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Exact Evaluation Method for Error Performance Analysis of Arbitrary 2-D Modulation OFDM Systems with CFO

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***Abstract*—**Orthogonal frequency division multiplexing (OFDM) has developed into a popular scheme for wideband digital communications used in consumer applications such as digital broadcasting, wireless networking and broadband internet access. In the OFDM system, carrier frequency offset (CFO) causes intercarrier interference (ICI) which significantly degrades the system error performance. In this paper we provide an exact evaluation method for error performance analysis of arbitrary 2-D modulation OFDM systems with CFO, and analyze the effect of CFO on error performance.

***Keywords*—**Carrier frequency offset, Probability of error, Inter-channel interference, Orthogonal frequency division multiplexing

I. Introduction

Wireless communication technologies have been increasingly used in consumer devices such as smartphones, digital TV, and wireless internet terminals. Orthogonal frequency division multiplexing (OFDM) has shown many advantages, including low-complexity equalization in multipath fading channels and adaptability/scalability to channel conditions, enabling a number of wireless communication standard systems such as DAB, DVB-T, WLAN (802.11 a/g/n), WPAN (802.15.3a), WMAN (802.16 a/d/e/m), MBWA (802.20), and LTE.

Various impediments in wireless systems degrade communication between transmitters and receivers. The Doppler effect arises from relative motion between transmitters and receivers and from the frequency instability of oscillators at transmitters or receiver; inevitably, it causes carrier frequency offset (CFO). The presence of CFO destroys the orthogonality among subcarriers; thus, resulting in inter-channel interference (ICI) [1]-[4]. Consequently, CFO is one of the chief detriments to system error performance.

Accurate analysis of the error probability of OFDM systems with CFO is an important topic, and many studies have recently been added to the literature [5]-[9]. Zhao and Haggman presented an exact BER expression of BPSK in an additive white Gaussian noise (AWGN) channel using moments of ICI distribution [5]; Sathananthan and Tellambura provided exact SER expressions, not a closed form, for BPSK, QPSK, and 16-QAM in an AWGN channel using the characteristic function of the ICI [6]; Rugini and Banelli derived BER expressions, not a closed form, for QPSK and 16-QAM in frequency-selective Rician and Rayleigh fading channels [7]. More recently, Dharmawanasa, Rajatheva, and Minn discussed closed-form SER/BER expressions, useful for lower CFO values, for BPSK and QPSK in AWGN, frequency-flat, and selective Rayleigh fading channels [8]; Mahesh and Chaturvedi derived the closed-form BER expression, valid for all CFO values, for BPSK in frequency-flat and selective Rayleigh fading channels [9]. The results of existing methods are exact, but are limited to BPSK, QPSK, and/or 16-QAM schemes. Although the high-order modulation schemes with modulation order greater than 16 have been adopted as standard modulation schemes in wireless communication systems such as WLAN (802.11 a/g/n), WMAN (802.16 a/d/e/m), MBWA (802.20), and LTE, an exact evaluation method for error performance analysis in a closed form for the high-order modulated OFDM signals with CFO has not yet been addressed and analyzed even in frequency-flat fading channels.

In this paper, we provide an exact evaluation method for error performance analysis of an arbitrary 2-D modulated OFDM signal with CFO. Our method is presented in a closed form including the 2-D Gaussian Q-function, based on the method of [10]. We first analyze the effect by CFO on the received OFDM symbol. Then, using the method of [10], we propose a general method to exactly evaluate the error probability of an arbitrary 2-D modulated OFDM signal with CFO. To verify the validity of the provided method, we also conduct computer simulations and analyze their results.

II. Exact Error Probability of an Arbitrary 2-D Modulated OFDM Signal with CFO

Assume that $D_k \in \mathbf{t\_s} = [t\_s_1\ t\_s_2\ \ldots\ t\_s_M]$ is the complex symbol modulated by an arbitrary 2-D M-ary modulator, transmitted on subcarrier k in the OFDM system, where $\mathbf{t\_s}$ is a set of M signal points, $t\_s_i = \zeta_i \sqrt{E_s} e^{j\psi_i}, i = 1, 2, \ldots, M$ transmitted through the OFDM system and the components. Then D_k can be defined as

$$D_k = \zeta_{i,k} \sqrt{E_s} e^{j\psi_{i,k}},\ k = 0, 1, \ldots, N_s - 1,\ \ i = 1, 2, \ldots, M \qquad (1)$$

where $\zeta_{i,k}\sqrt{E_s}$ and $\psi_{i,k}$ are the amplitude and phase of the i-th

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signal point transmitted on subcarrier *k*, E_s is the average symbol energy, $\zeta_{i,k}$ is a scale factor which varies with the position of the signal point, and N_s is the number of subcarriers.

