World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:2, No:1, 2008

Unsupervised Image Segmentation Based on Fuzzy Connectedness with Sale Space Theory

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***Abstract*—**In this paper, we propose an approach of unsupervised segmentation with fuzzy connectedness. Valid seeds are first specified by an unsupervised method based on scale space theory. A region is then extracted for each seed with a relative object extraction method of fuzzy connectedness. Afterwards, regions are merged according to the values between them of an introduced measure. Some theorems and propositions are also provided to show the reasonableness of the measure for doing mergence. Experiment results on a synthetic image, a color image and a large amount of MR images of our method are reported.

***Keywords*—**Image segmentation, unsupervised image segmentation, fuzzy connectedness, scale space.

I. INTRODUCTION

IMAGE segmentation, also referred to recognize objects in an image in some cases, is a difficult problem. Unsupervised segmentation of image is even more like a nettlesome one. However, unsupervised segmentation often plays an important role in some applications like in content based image retrieval, etc.

The fuzzy connectedness and its extensions [1]-[5] have been effectively utilized to do segmentations in several applications. Fuzzy connectedness can address the graded composition of intensity values and hanging togetherness of image elements in object regions.

It hasn't been publicly reported for fuzzy connectedness to do unsupervised segmentation so far. We have done some researches on it and ever submitted a paper about its accomplishment by means of scales space [6]. In that paper, the scale space theory is utilized to assess automatically the underlying structures within an image data which consists of intensity/feature values of pixels. The estimated number of underlying structures is considered as the one of objects in segmentation by fuzzy connectedness. With the reference value of each structure, seeds are specified for each object. Then, using the fuzzy connected object delineation method in the case of multiple seeds [4], all objects are extracted with the specified seeds. However, in the further researches, we found that it is very difficult for scale space method to determine the underlying structures each of which corresponds definitely to one object because scale space theory considers only the intensity/feature values of pixels in an image. The seeds specified by scale space relative to one structure are sometimes far away in the 2 dimensional space on the image though they are near in intensity/feature space, and possibly correspond to different objects. In another case, if there exists color gradation in an object, the object may be divided into more than one underlying structures. In both the cases, objects will be extracted wrongly because of the incorrect specification of object seeds for fuzzy connectedness.

In this paper, we solve that problem in a much different mechanism of segmentation. We utilize the seeds specified by scale space, but neglect the information of which object each seed is assigned to in order to avoid the possible wrong assignation. Each seed is considered as belonging to a different part of object. The number of segmented regions equalizes to the one of seeds. The segmented region of each seed is acquired by the method in [3]. Then we construct a measure between any two segmented regions to determine the degree of their belonging to a same object, and some of the segmented regions are merged because they look more like belonging to a same object according to the measure values.

II. RERATED THEORIES TO FUZZY CONNECTEDNESS

In this paper, we use $\mu_\kappa(c,d)$ to denote the affinity between two given pixels c and d in an image (or a volume), and $\mu_K(c,d)$ to denote the fuzzy connectedness between them. The coordinates of the centre of a pixel is denoted by Z^n, where n denotes the corresponding dimension of the coordinate space. The image (or a volume) domain in the coordinate space is denoted by $C=\{c|-h_i\le c_i\le h_i, \text{for } h\in Z_+^n, c\in Z^n\}$, where Z_+^n is the set of n-tuples of positive integers.

The two object extracting methods in [2][3] have both used iterative strategy, and considered the fact that, to determine the fuzzy connectedness of an arbitrary pixel, the segmented spatial domain of other objects in last iteration should be simultaneously based on. That consideration can lead to a more practical result. In this paper, we call the considered fact the *mutually spatial influence* (*MSI* for short) of objects to extraction.

We call, in the rest of the paper, the fuzzy affinity and fuzzy connectedness defined with considering *MSI*, the *MSI* fuzzy affinity and *MSI* fuzzy connectedness, in order to distinguish them with the ones defined originally without considering *MSI*

Manuscript received May 6, 2004. The research reported in this paper is supported by China National Science Foundation, No: 30170274.

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[1]. The kind of extracting method as in [2][3], which does extraction with the *MSI* fuzzy affinity and *MSI* fuzzy connectedness, is called *MSI_FOE*.

