

Fast Complex Valued Time Delay Neural Networks

Hazem M. El-Bakry and Qiangfu Zhao

Abstract—Here, a new idea to speed up the operation of complex valued time delay neural networks is presented. The whole data are collected together in a long vector and then tested as a one input pattern. The proposed fast complex valued time delay neural networks uses cross correlation in the frequency domain between the tested data and the input weights of neural networks. It is proved mathematically that the number of computation steps required for the presented fast complex valued time delay neural networks is less than that needed by classical time delay neural networks. Simulation results using MATLAB confirm the theoretical computations.

Keywords—Fast Complex Valued Time Delay Neural Networks, Cross Correlation, Frequency Domain

I. INTRODUCTION

IN recent years, complex-valued neural networks have widened the scope of application in optoelectronics, imaging, remote sensing, quantum neural devices and systems, spatiotemporal analysis of physiological neural systems, and artificial neural information processing [6,10]. The main objective of this paper is to reduce the response time of complex valued time delay neural networks. The idea is to perform the testing process in the frequency domain instead of time domain. This is our idea which was applied successfully for sub-image detection using fast neural networks proposed in [1,2,3]. Furthermore, such idea was presented for fast face detection [5,7,9]. Moreover, the same principles were applied for fast iris detection [8]. Another idea to further increase the speed of fast neural networks through image decomposition was suggested in [7].

Fast neural networks for detecting a certain code in one dimensional serial stream of sequential data was described in [4]. Compared with conventional neural networks, fast neural networks based on cross correlation between the tested data and the input weights of neural networks in the frequency domain showed a significant reduction in the number of computation steps required for certain data detection [1,2,3,4,5,7,8,9]. Here, we make use of our theory on fast neural networks implemented in the frequency domain to increase the speed of complex valued time delay neural networks. The idea of moving the testing process from the time domain to the frequency domain is applied to complex valued time delay neural networks. Theoretical and practical results show that the proposed fast complex valued time delay neural networks are faster than classical complex valued time delay neural networks. In section II, our theory on fast neural networks for detecting certain data in one dimensional matrix is described. Experimental results for fast complex valued time delay neural networks are presented in section III.

II. THEORY OF FAST NEURAL NETWORKS BASED ON CROSS CORRELATION IN THE FREQUENCY DOMAIN

Finding a certain code/data in the input one dimensional matrix is a searching problem. Each position in the input matrix is tested for the presence or absence of the required code/data. At each position in the input matrix, each sub-matrix is multiplied by a window of weights, which has the same size as the sub-matrix. The outputs of neurons in the hidden layer are multiplied by the weights of the output layer. When the final output is high, this means that the sub-matrix under test contains the required code/data and vice versa. Thus, we may conclude that this searching problem is a cross correlation between the matrix under test and the weights of the hidden neurons.

The convolution theorem in mathematical analysis says that a convolution of f with h is identical to the result of the following steps: let F and H be the results of the Fourier Transformation of f and h in the frequency domain. Multiply F and H in the frequency domain point by point and then transform this product into the spatial domain via the inverse Fourier Transform. As a result, these cross correlations can be represented by a product in the frequency domain. Thus,

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by using cross correlation in the frequency domain, speed up in an order of magnitude can be achieved during the detection process [1,2,3,4,5,7,8,9].

In the detection phase, a sub matrix I of size 1xn (sliding window) is extracted from the tested matrix, which has a size 1xN, and fed to the neural network. Let X_i be the vector of weights between the input sub-matrix and the hidden layer. This vector has a size of 1xn and can be represented as 1xn matrix. The output of hidden neurons h_i can be calculated as follows:

$$h_i = g\left(\sum_{k=1}^n X_i(k)I(k) + b_i\right) \quad (1)$$

where g is the activation function and b_i is the bias of each hidden neuron (i). Equation 1 represents the output of each hidden neuron for a particular sub-matrix I. It can be obtained to the whole matrix Z as follows:

$$h_i(u) = g\left(\sum_{k=-n/2}^{n/2} X_i(k) Z(u+k) + b_i\right) \quad (2)$$

Eq.2 represents a cross correlation operation. Given any two functions f and d, their cross correlation can be obtained by:

$$f(x) \otimes d(x) = \left(\sum_{n=-\infty}^{\infty} f(x+n)d(n)\right) \quad (3)$$

Therefore, Eq. 2 may be written as follows [1]:

$$h_i = g(Z \otimes X_i + b_i) \quad (4)$$

where h_i is the output of the hidden neuron (i) and $h_i(u)$ is the activity of the hidden unit (i) when the sliding window is located at position (u) and $(u) \in [N-n+1]$.

