

Estimation of Shock velocity and pressure of detonations and finding their flow parameters

Mahmoud Zarrini and R. N. Pralhad

Abstract—In this paper, mathematical modeling of detonation in the ground is studied. Estimation of flow parameters such as velocity, maximum velocity, acceleration, maximum acceleration, shock pressure as a result of an explosion in the ground have been computed in an appropriate dynamic model approach. The variation of these parameters with the diameter of detonation place (L), density of earth or stone (ρ), time decay of detonation (T), peak pressure (P_m), and time (t) have been analyzed. The model has been developed from the concept of underwater explosions [Refs. [1]-[3]] with appropriate changes to the present model requirements.

Keywords—shock velocity, detonation, shock acceleration, shock pressure.

I. INTRODUCTION

SINCE explosive detonations are very dangerous for nature, preventing the destructive effects of explosive detonation on nature is necessary (Refs. [4] and [5]). It can be done through mathematical modeling: the estimation of the field of detonation. To find the model of detonation, we have used the concepts of under water explosions [Refs. [1]-[3] and [6]] and conservation equations of fluid dynamic such as: mass, momentum and energy equations.

II. ANALYSIS

The equation for motion of particles on the ground and prone of explosive detonation can be written as

$$2P(t) - \rho c V(t) = m \frac{dV}{dt} \quad (1)$$

Here $P(t)$ is the shock pressure (as a result of an explosion), c is the sound velocity, $V(t)$ is the velocity of the medium [mass of the particles], m is the body mass per unit area. Estimation of $P(t)$ is quite difficult and no mathematical relation exists so far in the literature relating to particle condition. Recently Ramajeyathilagama, Hollyer and Hwang-Fuu [Refs. [1]-[3]] proposed empirical relation for the shock pressure with respect to time for underwater explosion.

$$P(t) = P_m \exp\left(-\frac{t}{T}\right) \quad (2)$$

where

$$P_m = P_0 \cdot r^{-1.64} \quad (3)$$

Mahmoud Zarrini is Research Scholar in Department of Mathematics, University of Pune, Pune, India. He is with Ministry of Education, Hamedan District, Hamedan, Iran . e-mail: (m_zarrini@yahoo.co.in).

R.N. Pralhad is with Department of Applied Mathematics, Defence Institute of Advanced Technology, Girinagar, pune, India.

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And P_m is the peak pressure at distance r , P_0 is the reference pressure, t is the time since the shock front arrived at the target point and T is the time decay. In the proposed studies the relation (2) has been adopted with an assumption that the peak pressure arriving as a result of impact of shock wave will be equivalent to that of the pressure produced by the explosion of TNT and its value after the range of 3 meters distance apart. Here we introduce characteristic mass ratio (z) parameter which is defined as;

$$z = \frac{m}{\rho c T} \quad (4)$$

Time decay factor T has been used for solving equation of motion [Eqn. (1)]. Substituting of equation of (4) in equation (1), we get

$$\frac{dV}{dt} + \frac{1}{z \cdot T} V(t) = \frac{2P_m}{m} \cdot \exp\left(-\frac{t}{T}\right) \quad (5)$$

The solution of Equation (5) with initial condition $V(0) = 0$ [It is assumed that velocity exist only after impacting on the particle, hence before impact the velocity is assumed as zero] can be written as

$$V(t) = \frac{2P_m}{\rho c(z-1)} \left(\exp\left(-\frac{t}{z \cdot T}\right) - \exp\left(-\frac{t}{T}\right) \right) \quad (6)$$

Re Substituting of eqn. (4) in eqn. (6), $V(t)$ simplifies to

$$V(t) = \frac{2P_m \cdot T}{\rho(L - c \cdot T)} \left(\exp\left(-\frac{c \cdot t}{L}\right) - \exp\left(-\frac{t}{T}\right) \right) \quad (7)$$

A. Estimation of Maximum Velocity

The concept of maximum velocity has been introduced here in analogy with the formation of the temporary cavity in the ground at the time of explosive particles impact and subsequent penetration mechanism. In order to know, this formation of maximum and minimum cavitations, subsequent knowledge velocity and pressure are required. Having known the detailed aspect of the cavities, one can calculate the damage caused by the explosive particles to the surrounding tissues being attacked. In view of its importance maximum velocity is calculated by taking $\frac{dV}{dt} = 0$ and solving for the time ' t' '. The simplified form of t' (t_v) for the maximum velocity can be written as,

$$t_v = \frac{L \cdot T}{L - c \cdot T} \ln \left(\frac{L}{c \cdot T} \right) \quad (8)$$