III. SEEDS SPECIFICATION BY SCALE SPACE THEORY

Here we introduce briefly the content of seed specification by scale space. For more detailed information, please refer to [6] and [7].

The method first nonparametrically estimates the probability density function of possibly multiple dimensional feature vectors, and then extracts the reference features for each cluster through finding out the approximation of the genuine peaks of the feature data with Gaussian filtering and zero crossing, and, finally determines the cluster to which a datum belongs with gradient ascent criteria.

Pixel whose features vector locates near enough to the reference features of one cluster is labelled as the corresponding candidate seed. In C, all the connected components, in each of which elements are all seeds and have the same label, are, if their areas exceed a threshold, candidate seed regions, denoted by $R = \{r_1, r_2, \cdots, r_{m'}\}$. For a candidate seed region $r_i (1 \le i \le m')$, we use the element which locates nearest to the centroid of the seed region area as the corresponding seed element and denote it by s_i'. So we get candidate seed elements $S' = \{s_1', \cdots, s_{m'}'\}$ from R.

It has been proven that seed elements should be within the region of the corresponding physical object in the scene, i.e. not on boundary, in order to do extractions correctly. Consequently, every element in S' needs to be judged valid.

Unfortunately, the region(s) of an object can't be known before segmentation is finished. Consequently, candidate seed elements can't be judged valid so directly. However, please note that there are many segmentation methods which partition an image through finding the boundary between regions of objects. Their criterions on determining if an element is on boundary can be used here to judge whether a candidate seed locates on boundary. In fact, for a candidate seed $s_i' \in S'$, we can compute its features value's second order directional derivates in transverse and longitudinal directions and take their average value. If the average value exceeds a threshold, we think the seed element locates on boundary and so is invalid. The set of valid seed elements are denoted by $S = \{s_1, \cdots, s_m\} (m \le m')$.

Please note that if we use first order directional derivate instead of the second order one, a seed element which locates in area of heavily graded composition may be judged invalid. That violates the idea of fuzzy connectedness, that there may be graded composition within and object.

IV. REGIONS EXTRACTION AND MERGENCE

As discussed in the introduction, seed specified by scale space method may be assigned to a wrong object if judged only by the distance between the features vector of a pixel and the reference one of a cluster in the scale space analysis. For example, Fig. 1(a) shows a color photographic image of a sculpture. The two seeds s_1, s_2 shown in Fig. 1(b) belong apparently to two different objects which are "sculpture" and "trees" respectively. Otherwise, the color values of the two seeds are very similar. The two seeds are easy to be assigned wrongly to a same object by scale space theory. With the wrongly decided seeds, incorrect extraction results are bound to be acquired through the object delineation method in [4].

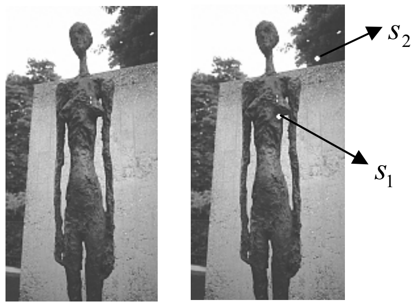

Fig. 1: (a) A digital color photographic image of sculpture. (b) The two seeds s_1, s_2 shown by the two red dots.

In this paper, all the specified seeds are supposed to belong to a distinct object. The region which corresponds to each seed is extracted by the relative object extraction approach in [3]. Then the regions are merged if judged to belong to a same object according to the values of an introduced measure which implies the closeness of any two regions.

Definition 1. The set of pixels which have been determined to belong to the same object as done for a seed s, is called the *territory* of s, and denoted by T_s.

The similar definitions to *territory* are "core part" of object in [3] and "$\psi - connected$" component in [2].

Definition 2. For a given relation of fuzzy pixel adjacency α, for two seeds s_1 and s_2, providing the value of α between one pixel a in T_{s_1} and the other one b in T_{s_2} to be larger than zero, i.e. $\mu_\alpha(a,b) > 0$, we call s_1 and s_2 are *adjacent* for *territory*, and a,b the *corresponding adjacent elements/pixels* of T_{s_1} and T_{s_2}.