Now, the above cross correlation can be expressed in terms of one dimensional Fast Fourier Transform as follows [1]:

$$Z \otimes X_i = F^{-1}\left(F(Z) \bullet F^*(X_i)\right) \quad (5)$$

Hence, by evaluating this cross correlation, a speed up ratio can be obtained comparable to conventional neural networks. Also, the final output of the neural network can be evaluated as follows:

$$O(u) = g\left(\sum_{i=1}^q w_o(i) h_i(u) + b_o\right) \quad (6)$$

where q is the number of neurons in the hidden layer. $O(u)$ is the output of the neural network when the sliding window located at the position (u) in the input matrix Z.

The complexity of cross correlation in the frequency domain can be analyzed as follows:

- 1- For a tested matrix of 1xN elements, the 1D-FFT requires a number equal to $N \log_2 N$ of complex computation steps. Also, the same number of complex computation steps is required for computing the 1D-FFT of the weight matrix at each neuron in the hidden layer.
- 2- At each neuron in the hidden layer, the inverse 1D-FFT is computed. Therefore, q backward and $(1+q)$ forward transforms have to be computed. Therefore, for a given matrix under test, the total number of operations required to compute the 1D-FFT is $(2q+1)N \log_2 N$.
- 3- The number of computation steps required by fast neural networks is complex and must be converted into a real version. It is known that, the one dimensional Fast Fourier Transform requires $(N/2) \log_2 N$ complex multiplications and $N \log_2 N$ complex additions. Every complex multiplication is realized by six real floating point operations and every complex addition is implemented by two real floating point operations. Therefore, the total number of computation steps required to obtain the 1D-FFT of a 1xN matrix is:

$$\rho = 6((N/2) \log_2 N) + 2(N \log_2 N) \quad (7)$$

which may be simplified to:

$$\rho = 5(N \log_2 N) \quad (8)$$

- 4- Both the input and the weight matrices should be dot multiplied in the frequency domain. Thus, a number of complex computation steps equal to qN should be considered. This means $6qN$ real operations will be added to the number of computation steps required by fast neural networks.
- 5- In order to perform cross correlation in the frequency domain, the weight matrix must be extended to have the same size as the input matrix. So, a number of zeros = $(N-n)$ must be added to the weight matrix. This requires a total real number of computation steps = $q(N-n)$ for all neurons. Moreover, after computing the FFT for the weight matrix, the conjugate of this matrix must be obtained. As a result, a real number of computation steps = qN should be added in order to obtain the conjugate of the weight matrix for all neurons. Also, a number of real computation steps equal to N is required to create butterflies complex numbers $(e^{-jk(2\pi n/N)})$, where $0 < K < L$.

These (N/2) complex numbers are multiplied by the elements of the input matrix or by previous complex numbers during the computation of FFT. To create a complex number requires two real floating point operations. Thus, the total number of computation steps required for fast neural networks becomes:

$$\sigma = ((2q+1)(5N \log_2 N) + 6qN + q(N-n) + qN + N) \quad (9)$$

which can be reformulated as:

$$\sigma = ((2q+1)(5N \log_2 N) + q(8N-n) + N) \quad (10)$$

6- Using sliding window of size 1xn for the same matrix of 1xN pixels, (q(2n-1)(N-n+1)) computation steps are required when using classical neural networks for certain code detection. The theoretical speed up factor η can be evaluated as follows [11]:

