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A Heuristic Statistical Model for Lifetime Distribution Analysis of Complicated Systems in the Reliability Centered Maintenance

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***Abstract*—**A heuristic conceptual model for to develop the Reliability Centered Maintenance (RCM), especially in preventive strategy, has been explored during this paper. In most real cases which complicity of system obligates high degree of reliability, this model proposes a more appropriate reliability function between life time distribution based and another which is based on relevant Extreme Value (EV) distribution. A statistical and mathematical approach is used to estimate and verify these two distribution functions. Then best one is chosen just among them, whichever is more reliable. A numeric Industrial case study will be reviewed to represent the concepts of this paper, more clearly.

***Keywords*—**Lifetime distribution, Reliability, Estimation, Extreme value, Improving model, Series, Parallel.

I. Introduction

The Extreme Value (EV) theory which is related to statistical behavior of maximum and minimum value in a random sample was suggested by Fisher and Tippett [1] initially, and extended by Gumbel [2] through determining the approximate distribution function of $X(1)$ and $X(n)$ as the extreme values (minimum and maximum) of a statistical random sample, $X_1, X_2, \ldots, X_n$.

The intricate models like EV type II and type III, were represented and adapted by Ang and Tang as scientific and applied extension of this theory [3]. Kotz and Nagarajah listed over 50 application ranging from accelerated life testing through to earthquakes, foods horse racing, rainfall, queues in systems, sea currents, wind speeds, and track race records [4]. Developing the EV theory in reliability applications, studied through kinds of mathematical modeling by Alkallut and Lye et al [5, 6] and various dynamic evaluating of systems reliability [7, 8, 9, 10, 11, 12].

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This paper is going to explore a new application of EV distributions to estimate reliability function for complicated systems. It is chiefly useful in the Reliability Centered Maintenance (RCM) planning.

Therefore in section 2 some of the mathematical and statistical features of EV distributions are briefly reviewed and followed by introducing our heuristically applied algorithm in section 3. In section 4 a case study is used to deliberate the proposed model.

II. Some Features of Extreme Value Distributions

Suppose in a random sample like $X_1, X_2, \ldots, X_n$, $X(1)$ and $X(n)$ are minimum and maximum value, respectively.

Equation (1) and (2), demonstrate the approximate probability density functions (PDF) of these variables [2].

$$f_{EV}^{x(1)}(x;\gamma,\delta) = \frac{1}{\delta}\exp\left(\frac{x-\gamma}{\delta} - \exp\left(\frac{x-\gamma}{\delta}\right)\right), \quad -\infty < x < +\infty \quad , \quad \delta > 0 \tag{1}$$

$$f_{EV}^{x(n)}(x;\gamma,\delta) = \frac{1}{\delta}\exp\left(-\left(\frac{x-\gamma}{\delta}\right) - \exp\left(-\frac{x-\gamma}{\delta}\right)\right), \quad -\infty < x < +\infty \quad , \quad \delta > 0 \tag{2}$$

Where γ and δ are location and scale parameters, respectively.

The PDF curves are illustrated in Fig. 1 and Fig. 2, where $\gamma = 0$ and $\delta = 1, 2, 3$. Some of useful statistical features of these PDFs are represented in Table I [13].

TABLE I
SOME STATISTICAL FEATURES OF EV VARIABLES

Terms	Minimum State, $f_{EV}^{x(1)}$	Maximum State, $f_{EV}^{x(n)}$
Mean	$\mu = \gamma - 0.5772\,\delta$	$\mu = \gamma + 0.5772\,\delta$
Standard Deviation	$\sigma = \frac{\pi}{\sqrt{6}}\delta$	$\sigma = \frac{\pi}{\sqrt{6}}\delta$
Skewness	$S.K = -5.4$	$S.k = 5.4$
Kurtosis	$K = 1.14$	$K = 1.14$

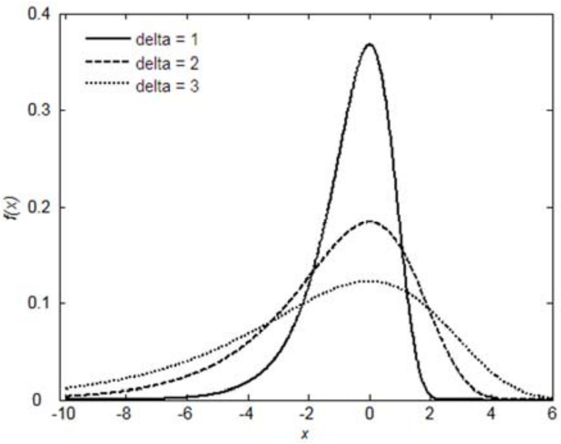

Fig. 1 PDF curves of EV variable in the minimum case, *X(1)*, where $\gamma = 0$ and $\delta = 1, 2, 3$