Definition 3: For two seeds s_1 and s_2, for a given κ and a threshold θ, for two pixels a and b which are the *corresponding adjacent elements* of T_{s_1} and T_{s_2}, if affinity value $\mu_\kappa(a,b)$ exceeds θ, we call the territories of s_1 and s_2 are *adherent*, and a,b the corresponding *adherent components* of s_1 and s_2.

Definition 4: For two seeds s_1 and s_2, $s_1 \neq s_2$ whose *territories* are *adjacent*, for a given affinity κ, if a, b are any two *corresponding adjacent elements* of T_{s_1} and T_{s_2} respectively, we define the *adherent strength* of s_1 and s_2 through a, b, denoted by $AS(s_1, s_2; a, b)$, as $AS(s_1, s_2; a, b) = \min(\mu_{K_{s_1}}(a), \mu_{K_{s_2}}(b), \mu_\kappa(a,b))$, where $\mu_{K_{s_1}}(a), \mu_{K_{s_2}}(b)$ are all *MSI* fuzzy connectedness. We also define the *adherent strength* of s_1 and s_2 as the maximum value of $AS(s_1, s_2; a, b)$ in all the *corresponding adjacent elements*, and denote it by $AS(s_1, s_2)$. If $s_1 = s_2$, we set $AS(s_1, s_2) = 1$, and there is no meaning for $AS(s_1, s_2; a, b)$.

The measure of *adherent strength* between regions is to be used to do mergence of segmented regions determined with seeds specified by scale space theory. In the left part of this section, the measure will be proven to be reasonable to do the mergence.

Theorem 1: Given fuzzy affinity κ, given a set of seeds $S = \{s_1, s_2, \cdots, s_m\}$, providing there is a section of path [1] denoted by $\overset{\frown}{cd}$, $c = s_l$ (for one $l, 1 \le l \le m$), $d \in C$, and to any pixel $e \in \overset{\frown}{cd}$, $\mu_K(s_l, e) > \mu_K(s_{l'}, e)(1 \le l' \le m, l' \neq l)$, then we have $\overset{\frown}{cd} \in T_{s_l}$ with any objects extracting method of the kind *MSI_FOE*.

Proof: It is obvious reasonable with considering that, in each extracting method of the kind *MSI_FOE*, firstly, the *MSI* affinity relation between two arbitrary pixels is defined on the path which locates totally in or "mainly" in the *territory* of a seed got in the last iteration, secondly, the seed is in the *territory* of itself at first, thirdly, there is the property of competition to do extractions, and fourthly, $\overset{\frown}{cd}$ is connected with s_l. □

Theorem 2: Given fuzzy affinity κ, given a set of seeds $S = \{s_1, s_2, \cdots, s_m\}$, suppose we have got the *territory* of seeds with a given *MSI_FOE* method, providing $\mu_K(s_{l_3}, s_{l_4}) < \mu_K(s_{l_1}, s_{l_2})$ ($1 \le l_1, l_2 \le m$, for all $1 \le l_3, l_4 \le m$, $l_1 \neq l_2, l_3 \neq l_4$, nonordered pairs: $(l_3, l_4) \neq (l_1, l_2)$),then all the pixels on the best path [1] without considering *MSI* between s_{l_1} and s_{l_2} locate entirely in the territories of s_{l_1} and s_{l_2} with any objects extraction method of the kind *MSI_FOE*.

Proof: c is any arbitrary pixel on the best path corresponding to $\mu_K(s_{l_1}, s_{l_2})$.

We can assure that $\mu_K(s_{l_1}, c) > \mu_K(s_l, c)$, otherwise $\mu_K(s_l, s_{l_2}) \ge \mu_K(s_{l_1}, s_{l_2})$, that's contradictory to the assumptions of the theorem. Because s_l can be anyone in S excluding s_{l_1} and s_{l_2}, we know that only $\mu_K(s_{l_2}, c) \ge \mu_K(s_{l_1}, c)$ can exist.