Maximum velocity V_{max} be computed by the substituting ' t' ' in eqn. (7)

$$V_{max} = \frac{2P_m}{\rho \cdot c} \left(\frac{L}{c \cdot T} \right)^{\frac{L}{c \cdot T - L}} \quad (9)$$

B. Estimation of Acceleration and Maximum Acceleration

Acceleration (a) and maximum acceleration (a_{max}) have been derived in a similar way to that of velocity.

$$a(t) = \frac{2P_m}{\rho(L - c \cdot T)} \left(-\frac{c \cdot T}{L} \cdot \exp\left(-\frac{c \cdot t}{L}\right) - \exp\left(-\frac{t}{T}\right) \right) \quad (10)$$

$$t_a = \frac{2L \cdot T}{L - c \cdot T} \ln \left(\frac{L}{c \cdot T} \right) \quad (11)$$

$$a_{max} = \frac{2P_m \cdot cT}{\rho L(L - c \cdot T)} \left(\left(\frac{L}{c \cdot T} \right)^{\frac{L+c \cdot T}{c \cdot T - L}} - \left(\frac{L}{c \cdot T} \right)^{\frac{2c \cdot T}{c \cdot T - L}} \right) \quad (12)$$

where t_a is the time taken for acceleration to reach maximum value.

C. Estimation of Shock pressure

Estimation of shock pressure which is one of the most important parameters in the explosive dynamics has been obtained by the following approach. We have [Ref. [7]]

$$Pressure = \frac{F}{A} = \rho \cdot L \cdot a$$

Where F is the force, A is an area. Using relation for acceleration from equation (10), the relation for shock pressure (P^*) can be estimated as:

$$P^* = \frac{2LP_m}{(L - c \cdot T)} \left(-\frac{c \cdot T}{L} \cdot \exp\left(-\frac{c \cdot t}{L}\right) - \exp\left(-\frac{t}{T}\right) \right) \quad (13)$$

III. RESULTS

The flow parameters such as velocity, maximum velocity, shock pressure, acceleration and maximum acceleration have been computed. The data required for the computation purpose has been taken from Refs. [[1]-[3], [8] and [9]]. The results have been shown in Figs. 1-5.

TABLE I
 DATA OF FLOW PARAMETERS

Symbol	Description	Range	Typical Value
P_m	Peak pressure (TNT)	200 – 300	300 <i>pa</i>
r	Distance of detonation	2 – 9	3 <i>m</i>
T	Time decay	2 – 15	3 <i>s</i>
t	Arrival time	0 – 2	0.01 <i>s</i>
ρ	Density	2000 – 6000	3000 <i>kg/m</i> ³
L	Diameter of detonation	0.5 – 5	2 <i>m</i>
c	Sound speed in medium	3000 – 6000	4000 <i>m/s</i>

