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The Application of Real Options to Capital Budgeting

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***Abstract*—**Real options theory suggests that managerial flexibility embedded within irreversible investments can account for a significant value in project valuation. Although the argument has become the dominant focus of capital investment theory over decades, yet recent survey literature in capital budgeting indicates that corporate practitioners still do not explicitly apply real options in investment decisions. In this paper, we explore how real options decision criteria can be transformed into equivalent capital budgeting criteria under the consideration of uncertainty, assuming that underlying stochastic process follows a geometric Brownian motion (GBM), a mixed diffusion-jump (MX), or a mean-reverting process (MR). These equivalent valuation techniques can be readily decomposed into conventional investment rules and "option impacts", the latter of which describe the impacts on optimal investment rules with the option value considered. Based on numerical analysis and Monte Carlo simulation, three major findings are derived. First, it is shown that real options could be successfully integrated into the mindset of conventional capital budgeting. Second, the inclusion of option impacts tends to delay investment. It is indicated that the delay effect is the most significant under a GBM process and the least significant under a MR process. Third, it is optimal to adopt the new capital budgeting criteria in investment decision-making and adopting a suboptimal investment rule without considering real options could lead to a substantial loss in value.

***Keywords*—**real options, capital budgeting, geometric Brownian motion, mixed diffusion-jump, mean-reverting process

I. Introduction

THE literature on capital budgeting suggests two important facts in theory and practice: first, conventional capital budgeting techniques are shown to have various theoretical shortcomings yet still have widespread applications in practice ; second, real options techniques are often considered as relatively sophisticated analysis tools, yet most firms do not make explicit use of real options techniques to evaluate capital investments.

This paper aims to bridge the theory-practice gap by translating real options valuation into current capital budgeting practices. The research purpose is therefore two-fold. On one hand, we would like to explore how real options decision criteria can be transformed into equivalent capital budgeting criteria such as NPV, profitability index, hurdle rate, and (discounted) payback under the consideration of uncertainty. On the other hand, we would like to propose heuristic investment rules in terms of capital budgeting practices to proxy for the inclusion of real options valuation. We then demonstrate how rules of thumb under incomplete information can approximate managerial flexibility into capital budgeting techniques across different types of projects and provide results which are close to optimal investment decisions.

The literature on bridging the real options approach and capital budgeting techniques are seen in Dixit [5], Ingersoll and Ross[8], Boyle and Guthrie [2], McDonald [12], and Wambach [18]. Dixit [5] suggests the optimal investment rule with the option of waiting to invest can be expressed as a constant hurdle rate for time-homogeneous cash flows in an infinite horizon framework. Ingersoll and Ross [8] argue that management should set corporate hurdles rates above the cost of capital to recognize the gains of waiting. McDonald [12] investigates whether arbitrage investment criteria such as hurdle rates and profitability indexes can proxy for the use of real options techniques. He finds that under the GBM assumption, a profitability index of 1.5 or alternatively a hurdle rate of 20% can provide a reasonable approximation to the optimal trigger across different characteristics of projects. Along the line of research, Boyle and Guthrie [2] and Wambach [18] propose a similar approach to equivalent investment rules for payback and hurdle rate under the option of waiting, assuming that the underlying process follows a GBM and that projects have time-homogenous cash flows. Compared to standard capital budgeting criteria, the modified investment rules tend to have a lower payback trigger or a higher hurdle rate trigger.

The rest of the paper is organized into the following sections: Section 2 illustrates how real options valuation can be approximated by capital budgeting techniques. Section 3 extends the idea from Section 2 and shows that conventional capital budgeting techniques can explicitly integrate the value of deferment options into become modified investment rules under alternative stochastic processes. Section 4 conducts a numerical analysis, which shows optimal investment rules under various stochastic processes. Also, a comparison of modified capital budgeting criteria under various stochastic processes is presented. Concluding remarks are given in Section 5.

II. Real Options and Capital Budgeting Criteria

The standard investment theory under uncertainty is to explore the optimal timing to pay an investment cost, , in return for an irreversible project whose value, , is a major source of uncertainty. Since the investment opportunity is normally assumed to exist infinitely in order to derive closed-form solutions, the investment timing problem turns out to be the optimal stopping problem in searching for the optimal investment trigger, . In the presence of options of waiting, is found to be greater than . In this section, we first derive modified capital budgeting criteria, e.g. NPV, profitability index, hurdle rate, and payback, in the explicit expressions of without specifying any particular stochastic process. We then assume a specific stochastic process such as GBM, mixed diffusion-jump, and mean reversion, to examine how the forms of modified capital budgeting criteria are influenced by the stochastic process.