Suppose there is a distinct pixel d from c on the best path from s_{l_1} to s_{l_2}, and d is nearer to s_{l_2} than c. We have $\mu_K(s_{l_1}, c) \not< \mu_K(s_{l_2}, c)$ if $\mu_K(s_{l_1}, d) \ge \mu_K(s_{l_2}, d)$, because it can be obviously seen that $\mu_K(s_{l_1}, c) \ge \mu_K(s_{l_1}, d)$ and $\mu_K(s_{l_2}, c) \le \mu_K(s_{l_2}, d)$, such that $\mu_K(s_{l_1}, c) \ge \mu_K(s_{l_2}, c)$. So there must be a pixel e on the path such that, on the section of the best path form s_{l_1} to e (denoted by $\overset{\frown}{s_{l_1}e}$), any pixel's fuzzy connectedness from s_{l_1} is larger than from s_{l_2}, and on the section from s_{l_2} to e (denoted by $\overset{\frown}{s_{l_2}e}$), it is larger from s_{l_2} than from s_{l_1}. We can go a step further to get that, for any pixel c on $\overset{\frown}{s_{l_1}e}$, $\mu_K(s_{l_1}, c) \ge \mu_K(s_l, c)$ (for all $1 \le l \le m, l \neq l_1$), and any pixel d on $\overset{\frown}{s_{l_2}e}$, $\mu_K(s_{l_2}, d) \ge \mu_K(s_l, d)$(for all $1 \le l \le m, l \neq l_2$).

With theorem 1, we can surely get $\overset{\frown}{s_{l_1}e} \in T_{s_{l_1}}$ and $\overset{\frown}{s_{l_2}e} \in T_{s_{l_2}}$ using any extracting method of the kind *MSI_FOE*.

Please note that all the relations of fuzzy connectedness used above haven't considered the factor of *MSI*. □

Proposition 2.1: Given fuzzy affinity κ and a set of seeds $S = \{s_1, s_2, \cdots, s_m\}$, suppose we have got the *territory* of seeds with a given *MSI_FOE* method, providing: for $1 \le l_1, l_2, l_3 \le m$, $\mu_K(s_{l_4}, s_{l_5}) < \mu_K(s_{l_1}, s_{l_2})$, $1 \le l_4, l_5 \le m$, $l_5 \neq l_4$, nonordered pair: $(l_4, l_5) \neq (l_1, l_2)$ and $\mu_K(s_{l_4}, s_{l_5}) < \mu_K(s_{l_1}, s_{l_3})$, $1 \le l_4, l_5 \le m$, $l_5 \neq l_4$, where nonordered pairs: $(l_4, l_5) \neq (l_1, l_3)$ and $(l_4, l_5) \neq (l_1, l_2)$, and if merge s_{l_1} with s_{l_2} to $s_{l_{12}}$, $T_{s_{l_1}}$ with $T_{s_{l_2}}$ to $T_{s_{l_{12}}}$ as the *territory* of $s_{l_{12}}$, then the best path from s_{l_3} to $s_{l_{12}}$ is the one from s_{l_3} to s_{l_1}, and all the pixels on the best path locate in $T_{s_{l_{12}}}$ and $T_{s_{l_3}}$.

Proof: Please note that, from the assumptions of the proposition and the proof of theorem 2, to an arbitrary seed c on best path $\overset{\frown}{o_{l_1}o_{l_3}}$ from s_{l_1} to s_{l_3} (we can also say from $s_{l_{12}}$ to s_{l_3}), the fuzzy connectedness from s_{l_1} or s_{l_2} (we can also say from $s_{l_{12}}$), to s_{l_3}, however not other seeds, may be the largest one. The remainder of the proof is similar to the one of theorem 2. □

Proposition 2.2: Given fuzzy affinity κ and a set of seeds $S = \{s_1, s_2, \cdots, s_m\}$, suppose we have got the *territory* of seeds with a given *MSI_FOE* method, providing: $\mu_K(s_{l_3}, s_{l_4}) < \mu_K(s_{l_1}, s_{l_2})$, where $1 \le l_1, l_2 \le m$, $l_1 \ne l_2$, for all $1 \le l_3, l_4 \le m$, $l_3 \ne l_4$, nonordered pairs: $(l_3, l_4) \ne (l_1, l_2)$, then $AS(s_{l_1}, s_{l_2}) = \mu_K(s_{l_1}, s_{l_2})$, and $AS(s_{l_1}, s_{l_2}) > AS(s_{l_3}, s_{l_4})$.