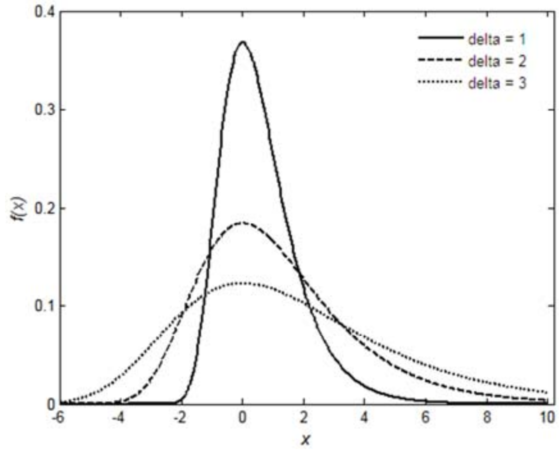

Fig. 2 PDF curves of EV variable in the maximum case, *X(n)*, where $\gamma = 0$ and $\delta = 1, 2, 3$

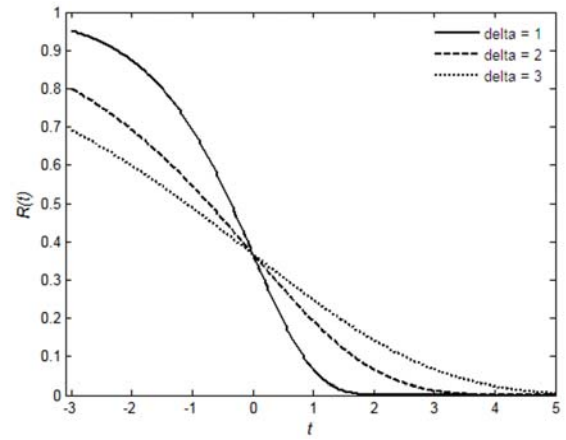

Fig. 3 Reliability function curves related to X(1), where $\gamma = 0$ and $\delta = 1, 2, 3$

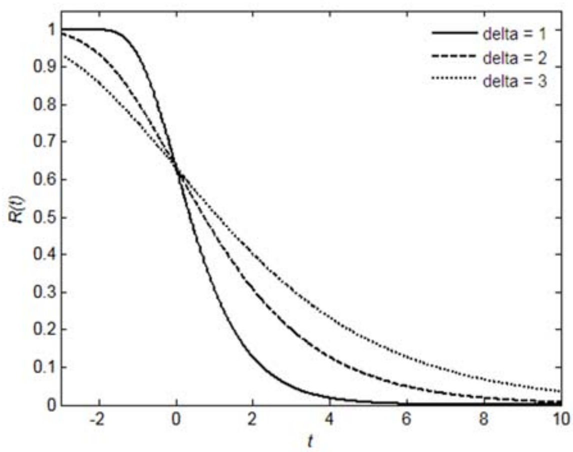

Fig. 4 Reliability function curves related to X(n), where $\gamma = 0$ and $\delta = 1, 2, 3$

Reliability function, R(t), can be obtained using (1) and (2), as calculated by Karbasian and Mahdavi [14]:

- Mathematical model of reliability function for minimum value in a random sample, $X(1)$, is following and its related curves are shown in Fig. 3.

$$R(t) = P(x \geq t) = \int_t^{\infty} f_{EV}^{x(1)}(x;\gamma,\delta)dx = \exp\left(-\exp\left(\tfrac{t-\gamma}{\delta}\right)\right) \tag{3}$$

- Mathematical model of reliability function for maximum value in a random sample, $X(n)$, is following and its related curves are shown in Fig. 4.

$$R(t) = P(x \geq t) = \int_t^{\infty} f_{EV}^{x(n)}(x;\gamma,\delta)dx = 1 - \exp\left(-\exp\left(-\tfrac{t-\gamma}{\delta}\right)\right) \tag{4}$$

III. IMPROVING MODEL DEFININIG

Considering a system composes of *n* components, $C_1, C_2, \ldots, C_n$, two simplest known basic arrangements are Parallel and Series like Fig. 5 and Fig. 6, and their reliability functions are definable according to (5) and (6), respectively [15]. Where, $R_{sys}(1)$ is reliability function of series system, $R_{sys}(n)$ is reliability function of parallel system and R_i is reliability of *i*th component.

$$R_{sys}(1) = \prod_{i=1}^{n} R_i \tag{5}$$

$$R_{sys}(n) = 1 - \prod_{i=1}^{n} (1 - R_i) \tag{6}$$

Suppose random sample $X_1, X_2, \ldots, X_n$ is available and X_i is working time of component C_i. Subsequently, $X(i)$ is working time of *i*th failed part. Let name this component $C(i)$.