In conventional capital budgeting, the NPV rule states that the investment should be undertaken when the NPV is greater than zero. However, this criterion only works in the absence of real options. Under uncertainty, when the project is allowed to delay, the modified NPV rule should justify the loss of option of waiting when launching the investment opportunity. This means that the actual costs of initiating the project are not only the investment cost but also the opportunity cost due to the loss of options. We use the superscript * to denote the modified investment rules. Therefore, the new NPV rule under the consideration of options of waiting should be modified as shown below:

$$NPV^* = V^* - I - F(V^*) \tag{1}$$

where $F(V^*)$ denotes the option value when project value equals V^*.

It is important to note that the modified NPV rule should be less than the conventional NPV rule for the reasons that $F(V^*)$ should be greater than zero when $V^* > V$.

Profitability Index, denoted by Π, is defined as the benefit/cost ratio or Tobin's "q" ratio associated with the project. In the absence of managerial flexibility, the project is undertaken when PI is greater than 1. Since the project is taken at the point of V^* under the consideration of real options, the new PI rule should be changed as follows:

$$\Pi^* = \frac{V^*}{I} \tag{2}$$

Π^*, by definition, can be interpreted as a unit optimal trigger and is greater than 1 due to $V^* > I$.

Now we suppose the project can generate infinite cash flows once the project is undertaken. Expressed by instantaneous time-homogeneous cash flows[1], π, the project value thus takes the form of the classic Gordon model:

$$V = E\left(\int_0^\infty \pi_s e^{(\alpha-\mu)s} ds\right) = \frac{\pi}{\mu-\alpha} \tag{3}$$

where α and μ denote growth rate and discount rate, respectively, and $\mu > \alpha$.

The convenience of the assumption of time-homogeneous cash flows provides a linear relationship between V and π, which ensures that V and π follow the same stochastic process with the same drift and volatility. Consequently, π is still allowed to fluctuate at the next instant as new information arrives. Equation (3) thus can be rearranged as follows:

$$\pi = V(\mu-\alpha) \tag{4}$$

In capital budgeting, the hurdle rate rule states that the project should be undertaken when the internal rate of return exceeds the arbitrary hurdle rate. With the time-homogeneous cash flows assumption, the hurdle rate rule can be transformed into the equivalent cash flow rule. Let γ denote hurdle rate. By definition, the hurdle rate rule must satisfy the following relationship:

$$\pi = I(\gamma-\alpha) \tag{5}$$

By equating Equation (4) and (5), we obtain the expression of γ as follows:

$$\gamma = \Pi(\mu-\alpha)+\alpha \tag{6}$$

If the project is taken at the point of $\Pi = 1$, then the hurdle rate is equal to the discount rate. However, if the project is taken at any point of $\Pi > 1$, the hurdle rate is greater than the discount rate.[2]

To derive the modified investment rules for hurdle rate and cash flows, we first substitute Equation (3) into Equation (2) and rearrange the terms for π^*. The modified cash flow rule, π^* can be expressed in terms of V^* or Π^* as follows:

$$\pi^* = V^*(\mu-\alpha) = \Pi^* I(\mu-\alpha) \tag{7}$$

With Π^* taking the place of Π in Equation (6), the modified hurdle rate rule can be derived as follows:

$$\gamma^* = \Pi^*(\mu-\alpha)+\alpha \tag{8}$$

Note that the modified investment rules of π^* and γ^* should be greater than the conventional investment rules of π and γ in the existence of positive option values.

The payback period rule has been one of the commonly used capital budgeting criteria.[3] Payback period is referred to as the time horizon in which the sum of expected cash flows returns the investment cost. In general, the payback rule favors the projects with shorter payback periods although the potential benefit of this concept is heavily disputed in the literature. One argument in support of the use of the payback rule is that to those firms which are short of capital, the payback rule may help recover the initial investment cost earlier. If we express payback period by time-homogeneous cash flows π, the payback rule must satisfy the following condition:

$$I = \pi\int_0^P e^{\alpha s} ds \tag{9}$$

[1] Time-homogenous cash flows can be seen as an equivalent transformation of non-time-homogenous cash flows, given other parameters held the same. Suppose there are project A and B under consideration. Both projects have two years of time horizon. These two projects are similar in every way except that the former has cash flows of $10 in both year 1 and 2 while the latter has $11 in year 1 and $8.9 in year 2. At the discount rate of 10%, they have exactly the same project value. Therefore, time-homogenous cash flows can represent non-time-homogenous cash flows with the advantage of easy mathematical treatment due to the linearity between project value and cash flows. With this viewpoint accepted, the feature of mean reversion or jump which is normally modeled with project value can also be modeled with time-homogenous cash flows.