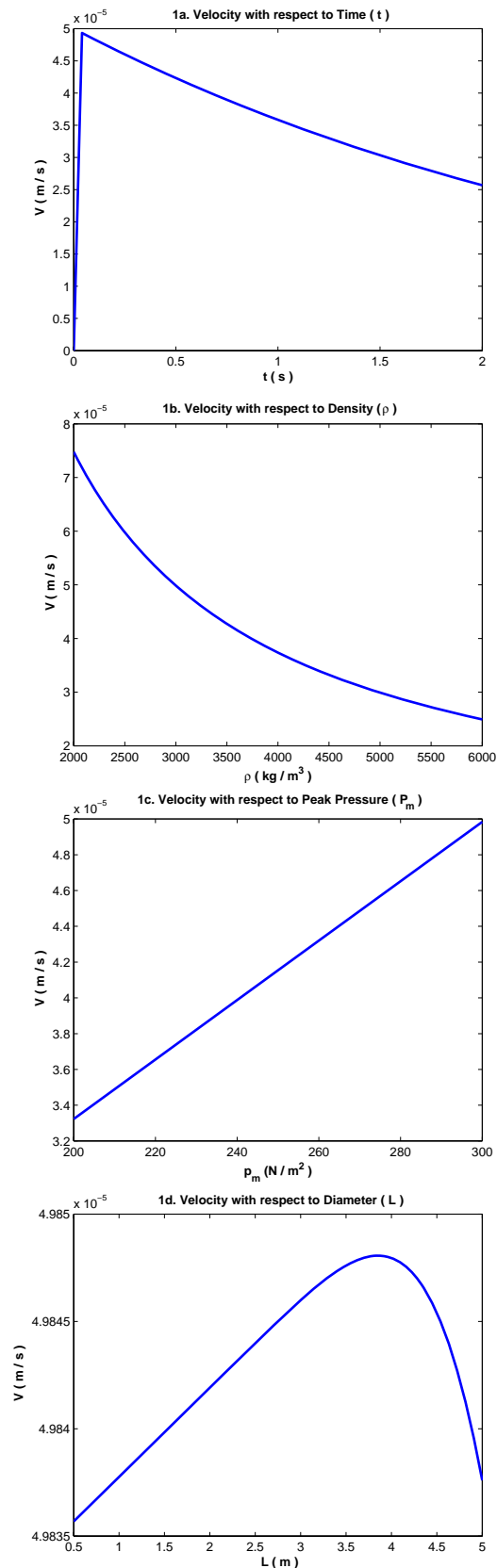


Fig. 1. Variation of velocity with respect to t, ρ, p_m and L

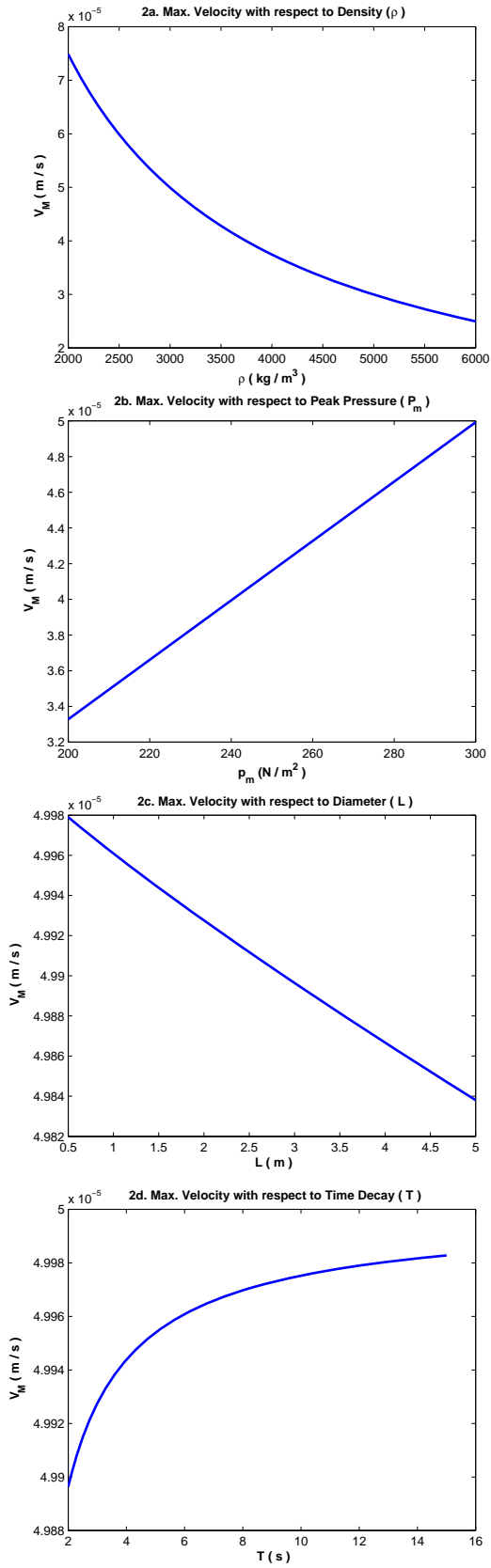


Fig. 2. Variation of maximum velocity with respect to ρ , P_m , L and T

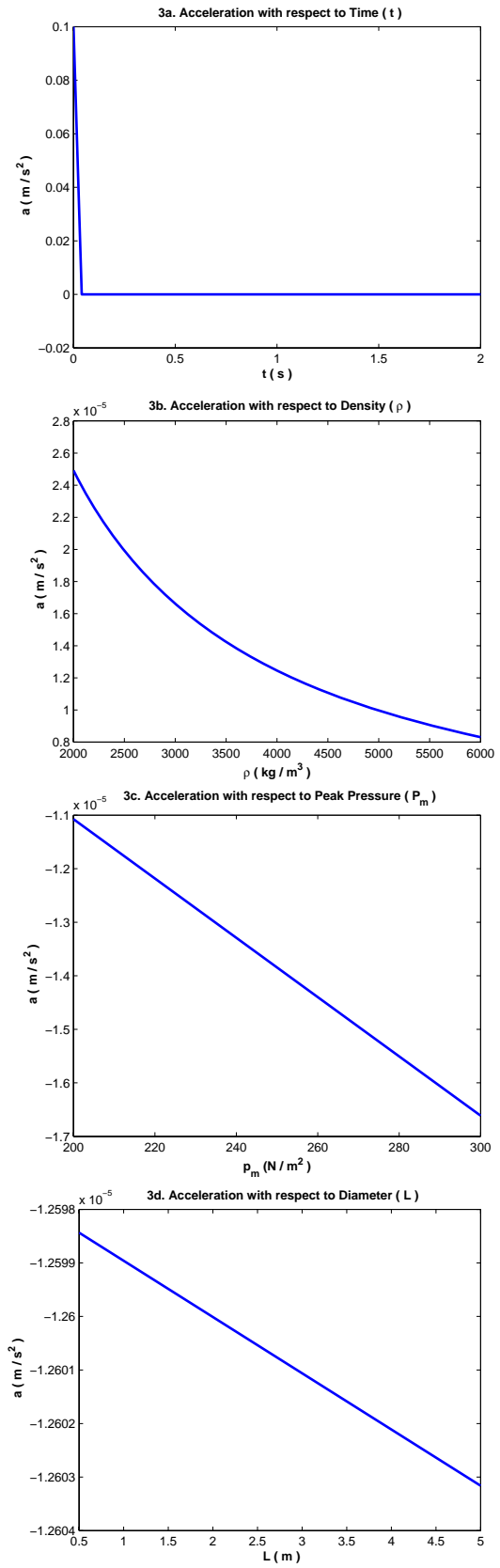


Fig. 3. Variation of acceleration with respect to t , ρ , P_m and L

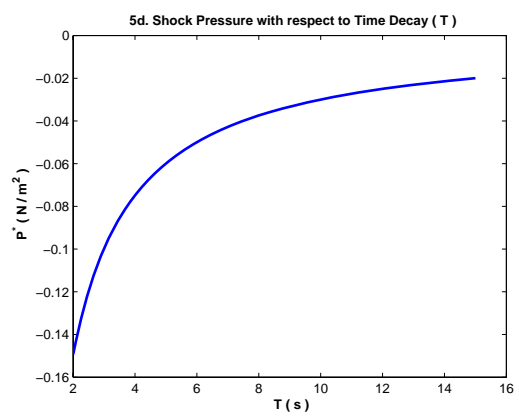
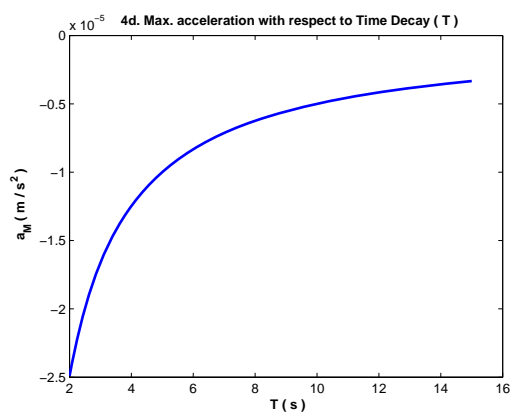
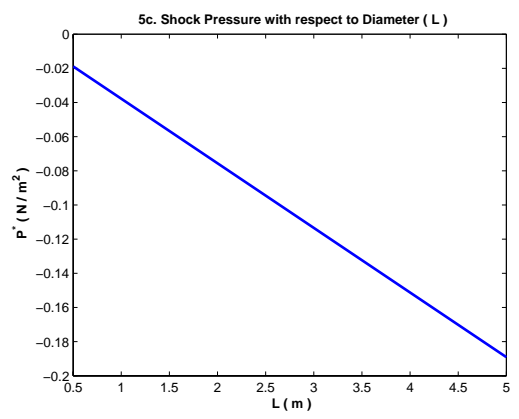
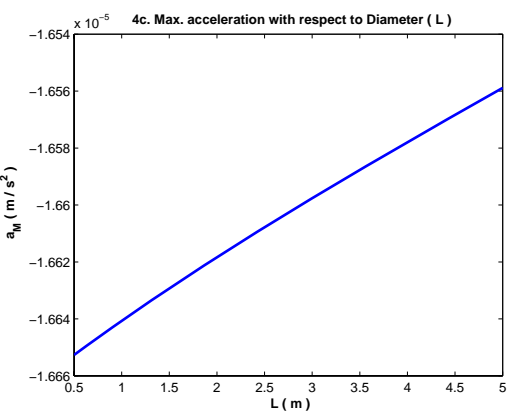
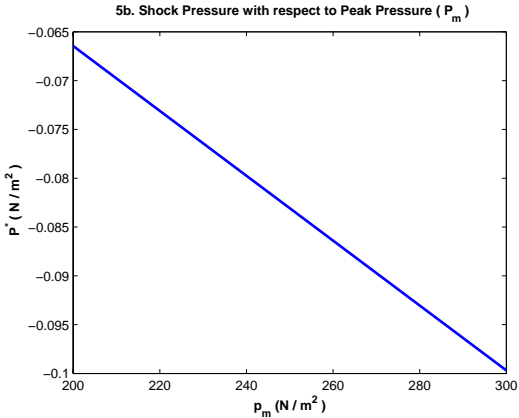
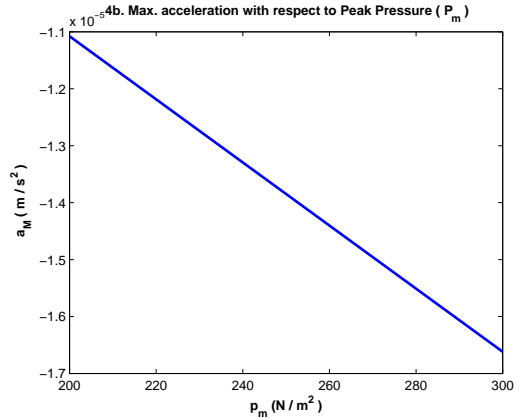
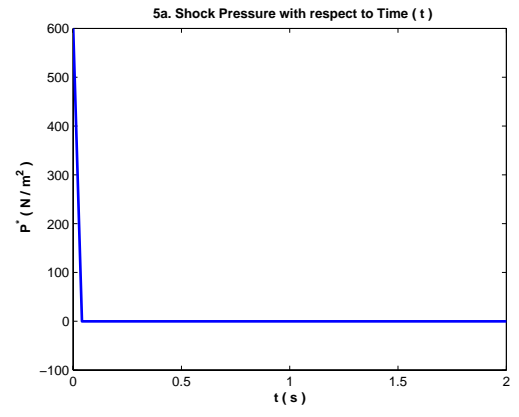
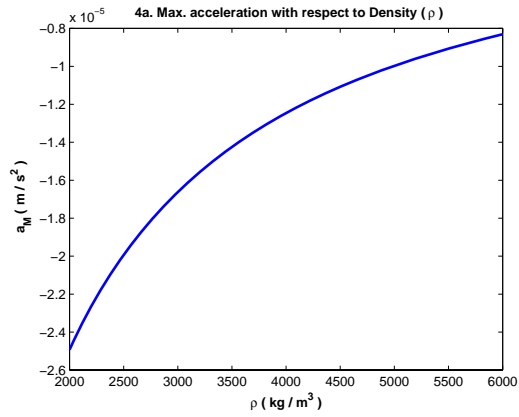


Fig. 4. Variation of maximum acceleration with respect to ρ , P_m , L and T

Fig. 5. Variation of Shock pressure with respect to t , P_m , L and T

IV. CONCLUSION

Mathematical model has been developed for the estimation of shock pressure in the present studies. The shock pressure is the pressure arriving on the particles as result of an explosion. The literature study indicates that, the experimental values are available for the pressure which is estimated at the distance of 100m [250 - 260 kb] however, the author could not locate any mathematical relation for the exact computation of the value at the site of an explosion. The initial detonation pressure which is required for the mathematical and computational approach has been taken from the Refs. [[1], [2] and [6]] which has been applied for the underwater explosions. The computed results have found to agree with the physical observation and science of deformation. The rupture mechanism of the tissue needs the basic values of the estimated parameters in the present studies. In view of the same, the present results can be used for the estimation of the damage which can cause to the particular organ once the initial parameters are estimated. There is a need to refine the peak pressure assumed via underwater explosions in the present studies. This will greatly enhance in estimation of the damage it can cause to the affected portion of the tissue.

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