Proof: By theorem 2, we know the best path from s_{l_1} to s_{l_2} for $\mu_K(s_{l_1}, s_{l_2})$ locate entirely in $T_{s_{l_1}}$ and $T_{s_{l_2}}$, consequently it can determine a value of *adherent strength* of s_{l_1} and s_{l_2} with the *corresponding adjacent spels*. Because the path is the best one between s_{l_1} and s_{l_2}, the value is sure to be the largest one, and consequently is the *adherent strength* of s_{l_1} and s_{l_2}. At the same time, the *adherent strength* between other pair of seeds can't exceed the value of fuzzy connectedness of them. So we know the proposition is right. □

From the above theorems and propositions, we see that a larger value of fuzzy connectedness between two seeds means a larger value of *adherent strength* between them. If we accept the principle adopted by all the segmentation methods based on fuzzy connectedness, like in [1], *adherent strength* is a reasonable measure to do mergence.

However, when the domain of an underlying object is actually composed of several separated regions, we can't merge correctly seeds with only *adherent strength*, because the strength value of two seeds may be very small or even zero because of their territories' separation from each other, though they belong to the same object. So we also have to recur to the measurement of feature distance. If the *adherent strength* of two seeds is too small to merge them, however their feature distance obtained from the elements in their territories is very small, we have also enough reasons to merge them.

Suppose seed's feature distance is $FD(s_i, s_j), s_i, s_j \in S = \{s_1, s_2, \cdots, s_m\}$ and $FD_{s\max} = \max_{s_i, s_j \in S}(FD(s_i, s_j))$. Because $FD(s_i, s_j)$ can be a value in any range, but $AS(s_i, s_j)$ takes its value in [0,1], the value of $FD(s_i, s_j)$ has to be adjusted in order to be comparable with $AS(s_i, s_j)$. We set

$$MS(s_i, s_j) = t \times AS(s_i, s_j) + (1-t)\left(1 - \frac{FD(s_i, s_j)}{FD_{s\max}}\right)(0 \le t \le 1)$$

as the measurement to merge seeds. Here we call it *mergence strength*. A higher value of *mergence strength* means more likely to merge the corresponding segmented regions.

If given the due number of objects, compute the *mergence strengths* and iteratively merge two regions whose value is the largest one, until getting the needed number of objects. If given a threshold of *adherent strength,* not the number of objects, we can merge the regions among which the *adherent strength* exceeds the value of the threshold.

V.EXPERIMTNTS AND RESULTS

In the first experiment, in order to show the effectiveness of doing mergence with the combination of *adherent strength* and feature distance, we did several experiments on the color photographic image shown in Fig. 1a. We specified 11 seeds, shown in Fig. 2a, and provided them different object ID. With setting $t = 0.5$, we extracted every object with the MSI_FOE method in [3]. All the 11 extracted objects before mergence are shown in Fig. 2d~2n. After mergence with mergence strength $MS(s_i, s_j)$, the sculpture and stone stele are shown in Fig. 2b and 3c. The 5 part of stone stele were correctly merged because of their smaller feature distances. We should explain here that, the adopted feature distance in the experiments is the Euclidean distance of the mean vectors of scenes with 3 orthogonal color components, namely, red, green, and blue for every spel in objects. In the experiments, we found the two parts of the trees were difficult to be merged.

Then we downloaded 3 sequences of images from the web [9], with noise=3%, noise=5%, and noise=7% respectively, and Modality=T_1, Protocol=ICBM, Phantom-name=normal, slice-thickness=1mm, INU=20. Each volume datum consists of $181 \times 217 \times 181$ voxels with a cubic resolution $1 \times 1 \times 1 mm^3$. All the images have been pre-processed to extract the intracranial volume. Seeds are specified with scale space method introduced in section 4. Then every region is extracted with the method in [3] for every seed. Afterwards, the values of *mergence strength* are computed between segmented regions with $t = 0.8$. At last regions are merged with the method introduced in section 4.