[2] Proof: Suppose the project is taken at $\Pi = 1 + z > 1$, where z denotes a number representing some arbitrary decision rule such that $z > 0$. By substituting $\Pi = 1 + z$ into $\gamma = \Pi(\mu-\alpha)+\alpha$, we obtain $\gamma = \mu + z(\mu-\alpha)$. Since $z > 0$ and $\mu > \alpha$ by design, we know $\gamma > \mu$.

[3] See Klammer [10], Klammer and Walker [11], Jog and Srivastava [9], Gilbert and Reichert [7], Bubsy and Pitts [3], and Arnold and Hatzopoulos [1].

where P denotes the payback period.[4]

Solving Equation (9) for P, we have the payback trigger as shown below:

$$P = \begin{cases} \dfrac{\ln\left[1+\dfrac{\alpha}{\Pi(\mu-\alpha)}\right]}{\alpha}, & \alpha \neq 0 \\ \dfrac{1}{\Pi\mu}, & \alpha=0 \end{cases} \tag{10}$$

To derive the modified payback rule under the consideration of options of waiting, we replace the profitability index Π in Equation (10) with the modified profitability index trigger Π^*. The new payback rule now becomes:

$$P^* = \begin{cases} \dfrac{\ln\left(1+\dfrac{\alpha}{\Pi^*(\mu-\alpha)}\right)}{\alpha}, & \alpha \neq 0 \\ \dfrac{1}{\Pi^*\mu}, & \alpha = 0 \end{cases} \tag{11}$$

Since Π^* is greater than Π, it is easy to see that the modified payback period is shorter than the conventional payback period. This means that under uncertainty and flexibility, one should defer investment until the market turns out to be favorable such that the project has a payback period not only shorter than the conventional payback period, P, but also the modified payback period, P^*.

One of the major critiques regarding the payback criterion is that the use of payback often ignores the time value of cash flows. The justification for this drawback is to introduce discount rate to become discounted payback, which is defined as the time horizon over which the present value of total expected cash flows equals the investment cost. Thus, the discounted payback criterion must satisfy the following condition:

$$I = \pi\int_0^{P^D} e^{(\alpha-\mu)s}ds \tag{12}$$

where P^D denotes the discounted payback period.
Equation (12) is solved as follows:

$$P^D = \frac{\ln\left[1+\dfrac{1}{\Pi-1}\right]}{\mu-\alpha} \tag{13}$$

With Π^* in Equation (2) in the place of Π in Equation (13), the modified discounted payback trigger is given below:

$$P^{D*} = \frac{\ln\left(1+\dfrac{1}{\Pi^*-1}\right)}{\mu-\alpha} \tag{14}$$

We have demonstrated how real option approach can be integrated into conventional capital budgeting rules. Equation (1), (2), (7), (8), (11), and (14) are the modified investment rules where the options of waiting are in place. To rationalize the benefits of options of waiting, management should defer investment until the project has a larger NPV than NPV^*, a higher profitability index than Π^*, a higher rate of returns than γ^*, higher expected cash flows than π^*, or alternatively a lower payback period than P^* or P^{D*}. Since V^* is greater than V, we can easily make a comparison between modified investment rules and conventional investment rules as shown in Table 1:

TABLE I THE COMPARISONS BETWEEN MODIFIED INVESTMENT RULES AND CONVENTIONAL INVESTMENT RULES

Modified Investment Rules	NPV^*	Π^*	π^*	γ^*	P^*	P^{D*}
Comparisons	$\wedge$	$\vee$	$\vee$	$\vee$	$\wedge$	$\wedge$
Conventional Investment Rules	NPV	Π	π	γ	P	P^D

Note: $\wedge$ = less than; $\vee$ = greater than

III. APPLYING REAL OPTIONS TO CAPITAL BUDGETING

In the section, the application of real options to capital budgeting under a specific stochastic process is discussed. The stochastic process of interest is geometric Brownian motion, mixed diffusion-jump, and mean-reverting process, as discussed below:

A. Modified Capital Budgeting Criteria under a GBM

Modern investment theory centers on searching for optimal investment trigger, V^*, such that the value of the investment opportunity, $F(V)$, is maximized. We assume that the project value, V, follows a GBM as follows:

$$dV = \alpha V dt + \sigma V dz \tag{15}$$

where σ and dz denote volatility and an increment of a standard Wiener process, respectively.

