_\mathrm{p} = (n_0 e^2/\epsilon_0 m_\mathrm{e})^{1/2}$ is the plasma frequency and we define the constants $\bar{\omega}=\omega/\omega_c$, $\gamma_3 = \sqrt{2}(v_\mathrm{d}/v_\mathrm{th})(1-\bar{\omega}^2)^{-1}$, $\gamma_4=\gamma_3/\bar{\omega}$, $v_\mathrm{th} = (2k_\mathrm{B}T_\mathrm{e}/m_\mathrm{e})^{1/2}$ being the thermal electron velocity and $v_\mathrm{d} = E/B_0$ the maximum value for the drift velocity.

Finally, to obtain numerical results, we take the longitudinal oscillations along the x-axis, namely, $\xi_y = 0$, $\xi_\perp = \xi_x = \xi$. Under this assumption we obtain

$$\epsilon(\xi,\bar{\Omega}) = 1 - \frac{\omega_p^2}{\omega_c^2}\frac{exp(-\xi^2)}{\xi^2}\sum_{m=-\infty}^{+\infty} J_m^2(\gamma_3\ \xi) \sum_{l=-\infty}^{+\infty}\frac{l}{\bar{\Omega}-l-m\bar{\omega}} I_l(\xi^2). \quad (43)$$

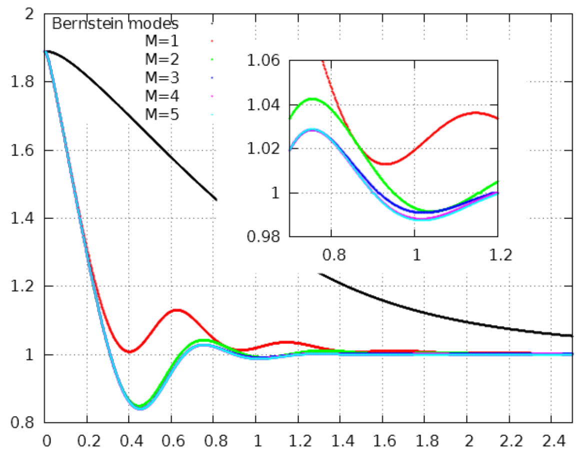
III. Dispersion Relation

The zeros of the dielectric function provide the dispersion relation for the longitudinal modes

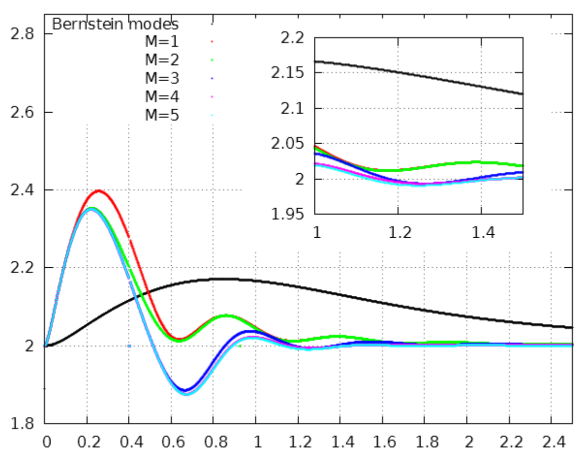
$$1 + 2\frac{\omega_p^2}{\omega_c^2}\frac{\exp(-\xi^2)}{\xi^2}\sum_{m=-\infty}^{\infty} J_m^2(\gamma_3\ \xi) \sum_{l=1}^{+\infty}\frac{l}{l^2-(\bar{\Omega}-m\bar{\omega})^2} I_l(\xi^2) = 0, \quad (44)$$

where we used the property $I_{-n}(x) = I_n(x)$ of the modified Bessel funtions.

The dispersion relation in (44) relates the variables $\bar{\Omega}$ and ξ by means of an implicit equation. To solve it, we create a routine in C language where we define the dielectric function, fix in it $\xi = \xi^*$ and then, through the Newton-Raphson method, determine its corresponding $\bar{\Omega}^*$, such that $\epsilon(\xi^*,\Omega^*) = 0$. In other words, we solve numerically the equation $\epsilon(\xi^*,\bar{\Omega}) = 0$ in the $\bar{\Omega}$ variable an then, sweeping values for ξ^*, we were able to plot the dispersion relation graphs. However, the function $\epsilon(\xi^*,\bar{\Omega})$ admits distinct roots Ω^* for different ranges of frequency. Thus, to obtain the graphs, we had to pass as parameter of the root finding routine the interval at which we are search for solution. For the plots we restrict ourselves to the first two ranges of frequencies. The functions J_m and I_l were implemented in such a way to prevent numerical divergence by means of a downward recurrence formula [11]. Their respective summations were truncated such that the higher orders, with negligible contributions, were not taken into account. For all plots we assume $L = 30$ as limit of summation in l. We used, for density and temperature, typical values of a gas discharge plasma, whilst the external radiation field was assumed in microwave range, in order to respect the dipole approximation assumed in our calculations.