$$\eta = \frac{q(2n-1)(N-n+1)}{(2q+1)(5N \log_2 N) + q(8N-n) + N} \quad (11)$$

III. EXPERIMENTAL RESULTS FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS

Complex valued neural networks have many applications in fields dealing with complex numbers such as telecommunications, speech recognition and image processing with the Fourier transform [6]. Complex valued neural networks means that the inputs, weights, thresholds and the activation function have complex values. Complex valued time delay neural networks accept serial input data with fixed size (n). Therefore, the number of input neurons equals to (n). Instead of treating (n) inputs, our new idea is to collect the input data together in a long vector (for example 100xn). Then the input data is tested by complex valued time delay neural networks as a single pattern with length L (L=100xn). Such test is performed in the frequency domain as described in section II. In this section, formulas for the speed up ratio with different types of inputs will be presented. The special case when the imaginary part of the inputs=0 (i.e. real input values) is considered. Also, the speed up ratio in case of one and two dimensional input matrix will be concluded. The operation of fast neural networks depends on computing the Fast Fourier Transform for both the input and weight matrices and obtaining the resulted two matrices. After performing dot multiplication for the resulted two matrices in the frequency domain, the Inverse Fast Fourier Transform is calculated for the final matrix. As the Fast Fourier Transform is already dealing with complex numbers, so there is no change in the number of computation steps required for fast neural networks.

Therefore, the speed up ratio in case of complex valued time delay neural networks can be evaluated as follows:

1) In case of real inputs

A) For one dimensional input matrix

Multiplication of (n) complex valued weights by (n) real inputs requires (2n) real operations. This produces (n) real numbers and (n) imaginary numbers. The addition of these numbers requires (2n-2) real operations. Therefore, the number of computation steps required by conventional neural networks can be calculated as:

$$\theta = (2q(2n-1)(N-n+1)) \quad (12)$$

The speed up ratio in this case can be computed as follows:

$$\eta = \frac{2q(2n-1)(N-n+1)}{(2q+1)(5N \log_2 N) + q(8N-n) + N} \quad (13)$$

The theoretical speed up ratio for searching of a short successive (n) data in a long input vector (L) using complex valued time delay neural networks is listed in Tables 1, 2, and 3. Also, practical speed up ratio for manipulating matrices of different sizes (L) and different in size weight matrices (n) is listed in Table 4 using 700 MHz processor and MATLAB.

B) For two dimensional input matrix

Multiplication of (n²) complex valued weights by (n²) real inputs requires (2n²) real operations. This produces (n²) real numbers and (n²) imaginary numbers. The addition of these numbers requires (2n²-2) real operations. Therefore, the number of computation steps required by conventional neural networks can be calculated as:

$$\theta = (2q(2n^2-1)(N^2-n^2+1)) \quad (14)$$

The speed up ratio in this case can be computed as follows:

$$\eta = \frac{2q(2n^2-1)(N^2-n^2+1)}{(2q+1)(5N^2 \log_2 N^2) + q(8N^2-n^2) + N} \quad (15)$$

The theoretical speed up ratio for detecting (nxn) real valued submatrix in a large real valued matrix (NxN) using complex

valued time delay neural networks is listed in Tables 5, 6, 7. Also, practical speed up ratio for manipulating matrices of different sizes (NxN) and different in size weight matrices (n) is listed in Table 8 using 700 MHz processor and MATLAB.

2) In case of complex inputs

A) For one dimensional input matrix

Multiplication of (n) complex valued weights by (n) complex inputs requires (6n) real operations. This produces (n) real numbers and (n) imaginary numbers. The addition of these numbers requires (2n-2) real operations. Therefore, the number of computation steps required by conventional neural networks can be calculated as:

$$\theta = (2q(4n-1)(N-n+1)) \quad (16)$$

The speed up ratio in this case can be computed as follows:

$$\eta = \frac{2q(4n-1)(N-n+1)}{(2q+1)(5N \log_2 N) + q(8N-n) + N} \quad (17)$$

The theoretical speed up ratio for searching of a short complex successive (n) data in a long complex valued input vector (L) using complex valued time delay neural networks is listed in Tables 9, 10, and 11. Also, practical speed up ratio for manipulating matrices of different sizes (L) and different in size weight matrices (n) is listed in Table 12 using 700 MHz processor and MATLAB.

b) For two dimensional input matrix

Multiplication of (n²) complex valued weights by (n²) real inputs requires (6n²) real operations. This produces (n²) real numbers and (n²) imaginary numbers. The addition of these numbers requires (2n²-2) real operations. Therefore, the number of computation steps required by conventional neural networks can be calculated as:

$$\theta = (2q(4n^2-1)(N^2-n^2+1)) \quad (18)$$

The speed up ratio in this case can be computed as follows:

$$\eta = \frac{2q(4n^2-1)(N^2-n^2+1)}{(2q+1)(5N^2 \log_2 N^2) + q(8N^2-n^2) + N} \quad (19)$$

The theoretical speed up ratio for detecting (nxn) complex valued submatrix in a large complex valued matrix (NxN) using complex valued neural networks is listed in Tables 13, 14, and 15. Also, practical speed up ratio for manipulating matrices of different sizes (NxN) and different in size weight matrices (n) is listed in Table 16 using 700 MHz processor and MATLAB.