For a project whose value follows a GBM, the literature has shown that the solution of $F(V)$ is as follows:[5]

$$F(V;V^*) = AV^{b_1} \tag{16}$$

where $A = \left(V^* - I\right)V^{*-b_1}$ (17)

$$b_1 = \left(\frac{1}{2} - \frac{r-(\mu-\alpha)}{\sigma^2}\right) + \sqrt{\left(\frac{r-(\mu-\alpha)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{18}$$

At the maximum of option values, the optimal investment trigger equals the sum of investment cost and the value of investment opportunity, which is also called the value-matching condition. By substituting Equation (16) into the value-matching condition, V^* is solved as follows:

$$V^*_{GBM} = \left(\frac{b_1}{b_1-1}\right)I \tag{19}$$

where V^*_{GBM} denotes the optimal GBM trigger.

Substituting V^*_{GBM} in Equation (19) into the modified capital budgeting criteria in Equation (1), (2), (7), (8), (11), and (14), we can obtain explicit expressions in terms of b_1 as follows:

$$\Pi^*_{GBM} = 1 + \frac{1}{b_1 - 1} \tag{20}$$

[4] If π is non-time-homogeneous, the payback rule satisfies the following condition:

$$I = \int_0^P \pi_s e^{\alpha s} ds$$

[5] See McDonald and Siegel [13], Dixit [4], and Pindyck [14].

$$\pi^*_{GBM} = (\mu-\alpha)I + \frac{1}{b_1-1}(\mu-\alpha)I$$
(21)

$$\gamma^*_{GBM} = \mu + \frac{1}{b_1-1}(\mu-\alpha) \quad (22)$$

$$P^*_{GBM} = \begin{cases} \dfrac{\ln\left(1+\dfrac{b_1\alpha}{(b_1-1)(\mu-\alpha)}\right)}{\alpha}, & \alpha \neq 0 \\ \dfrac{1}{\mu} - \dfrac{1}{b_1\mu}, & \alpha = 0 \end{cases} \quad (23)$$

$$P^{D*}_{GBM} = \frac{\ln(b_1)}{\mu-\alpha} \quad (24)$$

Note that these modified investment rules in Equation (20) -(24) have an important implication regarding how the options of waiting impact on conventional investment rules. For a stochastic process like a GBM, we can easily decompose the modified investment rules into two terms, one of which represents the conventional investment rules and the other term stands for the "option impact", which accounts for the impact on these investment rules in the presence of options of waiting. For the modified investment rules Π^*_{GBM}, π^*_{GBM}, and γ^*_{GBM}, the option impacts are $1/(b_1-1)$, $(\mu-\alpha)I/(b_1-1)$, and $(\mu-\alpha)/(b_1-1)$, respectively. Note that Equation (20) stands for unit optimal investment trigger, which makes $1/(b_1-1)$ become unit option value at the point of V^*_{GBM}. As the option value gets larger, these option impacts have a more significant, positive influence on the modified investment rules. For the modified payback rules, the option impacts are not readily observed. However, in the case of zero growth, i.e., $\alpha = 0$, the option impact of P^*_{GBM} is derived to be $-1/b_1\mu$, which indicates a lower optimal payback when there is a higher option value. The parameter b_1 has a number of important properties. First, b_1 must be greater than 1 when there is a positive option value.[6] Second, since b_1 is in the denominator of V^*_{GBM}, b_1 is inversely correlated with V^*_{GBM}. Consequently, b_1 is inversely correlated with Π^*_{GBM}, π^*_{GBM}, and γ^*_{GBM}, and is positively correlated with P^*_{GBM} and P^{D*}_{GBM}.