(a)



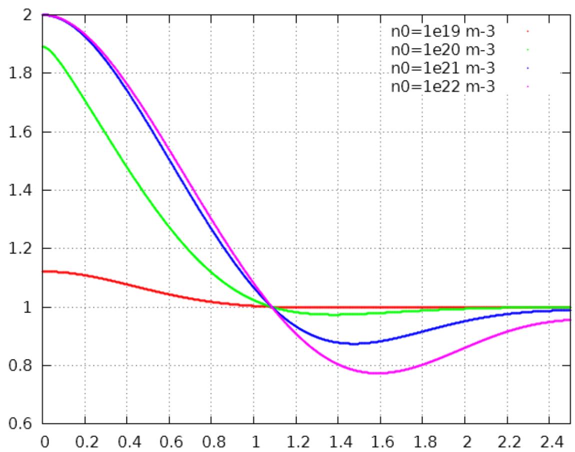
(b)

Fig. 1. Collective mode frequency as a function of the wave number for the first two gaps of frequency. The black line represents the Bernstein mode and the others refer to the distinct number of photons M involved in the process. For the plotting, we used a plasma with density $n_0 = 10^{19}\ \mathrm{m}^{-3}$ and temperature $k_\mathrm{B}T = 10^{-19}$ J. We assumed as values for the fields $E = 10\ \mathrm{V/m}$ and $B_0 = 2$ T.

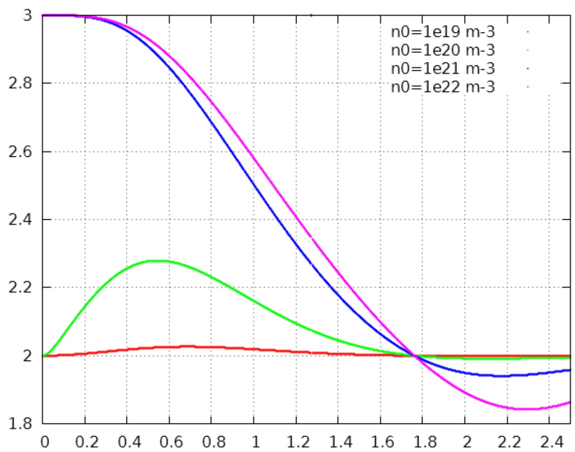
We notice, immediately, that when there is no radiation field ($\gamma_3 = 0$), the result shown in (44) reduces to the dispersion relation for the Bernstein modes as we should expect. The influence radiation is closely related to the factor γ_3, given by

$$\gamma_3 = \sqrt{2}\frac{v_{\rm d}}{v_{\rm th}}\frac{1}{1-\bar{\omega}^2}. \tag{45}$$

As we see in the dispersion relation graphs, Figures 3 and 4, as this factor increases the influence of radiation is more noticeable. From definition (45), we can check that as the radiation and cyclotron frequencies approach the same value ($\bar{\omega} \to 1$) they contributes for a larger value of γ_3. An increase in this parameter also occurs when the drift velocity prevails upon thermal velocity. On the other hand, for small values of γ_3 the curves tend to the Bernstein modes. We work here only the case $\omega \approx \omega_c$ by taking $|\omega - \omega_c| = 10^{-6}$ in our numerical code.



(a)



(b)

Fig. 2. We observe that the oscillations around the cyclotron harmonics are more pronounced in a denser plasma. A point to note is that the first cyclotron mode is independet of the plasma density, which is illustrated by the intercept point of the curves. For the plot, we fix the temperature $k_{\rm B}T = 1,6\times10^{-18}$ J and $M = 5$. For the fields, we assume the same values used previously. We analyse the following values of density: 10^{19} m^{-3}, 10^{20} m^{-3}, 10^{21} m^{-3} and 10^{22} m^{-3}.

To analyze the influence of the number of photons in the electronic transitions, we truncated the summation over m in Eq.(44) at a maximum number M of photons, namely, we change the summation limits to $m = -M$ and $m = M$. Thereby we obtain dispersion relations of the kind $\varepsilon_M(\xi, \bar{\Omega}) = 0$ which provided us the dispersion relation graphs $\bar{\Omega}$ vs. ξ in the Figures 1(a) and 1(b).