The complex received symbol Z_k passing through the FFT block in OFDM system with CFO can be expressed as [6]

$$Z_k = C_0 D_k + \sum_{l=0, l\neq k}^{N_s-1} C_{l-k} D_l + N_{I\_k} + jN_{Q\_k}, \; k = 0,1,\ldots,N_s - 1 \quad (2)$$

where N_{k_I} and N_{k_Q} are the noise components of subcarrier *k* on I/Q branches, which have the joint Gaussian distribution with zero mean, $E[N_{k\_I}^2] = E[N_{k\_Q}^2] = \sigma^2$, and $E[N_{k\_I} N_{k\_Q}] = 0$, where $E[\cdot]$ is the statistical expectation. The ICI coefficient C_k is given by [6]

$$C_k = \frac{\sin\left(\pi[k+\varepsilon]\right)}{N_s \sin\left(\pi(k+\varepsilon)/N_s\right)} \exp\left[j\pi\left(1 - \frac{1}{N_s}\right)(k+\varepsilon)\right] \quad (3)$$

where ε is the normalized frequency offset. Substituting (1) into (2) yields

$$Z_k = S_{k\_I} + jS_{k\_Q} + N_{k\_I} + jN_{k\_Q}, \; k = 0,1,\ldots,N_s - 1 \quad (4)$$

where S_{k_I} and S_{k_Q} are the received signal components on I/Q axes, respectively, expressible as

$$\begin{aligned} S_{k\_I} &= \zeta_{i,k}\sqrt{E_s}\left(Re(C_0)\cos\psi_{i,k} - Im(C_0)\sin\psi_{i,k}\right) \\ &+ \sum_{l=0,l\neq k}^{N_s-1}\left[\zeta_{i,l}\sqrt{E_s}\left(Re(C_{l-k})\cos\psi_{i,l} - Im(C_{l-k})\sin\psi_{i,l}\right)\right] \\ S_{k\_Q} &= \zeta_{i,k}\sqrt{E_s}\left(Im(C_0)\cos\psi_{i,k} + Re(C_0)\sin\psi_{i,k}\right) \\ &+ \sum_{l=0,l\neq k}^{N_s-1}\left[\zeta_{i,l}\sqrt{E_s}\left(Im(C_{l-k})\cos\psi_{i,l} + Re(C_{l-k})\sin\psi_{i,l}\right)\right] \end{aligned} \quad (5)$$

where $Re(C_k)$ and $Im(C_k)$ are the real and imaginary parts of C_k. Note that the second terms of S_{k_I} and S_{k_Q} induce the ICI: the complex symbols transmitted on $N_s - 1$ subcarriers interfere with the complex symbol transmitted on subcarrier *k*. Fig. 1 shows the geometry of the correct decision region for the received signal r_s_1 is R_{s_1}, when the signal transmitted on subcarrier *k* is t_s_1. CFO leads to ICI which causes distortion that shifts the received signal point on the constellation. I/Q noise components (N_{k_I}, N_{k_Q}) added to the transmitted signal also change the position of the received signal points, r_s_i, $i = 1,2,\ldots,M$.

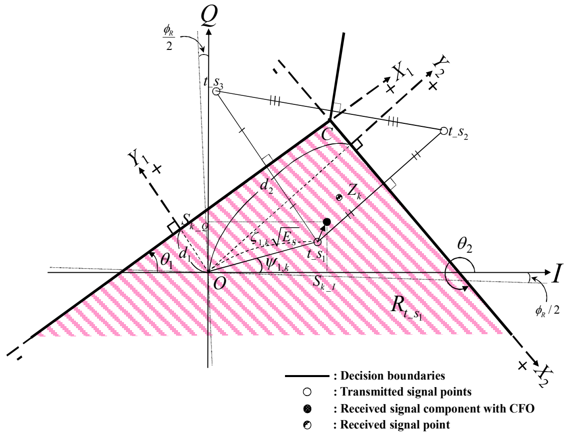

Fig. 1 Decision region and boundaries for ternary signal points

To obtain the conditional probability $P_k\{r\_s_1 \in R_{s_1} \mid S_t = t\_s_1\}$ that the signal point r_s_1 received through subcarrier *k* falls into the region R_{s_1}, we use the coordinate rotation and shifting technique well explained in [10] as follows:

$$\begin{bmatrix} X_j \\ Y_j + d_j \end{bmatrix} = \begin{bmatrix} \cos\theta_j & \sin\theta_j \\ -\sin\theta_j & \cos\theta_j \end{bmatrix} \begin{bmatrix} I \\ Q \end{bmatrix}, \quad j = 1,2 \quad (6)$$

where $d_j, j = 1,2$ are the distances from the origin to the $X_j, j = 1,2$ axes which denote the decision boundaries; I and Q have joint Gaussian distribution with $E[I] = S_{k\_I}$, $E[Q] = S_{k\_Q}$, $Var[I] = Var[Q] = \sigma^2$, and $COV[I\,Q] = 0$ where $Var[\cdot]$ and $COV[\cdot]$ denote the variance and covariance, respectively.

Rotational transformation (6) creates new axes Y_1 and Y_2, rotated $-\theta_1$ and θ_2 from Q axis and having joint Gaussian probability density function $f(y_1, y_2, \rho_{Y_1Y_2})$ with $E[Y_j] = S_{k\_Q}\cos\theta_j - S_{k\_I}\sin\theta_j - d_j, j = 1,2$, $Var[Y_j] = \sigma^2$, and $\rho_{Y_1Y_2} = \cos(\theta_1 - \theta_2)$ where $\rho_{Y_1Y_2}$ is the correlation coefficient between Y_1 and Y_2.

Consequently, the conditional probability $P_k\{r\_s_1 \in R_{t\_s_1} \mid S_t = t\_s_1\}$ can be obtained as [10]

$$\begin{aligned} P_k\left\{r\_s_1 \in R_{t\_s_1} \mid S_t = t\_s_1\right\} &= \int_{-\infty}^{0}\int_{-\infty}^{0} f\left(y_1, y_2, \rho_{Y_1Y_2}\right) dy_2 dy_1 \\ &= Q\left(\frac{E[Y_1]}{\sqrt{Var[Y_1]}}, \frac{E[Y_2]}{\sqrt{Var[Y_2]}}; \rho_{Y_1Y_2}\right), \end{aligned} \quad (7)$$

where $Q(x, y; \rho)$ is the 2-D joint Gaussian Q-function [11], [12].

When a symbol vector **x** is a set of baseband signals transmitted through $N_s - 1$ subcarriers causing ICI in the receiver, since each component of $\mathbf{x}$ has one of M signals $t\_s_i = \zeta_i \sqrt{E_s} e^{j\psi_i}$, $i = 1, 2, ..., M$, there are ${}_M\Pi_{N_s-1}$ possible symbol vectors $\mathbf{x}_g, g = 1, 2, 3, ..., {}_M\Pi_{N_s-1}$ with the length of $N_s - 1$ as follows:

$$\mathbf{x}_g = \left[x_{g,1}\, x_{g,2}\, x_{g,3} \ldots x_{g,N_s-1} \right], g = 1, 2, 3, ..., {}_M\Pi_{N_s-1} \quad (8)$$

where $x_{g,k} = \zeta_{i,k} \sqrt{E_s} e^{j\psi_{i,k}}, k = 1, 2, ..., N_s - 1$. Therefore, to obtain the exact error probability, we should average the error probabilities derived over all possible cases [6].

The exact average SER for a signal point t_s_1 transmitted on subcarrier k can be written as

$$P_{ser\_t\_s_1} = \frac{1}{{}_M\Pi_{N_s-1}} \sum_{g=1}^{{}_M\Pi_{N_s-1}} \left[\left(1 - P_k \left\{ r\_s_1 \in R_{t\_s_1} \mid S_t = t\_s_1 \right\} \right) \cdot P\left\{ t\_s_1 \right\} \right]$$
$$= \frac{1}{{}_M\Pi_{N_s-1}} \sum_{g=1}^{{}_M\Pi_{N_s-1}} \left[\left(1 - Q\left(\frac{E[Y_1]}{\sqrt{Var[Y_1]}}, \frac{E[Y_2]}{\sqrt{Var[Y_2]}}; \rho_{Y_1Y_2} \right) \right) \cdot P\left\{ t\_s_1 \right\} \right] \quad (9)$$

where $P\{t\_s_1\}$ is the a priori probability for the transmitted signal point.