B. Modified Capital Budgeting Criteria under a Mixed Diffusion-Jump

In this subsection, we extend the preceding analysis to the case in which project value (or cash flows) follows a mixed diffusion-jump process. This process is often specifically used to describe the situation in that the value of an investment opportunity can become worthless as potential competitors enter the market as first-movers.[7] In other words, the preemptive competitive effect may lead to the project value appropriated by the competitors, which thus can be characterized by a mixed diffusion-jump process.[8] A mixed diffusion-jump process is formalized as follows:

$$dV = \alpha V dt + \sigma V dz - V dq \quad (25)$$

where dq is the increment of a Poisson process with a mean arrival rate of λ and is expressed by

$$dq = \begin{cases} \phi & \text{with a probability of } \lambda dt \\ 0 & \text{with a probability of } 1-\lambda dt \end{cases} \quad (26)$$

where ϕ $(0 \le \phi \le 1)$ stands for the constant percentage of loss in V should the jump event, i.e. competitive arrivals, occur. Meanwhile, dq is assumed to be independent of dz, i.e., $E(dqdz) = 0$.

With the same boundary conditions as in the GBM model, McDonald and Siegel [13] and Dixit and Pindyck [6] have verified that if $\phi = 1$, the solutions of $F(V)$ and V^*_{MX} are exactly the same as Equation (16) and (19), respectively, except that b_1 is replaced with b_2.[9]

$$b_2 = \left(\frac{1}{2} - \frac{r-(\mu-\alpha)}{\sigma^2}\right) + \sqrt{\left(\frac{r-(\mu-\alpha)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} \quad (27)$$

The parameter b is similar to the parameter b_1 in the functional form except that the jump intensity λ gets added into the interest rate in the constant term. It is easy to see that b is equal to b_1 if $\lambda = 0$ and greater than b_1 if $\lambda > 0$. Since both b_1 and b_2 are inversely correlated with optimal investment trigger, for the same set of parameter values the relationship of $b_2 \ge b_1$ leads to the result of $V^*_{MX} \le V^*_{GBM}$, where the subscript MX denotes a mixed diffusion-jump process.

Since the optimal trigger under an MX process takes the same form as the one under a GBM with b_1 substituted by b_2, all the modified optimal investment rules are thus akin to those in the GBM model with b_2 in the place of b_1. The option impacts under an MX process are also similar to those under a GBM with b_1 replaced by b_2. Note that b_2 also bears the same properties as b_1. With other parameters held constant, the jump intensity is inversely correlated with V^*_{MX}. This means that a higher λ leads to a lower V^*_{MX} and, in turn, a lower Π^*_{MX}, π^*_{MX}, and γ^*_{MX}, and also a larger optimal payback of P^*_{MX} and P^{D*}_{MX}.

C. Modified Capital Budgeting Criteria under a Mean-Reverting Process

Dixit and Pindyck [6] introduce a specific mean-reverting process for the ease of deriving an analytical solution. The mean-reverting process can be expressed as the following form:

$$dV = \eta(\bar{V} - V)Vdt + \sigma V dz \quad (28)$$

where $\bar{V}$ and η denotes the long-run mean and the speed of mean reversion, respectively.

The solution of an investment opportunity under a mean-reverting process is given as follows:

[6] Since $V^* = \left(1+\frac{1}{b_1-1}\right)I$, $F(V^*) = \frac{I}{b_1-1}$. $\because \frac{F(V^*)}{I} = \frac{1}{b_1-1} > 0$, $\therefore b_1 > 1$.

[7] Trigeorgis [17] deals with the preemptive competitive effect by treating the competitors' actions as dividends which are the proportions of the project value appropriated by the competitors. His analysis is limited by the assumption that the erosion effect can be completely anticipated and quantified by the firm, which appears to be less realistic.

[8] See [13].

[9] If $\phi \neq 1$, the value of the investment opportunity is still the form of $F(V) = AV^b$. However, the solution needs to be found numerically together with the boundary conditions.

$$F(V;V_{MR}^*) = BV^\theta G(x;\theta,g) \tag{29}$$

where the subscript MR denotes a mean-reverting process,

$\theta = \frac{1}{2} + \frac{(\mu - r - \eta\bar{V})}{\sigma^2} + \sqrt{\left[\frac{(r-\rho+\eta\bar{V})}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}}$,

$x = \frac{2\eta}{\sigma^2}V$,

$g = 2\theta + \frac{2(r-\mu+\eta\bar{V})}{\sigma^2}$, and

$G(x;\theta,g) = 1 + \frac{\theta}{g}x + \frac{\theta(\theta+1)}{g(g+1)}\frac{x^2}{2!} + \frac{\theta(\theta+1)(\theta+2)}{g(g+1)(g+2)}\frac{x^3}{3!} + \cdots$.