Therefore a series system fails at time $X(1)$ (when $C(1)$ fails). Similarly this time for parallel system is $X(n)$ that failure in $C(n)$ occurs. So it will possible to simulate working time of series and parallel systems using $X(1)$ and $X(n)$ statistical characteristics.

Considering section 2, $X(1)$ and $X(n)$ meet EV distributions, $f_{EV}^{x(1)}$ and $f_{EV}^{x(n)}$ respectively.

This concept can be applied for estimating reliability function of system.

Particularly for systems are uncertain in the arrangement structure or reliability of components or both of them, an appropriate EV function can be investigated instead of the system's lifetime PDF.

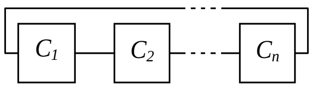

Fig. 5 Single structure of series system

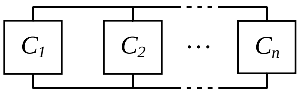

Fig. 6 Single structure of parallel system

In the following heuristic model, the mentioned capability has been systematically expanded. According to this model, for a series system, lifetime PDF and minimal EV function can be used for reliability evaluation which one is more reliable.

The same selective condition in parallel system is for maximal EV function, too. If arrangement of the system is not certain, selection can be done among all of them. Vice versa the lifetime distribution function of system can be estimated as strong as the reliability function which has been extracted by using this model. Obviously this function includes the most reliable lifetime distribution of system.

A numerical study is reviewed in the next section.

IV. Numerical Case Study

A complicated electromechanical system whit uncertain arrangement of components has been investigated based on introduced model at the following steps. So evidently, reliability function and most reliable lifetime distribution are expected to be obtained via this model.

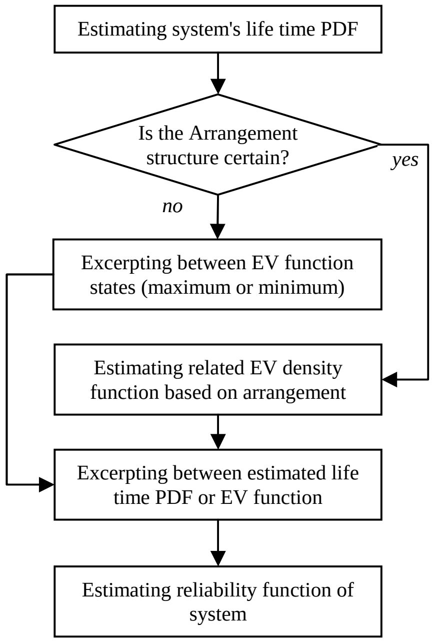

Fig. 7 The heuristic improving model

A. Estimating System's Lifetime PDF

A random sample in size $n = 48$ has been taken and represented in Table II.

Based on the Anderson-Darling goodness of fit test, Weibull density function with shape and scale parameters $\alpha = 2.36$ and $\beta = 3.95$ is selected. The probability sheet for fit test and its approximate curve is shown in Fig. 8. Estimated PDF is defined following

$$Wei\left(x;\alpha,\beta\right) = \frac{2.36}{3.95}\left(\frac{x}{3.95}\right)^{1.36} \exp\left(-\left(\frac{x}{3.95}\right)^{2.36}\right) \quad (7)$$

TABLE II
RANDOM SAMPLE DATA COLLECTION

5.04	3.13	2.31	4.02	4.55	2.35	3.17	5.62
3.66	1.32	4.77	2.52	2.31	2.81	0.8	1.44
3.18	4.42	1.12	7.75	3.55	2.10	3.90	1.87
4.36	2.68	3.93	7.35	4.36	4.10	3.29	3.43
7.23	2.02	1.46	3.28	5.17	4.58	2.20	3.64
4.69	2.78	5.75	3.52	3.16	2.44	3.23	1.27

B. Excerpting Between EV Functions

Some statistical descriptions of the collected data have been gathered in Table III. Considering these features and shape of estimated curve, the maximal EV function seems to be more appropriate. Using relations in Table I location and scale parameters obtained as $\gamma = 4.20$ and $\delta = 1.23$ and its PDF as (8).