From the graphs shown in Figures 1(a) and 1(b), we observe that, as the wave number increases, the presence of the radiation accelerates the mitigation of the modes toward the stationary cyclotron harmonic modes. Futhermore, the curves exhibit an oscillatory character not seen in the Bernstein modes. Another interestig aspect is that, for the first range of frequencies, there is no mode with frequency less than ω_c when a single photon involved in the electronic transitions is considered. The second range does not allow modes with frequency less than $2\cdot\omega_c$ when we consider up to two photons ($M \leq 2$) in the process. We may expect this behavior to repeat for more energetic harmonics, namely, that in the n-th branch there is no mode such that $\bar{\Omega} < n$ (or equivalently $\Omega < n\cdot\omega_c$) for $M \leq n$.

We also note by the inset in Figures 1(a) and 1(b), that, with the data used in the plotting, the curves for $M = 4$ and $M = 5$ practically do not differ and hence truncating the summation at $M = 5$ represents a good approximation.

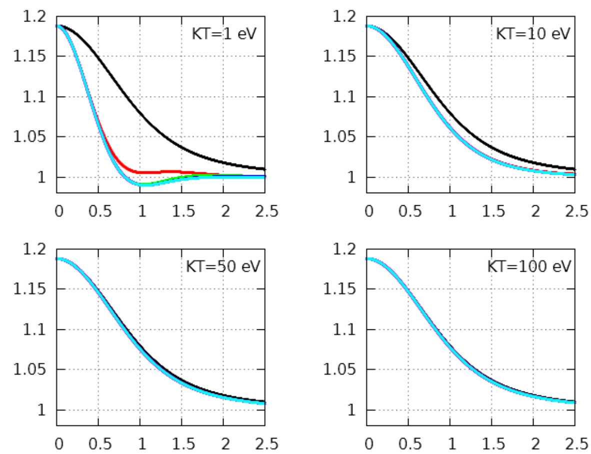


Fig. 3. Collective modes for different number of photons M and for different temperatures. We notice an approach of the curves for distinct values of M and that they tend to the Bernstein modes as the temperature increases. The red and light blue lines represent $M = 1$ and $M = 5$ respectively. The parameters for the plasma and for the fields were assumed as $n_0 = 10^{18}$ m^3, $E = 1,0$ Vm^{-1} and $B_0 = 0,5$ T.

It is clear in Figures 3 and 4 mainly for $k_BT = 1$ eV that multiphoton processes have to be taken into account in low temperatures plasmas. On the other hand, for higher temperatures, the curves for different values of M practically do not differ and, then, the multiphoton processes are less significant. We also note that as the temperature increases, all curves tend to the Bernstein modes (black line). This behavior can be justified because the influence of the temperature takes place only through the γ_3 factor in the Bessel function which, in turn, is proportional to v_{th}^{-1}. Thus, as temperature increases, we have that $J_m(\gamma_3\xi) \to \delta_{m,0}$. A physical interpretation

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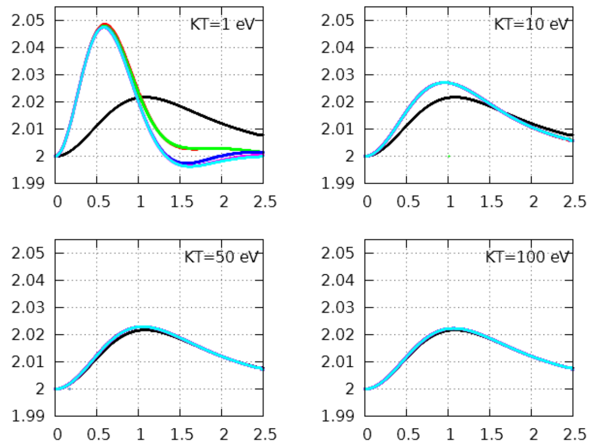


Fig. 4. The tending of the collective modes to the Bernstein modes and the approaching of the curves for differents values of M is also verified for the second range. We employed the same data used in the previously figure.

of this fact is that the energy stored in the radiation field becomes less expressive in high-energetic plasmas and, hence, this external field does not act as a mechanism able to excite new modes. In a quantum mechanical viewpoint, we say that the electron-photon interactions are suppressed by those which involve electrons and plasmons.