Since $G(x;\theta,g)$ is an infinite confluent hypergeometric function, there is no closed-form solution for V_{MR}^* . The coefficient, *B*, and V_{MR}^* must be solved numerically together with the same boundary conditions as in the GBM model.

To obtain the modified investment rules under a MR process in the expression of Equation (29), we first equate both of the unit optimal triggers under an MR process and under a GBM process as follows:

$$\frac{B^*(V_{MR}^*)^\theta G^*}{I} = \frac{1}{b_1 - 1} \tag{30}$$

where B^* denotes the coefficient B at the point of V_{MR}^* and $G^* = G(x^*;\theta,g)$.

$$b_1 = \frac{I + B^*(V_{MR}^*)^\theta G^*}{B^*(V_{MR}^*)^\theta G^*} \tag{31}$$

We then substitute Equation (31) into the modified GBM investment rules in Equation (20)-(24) for the modified MR investment rules as follows:

$$\Pi_{MR}^* = 1 + \frac{B(V^*)^\theta G^*}{I} \tag{32}$$

$$\pi_{MR}^* = (\mu-\alpha)I + \left[B^*(V_{MR}^*)^\theta G^*\right](\mu-\alpha) \tag{33}$$

$$\gamma_{MR}^* = \mu + \frac{B^*(V_{MR}^*)^\theta G^*}{I}(\mu-\alpha) \tag{34}$$

$$P_{MR}^* = \begin{cases} \dfrac{\ln\left(1 + \dfrac{I\alpha}{\left[I + B(V^*)^\theta G^*\right](\mu-\alpha)}\right)}{\alpha}, & \alpha \neq 0 \\[2ex] \dfrac{1}{\mu} - \dfrac{\left[B(V^*)^\theta G^*\right]}{\left[I + B(V^*)^\theta G^*\right]\mu}, & \alpha = 0 \end{cases} \tag{35}$$

$$P_{MR}^{D*} = \frac{\ln\left[\dfrac{I + B^*(V_{MR}^*)^\theta G^*}{B^*(V_{MR}^*)^\theta G^*}\right]}{\mu - \alpha} \tag{36}$$

The modified investment rules under an MR process can also be decomposed into the conventional investment rules and the "options impacts". The option impacts for Π_{MR}^* , π_{MR}^* , and γ_{MR}^* are $\left[B(V^*)^\theta G^*\right]/I$, $\left[B^*(V_{MR}^*)^\theta G^*\right](\mu-\alpha)$, and $\left[B^*(V_{MR}^*)^\theta G^*\right](\mu-\alpha)/I$, respectively. The option impact for P_{MR}^* when $\alpha = 0$ is $-\left[B(V^*)^\theta G^*\right] / \left[I + B(V^*)^\theta G^*\right]\mu$. As the options of waiting becomes larger, the option impacts are more positively significant for Π_{MR}^* , π_{MR}^* , and γ_{MR}^* , and more negatively significant for P_{MR}^* .

IV. Numerical Analysis

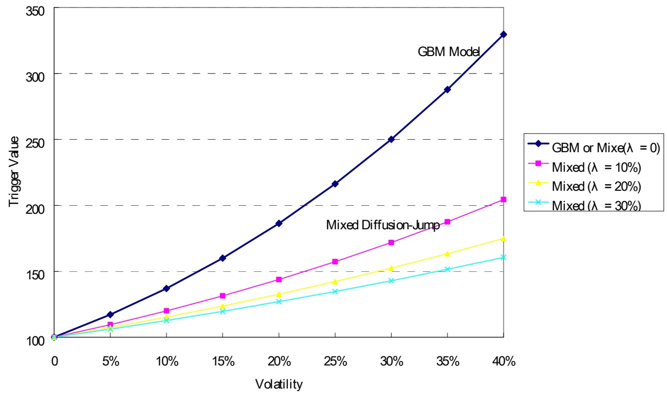
In the preceding subsections, the formulae for modified capital budgeting triggers under alternative processes are explicitly given. In this section, attention is directed to further investigate the relationships between modified investment rules and conventional investment rules under various stochastic processes by conducting a numerical comparative analysis based on a set of reasonable parametrical values. These stochastic processes of interest include GBM, mixed diffusion-jump, and mean-reverting processes. Since the solution of optimal investment trigger under a mean-reverting process is not closed-form, numerical analysis is necessary for comparing various investment triggers under alternative processes. Specifically, we focus on the effects on optimal investment rules of jumps and mean reversion.