We construct hard classifications by the maximum membership criterion according to the true partial volume fractions provided on the web and consider them as the standard segmentation results.

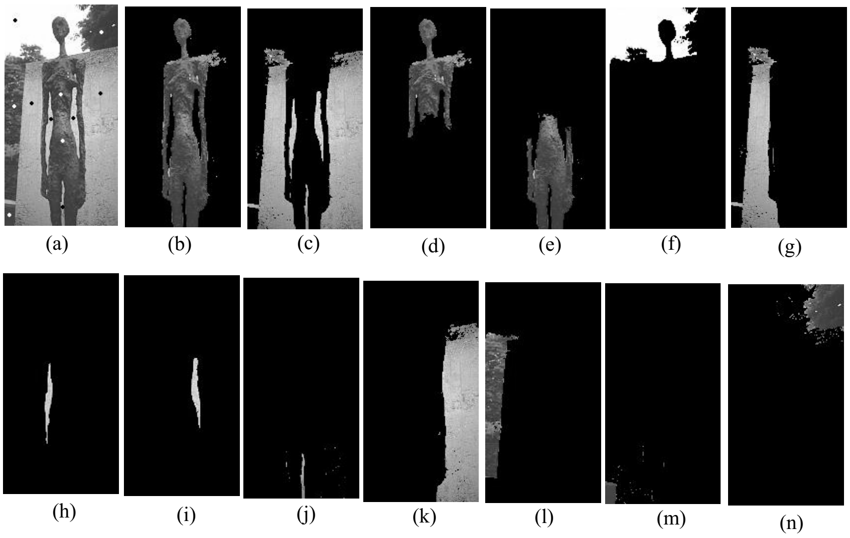

Figure 2: (a) Eleven seeds, white or black dots, specified manually in the digital color photographic image of a sculpture in a natural daylight scene. (b), (c) The "sculpture" and "stone stele" obtained after mergence with our defined mergence strength which combining *adherent strength* and feature distance. (d)~(n) The 11 objects' regions before mergence.

Then MCR values are computed in order to provide statistical values for our method's precision and as $MCR = N_{err} / N_{total}$, where N_{err} means the number of pixels misclassified, and N_{total} denotes the total number of pixels.

The statistical results are show in table 1.

TABLE I
ERROR MEASURES FROM SIMULATED MR BRAIN IMAGE RESULTS

MCR	WM	GM	CSF	Total
3% Noise	0.878%	1.521%	1.345%	3.755%
5% Noise	1.599%	2.234%	1.198%	4.878%
7% Noise	2.258%	2.543%	2.401%	7.500%

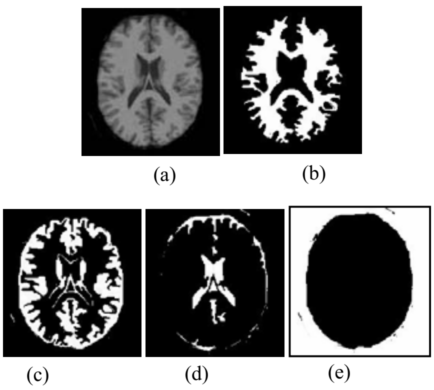

Fig. 3: (a) The original slice of simulated brain image. (b)~(e) The segmented results of white matter, gray matter, cerebrospinal fluid and background respectively

Fig. 3 shows the unsupervised segmentation results on one slice image with noise=3%. Fig. 3(a) is the original image. Fig. 3(b)~(e) show the segmented results of white matter, gray matter, cerebrospinal fluid and background respectively.

VI. CONCLUSION

In this paper, we depict how to do unsupervised segmentation based on fuzzy connectedness with scale space theory. We first provide the method to specify seeds with scale space theory in an unsupervised way. Then we construct a new measure, *adherent strength*, to merge segmented regions belonging to a same object and its validness is proven by some of our theoretical conclusions. The other more practical measure, *mergence strength,* is also provided for doing regions' mergence in applications, which unifies *adherent strength* and feature distance. Some experimental results of our methods are as well provided. In the future, we will apply our method on 3D volume segmentation.

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