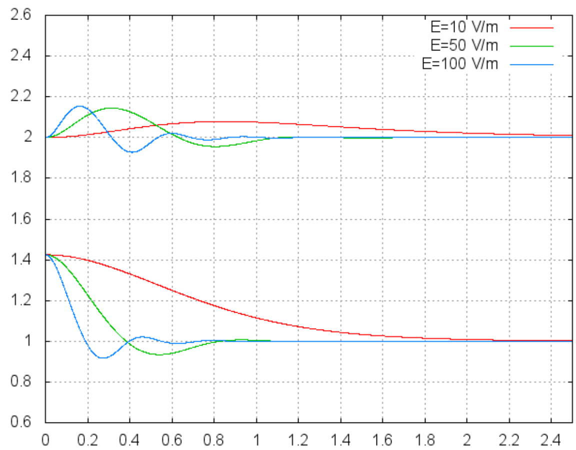
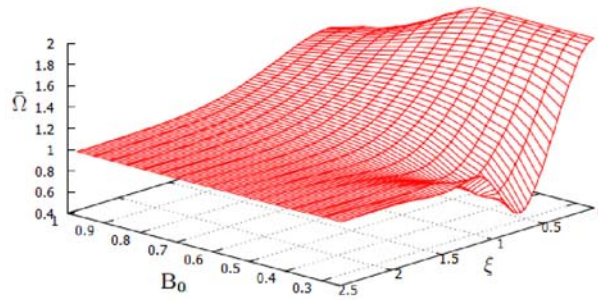


Fig. 5. We note that the decay of the electrostatic oscillations to the cyclotron mode is intensify as the magnitude of the electromagnetic field increases. We assume $n_0 = 10^{19}\ m^{-3}$, $k_B T = 150\ eV$ and $B_0 = 1,0\ T$.

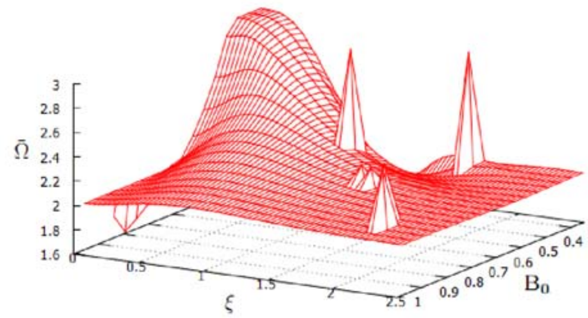
IV. Conclusion

Through mathematical tools of quantum mechanics we were able to describe the propagation of longitudinal modes of a classical magnetoplasma subjected to an external radiation field. The expression derived in Eq.(43) for the dielectric function reduces to that found by Bernstein [3] when the amplitude of the radiation field is taken to be zero.

We notice that the presence of radiation in the quasi-resonant regime ($\omega \approx \omega_c$) excites new electrostatic modes in the plasma since the energy stored in the radiation field is not so low to be neglected when compared to the thermal



(a)



(b)

Fig. 6. The magnetic field strength acts to smooth the oscillations around the cyclotron harmonics and cofinate the modes close to the cyclotron one. For the dispersion relation graph we used B_0 from $0.250\ T$ to $1.000\ T$, $M = 5$ and $E = 10\ Vm^{-1}$. The peaks in the second branch are due to numerical fluctuations in the program and therefore do not represent modes which deserve more interest.

energy of the plasma. For each range, we found, the possibility of electrostatic modes with frequency lower than the cyclotron harmonics frequencies. We also observed the importance of the multiphoton processes in the limit of low temperature plasmas.

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References

[1] E. G. Harris *Classical plasma phenomena from a quantum mechanical viewpoint*, Advances in plasma physcis, 3, 157-241, 1969.
[2] M. A. Amato and L. C. M. Miranda *Plasma wave instabillity in the field of an intense eletromagnetic wave*, Phys. of Fluids, 10, 1031-32, 1977.
[3] I. B. Bernstein *Waves in a plasma in a magnetic field*, Phys. Rev., 109, 10-21, 1958.
[4] S. Gruber and F. Bekefi *Excitation of longitudinal waves near electron-cyclotron harmonics*, Phys.of Fluids, 11, 122-33, 1968.
[5] B. V. Ribeiro and D. D. A. Santos and M. A. Amato and A. L. A. Fonseca *Collective modes in free plasmas subjected to a radiation field* work presented at ICPP-LAWPP 2010 in Santiago, Chile.
[6] L. D. Landau and E. Lifshitz *Quantum Mechanics, Non-Relativistic Theory*, Oxford, England: Pergamon Press, 1977.
[7] R. M. O. Galvão and R. C. M Miranda *Quantum theory of an electron in external fields using unitary transformations*, Am. J. Phys, 51, 729-33, 1982.

[8] J. Kremp and M. Schlanges and W.-D. Kraeft *Quantum Statistics of Nonideal Plasmas* Netherlands: Springer, 2005.
[9] G. M. Walters and E. G. Harris *Quantum-mechanical theory of nonlinear plasma phenomena in a strong magnetic field*, Phys. of Fluids, 11, 112-22, 1968.
[10] G. N. Watson *A Treatise on the Theory of Bessel Functions* 2rd ed. Cambridge University Press, 1944.
[11] W. H. Press and S. A. Teukolsky and W. T. Vetterling and B. F. Flannery *Numerical Recipes in C: The Art of Scientific Computing* 2rd ed. Cambridge University Press, 1992.