As illustrated earlier, when jump size, ϕ , equals 1, the mixed diffusion-jump model has a similar formula for option value and investment trigger to the GBM model, with jump intensity, λ , added into the parameter, b_2 . Since λ is in the numerator of b_2 , it is easy to find out $b_1 \le b_2$ for the same parameters values and, in turn, $V_{GBM}^* \ge V_{MX}^*$. Given $I = 100$, $\mu - \alpha = \delta = 5\%$, $r = 5\%$, Figure 1 displays the effects of various jump intensities on optimal investment triggers across increasing instantaneous volatilities. An important finding evident from the diagram is that the inclusion of jumps into consideration lowers the optimal investment trigger, V_{MX}^* . This negative effect on trigger price is even significant with λ increased. This negative effect can be described in terms of modified investment rules: the modified rules such as Π_{MX}^* , γ_{MX}^* , and π_{MX}^* are less than the GBM counterparts, Π_{GBM}^* , γ_{GBM}^* , and π_{GBM}^* ; and the optimal paybacks, P_{MX}^* and P_{MX}^{D*} , are greater than the GBM paybacks, P_{GBM}^* and P_{GBM}^{D*} . The implication to management is that when the market is highly competitive or the first-mover advantage is significant, optimal investment decisions should be initiated sooner than those under a GBM process. Note that the GBM line actually represents the mixed-jump case in that $\lambda = 0$. The more significant the competitive preemptive effect is, the higher λ becomes and thus the sooner the investment should be launched.

Next, we compare V_{GBM}^* and V_{MR}^* , where the subscript MR denotes a mean-reverting process. Schwartz [16] and other real options studies on mean reversion argue that when mean reversion is ignored in project evaluation, investment tends to be excessively delayed due to a overpriced investment trigger. This argument can be confirmed in our numerical analysis as shown in Figure 2, which exhibits the sensitivity of trigger price by varying mean-reverting speed and instantaneous volatility, given $I = \bar{V} = 100$, $\mu - \alpha = \delta = 5\%$, and $r = 5\%$.[10] It is

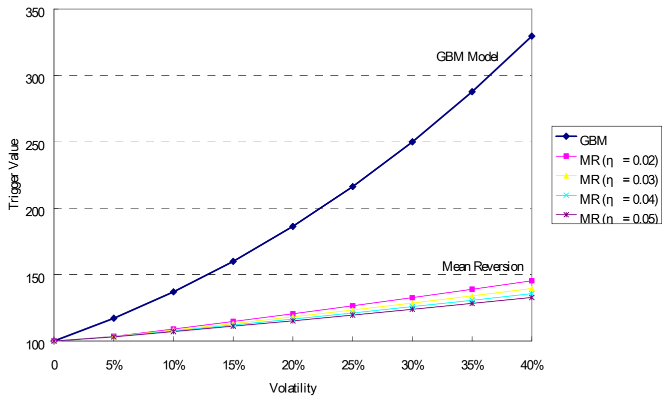
[10] Note that the long-run mean, $\bar{V}$, is set to be equal to 100 for two reasons. First, we are only interested in near at-the-money projects, i.e., $NPV \approx 0$, since real options are less influential in investment decision-making when involved with deep in-the-money or deep out-of-the-money projects. Second, as α may be substantially positive or negative in a disequilibrium setting, i.e., $V \neq \bar{V}$, to

apparent that the GBM trigger, V^*_{GBM}, is considerably greater than the mean reversion triggers, V^*_{MR}, even for a very slow mean-reverting speed. However, compared to V^*_{GBM}, V^*_{MR} is relatively insensitive to project volatility. This is possibly because mean reversion reduces the long-run volatility. Thus, even though instantaneous volatility is exactly equal in the models of both GBM and mean reversion, the long-run volatility under a mean reversion gets smaller as the mean-reverting speed becomes faster. [15]



Note: Other parameter values are $I = 100$, $\delta = 5\%$, $r = 5\%$.

Fig. 1 Optimal Triggers under GBM and Mixed Diffusion-Jump as a Function of Volatility



Note: Other parameter values are $I = 100$, $\bar{V} = 100$, $\delta = 5\%$, $r = 5\%$.