In general, the decision region of a transmitted signal point is a polygon that may be either closed or open [10], [13], and the decision region can be expressed as a linear combination of the basic shapes [10], [14]. Therefore, an exact error probability expression for a signal point with the polygonal decision region can be obtained by using the probability of (9). And the exact BER expression for an arbitrary 2-D modulated OFDM signal with CFO on subcarrier k can be easily obtained by using the result of [15] and extensions of the SER and BER expressions to various frequency-flat fading channels are straightforward applying the methods of [16]-[18] to the expressions.

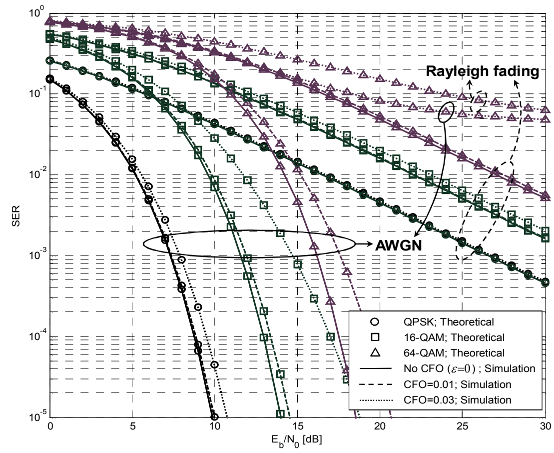

Fig. 2 BER for QPSK, 16-QAM, and 64-QAM-based-OFDM system with CFO ($\varepsilon = 0, 0.01, 0.03$) in AWGN and frequency-flat Rayleigh fading channels

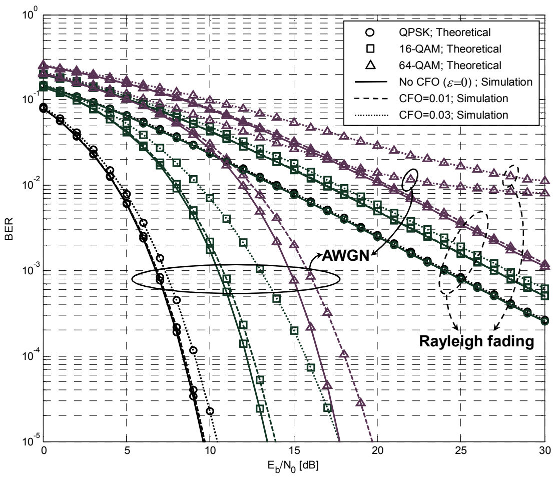

Fig. 3 BER for QPSK, 16-QAM, and 64-QAM-based-OFDM system with CFO ($\varepsilon = 0, 0.01, 0.03$) in AWGN and frequency-flat Rayleigh fading channels

III. NUMERICAL RESULTS

We consider QPSK, 16-QAM, and 64-QAM schemes that have been used mainly in a number of wireless devices in wireless communication/broadcasting systems such as Mobile WiMax, LTE, and DVB-T. Figs. 2 and 3 show the SER and BER curves, respectively, for QPSK, 16-QAM, and 64-QAM OFDM systems with $N_s = 64$ over AWGN and frequency-flat Rayleigh fading (m=1) channels when CFO values are 0, 0.01, and 0.03. From Figs. 2 and 3, we can confirm that as the order of modulation increases, the effect of CFO on the error performance becomes serious. When CFO=0.03, a 64-QAM OFDM signal is degraded to the point that error floors occur even in an AWGN channel. In addition, Figs. 2 and 3 show excellent matches between the results obtained from our exact expressions and computer simulations.

IV. CONCLUSIONS

We have provided an exact evaluation method, in a closed-form including the 2-D Gaussian Q-function, for the error performance analysis of arbitrary 2-D modulation OFDM systems in the presence of CFO in AWGN and fading channels. We also analyzed the effect of CFO on error performance. It is very useful in numerical evaluations of exact SER and BER for various cases of practical interest in communication devices adopting an arbitrary 2-D modulation format in OFDM systems.

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