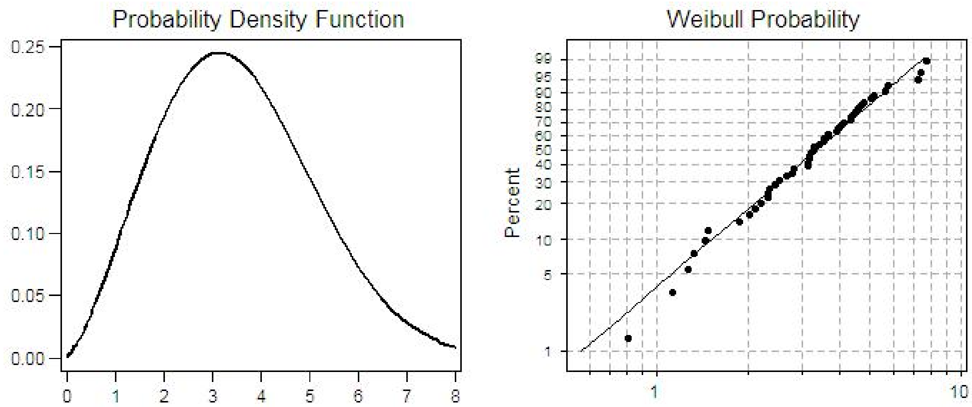

Fig. 8 Probability sheet and approximate PDF curve for Weibull

TABLE III
STATISTICAL DISCRIPTIONS OF COLLECTED DATA

Sample size (*n*)	48
Mean (μ)	3.49
Std. Dev. (σ)	1.58
Median (*Me*)	3.29
Skewness (*S.K*)	0.74
Kurtosis (*K*)	0.64

$$EV\left(\mathrm{x};\lambda,\delta\right)=\frac{1}{1.23}\exp\left(-\frac{x-4.20}{1.23}-\exp\left(-\frac{x-4.20}{1.23}\right)\right) \quad (8)$$

C. Excerpting Estimated Lifetime PDF or EV Function

Reliability functions based on the obtained PDFs can be calculated. The first one obtains by using (7) and second one by (4) and (8).

$$R_{Wei}(t)=\exp\left(-\left(\frac{t}{3.95}\right)^{2.36}\right) \quad (9)$$

$$R_{EV}(t)=1-\exp\left(-\exp\left(-\frac{t-4.20}{1.23}\right)\right) \quad (10)$$

Fig. 9 helps for final evaluating between R_{Wei} and R_{EV}. Apparently reliability function related to EV function (R_{EV}) is strongly more reliable than the other one (R_{Wei}), through the time axis. For example if $t=3$ then $R_{Wei}=0.602$ and $R_{EV}=0.924$.

D. Estimating reliability function of system

To make an accurate compare, the under curve area for both candidates R_{Wei} and R_{EV} can be evaluated using (9) and (10).

It's subsequently provided by

$$\Omega_1(T)=\int_0^\infty R_{Wei}(t)dt=11.86 \quad (11)$$

and

$$\Omega_2(T)=\int_0^\infty R_{EV}(t)dt=15.12 \quad (12)$$

where T is suppositive lifetime random variable.

Comparing Ω_1 and Ω_2 returns reliability function and most reliable lifetime distribution function of this system based on Extreme Value function, which are represented in (10) and (8), respectively.

On the other hand the last equations describe Mean Time to Failure (MTTF) of the system, based on both reliability functions. It means that $\Omega_1(T)=MTTF_{Wie}=11.86$ and $\Omega_2(T)=MTTF_{EV}=15.12$.

So there is a preference about $\frac{\Omega_2-\Omega_1}{\Omega_1}=27\%$ to select the EV based reliability function for this system.

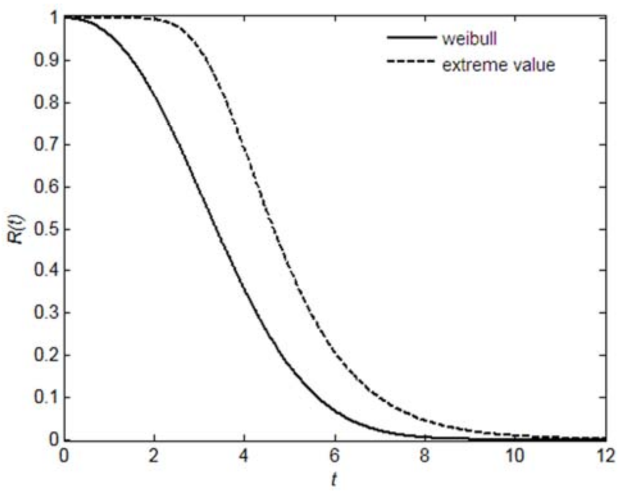

Fig. 9 Reliability Curves based on Weibull (R_{Wei}) and Maximal EV (R_{EV})

V. CONCLUSION

Estimating the reliability function of systems especially when arrangement of component or their reliability or both of them are not certain is the most important in the reliability centered maintenance (RCM).

A new concept which based on the Extreme Value functions properties was developed in this paper through improving model for reliability function of such systems. An aptly EV function can be drive better fitted distribution because of its intrinsic relation to arrangement.

In the numerical case study reliability function based on lifetime PDF of system was improved to more appropriate one based on related EV function using the mentioned model.

a strong statistical analysis based on the concept of this paper needs to make a total survey on all families of lifetime distributions and approve the mentioned model.

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