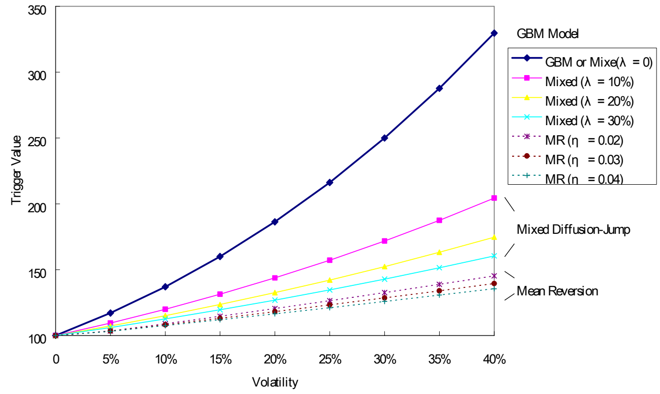
Fig. 2 Optimal Triggers under a GBM and a Mean-Reverting Process as a Function of Volatility

Figure 2 also indicates that when investment projects are characterized by a mean-reverting process, investments should be induced sooner than those under a GBM. Given the fact that V^*_{MR} is less than V^*_{GBM} for a reasonable set of parameters, it is obvious that the modified investment rules of mean reversion such as Π^*_{MR}, γ^*_{MR}, and π^*_{MR}, are less than the GBM counterparts

keep δ unchanged we need to adjust μ, which may produce a very unrealistic discount rate complicating the analysis.

while the optimal paybacks of mean reversion, P^*_{MR} and P^{D*}_{MR}, are greater than the GBM counterparts.

To compare the effects of jumps and mean reversion, Figure 1 and 2 are combined within the same frame into Figure 3, which displays the sensitivity of optimal trigger price to the changes in volatility among three alternative models. Figure 3 suggests that for a set of reasonable parameter values, both mean reversion and jumps have a significant influence on bringing down trigger price closer to the conventional triggers, indicating that investment under both cases should be launched sooner than that under a GBM. Furthermore, mean reversion may have a stronger power to induce investment than the competitive preemptive effect, given such a set of reasonable parameter values.



Note: Other parameter values are $I = 100$, $\bar{V} = 100$, $\delta = 5\%$, $r = 5\%$.

Fig. 3 A Comparison of Optimal Trigger Price under Alternative Processes

TABLE II THE COMPARISONS AMONG THE MODIFIED INVESTMENT RULES UNDER ALTERNATIVE PROCESSES

GBM Rules	Π^*_{GBM}	π^*_{GBM}	γ^*_{GBM}	P^*_{GBM}	P^{D*}_{GBM}
Comparisons	∨	∨	∨	∧	∧
MX Rules	Π^*_{MX}	π^*_{MX}	γ^*_{MX}	P^*_{MX}	P^{D*}_{MX}
Comparisons	∨	∨	∨	∧	∧
MR Rules	Π^*_{MR}	π^*_{MR}	γ^*_{MR}	P^*_{MR}	P^{D*}_{MR}
Comparisons	∨	∨	∨	∧	∧
Conventional Rules	Π	π	γ	P	P^D

Note: ∧ = less than; ∨ = greater than

To summarize the finding of this subsection, a comparison among the modified investment rules under alternative processes is presented in Table 2. Since the option impact under a GBM process is the most significant among three stochastic processes of interest, the GBM rules such as Π^*_{GBM},

π^*_{GBM}, and γ^*_{GBM} are thus larger than any of the MX and MR counterparts, and the GBM paybacks such P^*_{GBM} and P^{D*}_{GBM} are lower than the MX or MR payback rules, given a set of reasonable parameter values.

V. Conclusions

Real options theory suggests that managerial flexibility embedded within irreversible investments can account for a significant value in project valuation. Although the argument has become the dominant focus of capital investment theory over decades, yet recent survey literature in capital budgeting indicates that corporate practitioners still do not explicitly apply real options in investment decisions. In this paper, we explore how real options decision criteria can be transformed into equivalent capital budgeting criteria under the consideration of uncertainty, assuming that underlying stochastic process follows a geometric Brownian motion, a mixed diffusion-jump, or a mean-reverting process. These equivalent valuation techniques can be readily decomposed into conventional investment rules and "option impacts", the latter of which describe the impacts on optimal investment rules with the option value considered. Based on numerical analysis and Monte Carlo simulation, three major findings are derived. First, it is shown that real options could be successfully integrated into the mindset of conventional capital budgeting. Second, the inclusion of option impacts tends to delay investment. It is indicated that the delay effect is the most significant under a GBM process and the least significant under a MR process. Third, it is optimal to adopt the new capital budgeting criteria in investment decision-making and adopting a suboptimal investment rule without considering real options could lead to a substantial loss in value.

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