For one dimensional matrix, from Tables 1, 2,3,4,9,10,11, and 12 we can conclude that the response time for vectors with small lengths are faster than those which have larger lengths. For example, the speed up ratio for the vector of length 10000 is faster that of length 1000000. The number of computation steps required for a vector of length 10000 is much less than that required for a vector of length 40000. So, if the vector of length 40000 is divided into 4 shorter vectors of length 10000, the number of computation steps will be less than that required for the vector of length 40000. Therefore, for each application, it is useful at the first to calculate the optimum length of the input vector. The same conclusion can be drawn in case of processing the two dimensional input matrix as listed in Tables 5,6,7,8,13,14,15, and 16. From these tables, it is clear that the maximum speed up ratio is achieved at image size (N=200) when n=20, then at image size (N=300) when n=25, and at image size (N=400) when n=30. This confirms our previous results presented in [7] on fast subimage detection based on neural networks and image decomposition. Using such technique, it was proved that the speed up ratio of neural networks becomes faster when dividing the input image into many subimages and processing each subimage in the frequency domain separately using a single fast neural processor.

IV. CONCLUSION

A new fast complex valued time delay neural networks has been presented. Theoretical computations have shown that fast complex valued time delay neural networks requires fewer computation steps than conventional one. This has been achieved by applying cross correlation in the frequency domain between the input data and the input weights of complex valued time delay neural networks. Simulation results have confirmed this proof by using MATLAB. This algorithm can be successfully applied for any application uses complex valued time delay neural networks.

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TABLE 1

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (1D-REAL VALUES INPUT MATRIX, n=400)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 10000 | 4.6027e+008 | 4.2926e+007 | 10.7226 |
| 40000 | 1.8985e+009 | 1.9614e+008 | 9.6793 |
| 90000 | 4.2955e+009 | 4.7344e+008 | 9.0729 |
| 160000 | 7.6513e+009 | 8.8219e+008 | 8.6731 |
| 250000 | 1.1966e+010 | 1.4275e+009 | 8.3823 |
| 360000 | 1.7239e+010 | 2.1134e+009 | 8.1571 |
| 490000 | 2.3471e+010 | 2.9430e+009 | 7.9752 |
| 640000 | 3.0662e+010 | 3.9192e+009 | 7.8237 |
| 810000 | 3.8812e+010 | 5.0442e+009 | 7.6945 |
| 1000000 | 4.7921e+010 | 6.3201e+009 | 7.5823 |

TABLE 2

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (1D-REAL VALUES INPUT MATRIX, n=625)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 10000 | 7.0263e+008 | 4.2919e+007 | 16.3713 |
| 40000 | 2.9508e+009 | 1.9613e+008 | 15.0452 |
| 90000 | 6.6978e+009 | 4.7343e+008 | 14.1474 |
| 160000 | 1.1944e+010 | 8.8218e+008 | 13.5388 |
| 250000 | 1.8688e+010 | 1.4275e+009 | 13.0915 |
| 360000 | 2.6932e+010 | 2.1134e+009 | 12.7433 |
| 490000 | 3.6674e+010 | 2.9430e+009 | 12.4612 |
| 640000 | 4.7915e+010 | 3.9192e+009 | 12.2257 |
| 810000 | 6.0655e+010 | 5.0442e+009 | 12.0247 |
| 1000000 | 7.4893e+010 | 6.3201e+009 | 11.8500 |

TABLE 3

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (1D-REAL VALUES INPUT MATRIX, n=900)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 10000 | 9.823 e+008 | 4.2911e+007 | 22.8933 |
| 40000 | 4.2206e+009 | 1.9612e+008 | 21.5200 |
| 90000 | 9.6176e+009 | 4.7343e+008 | 20.3149 |
| 160000 | 1.7173e+010 | 8.8217e+008 | 19.4671 |
| 250000 | 2.6888e+010 | 1.4275e+009 | 18.8356 |
| 360000 | 3.8761e+010 | 2.1134e+009 | 18.3409 |
| 490000 | 5.2794e+010 | 2.9430e+009 | 17.9385 |
| 640000 | 6.8985e+010 | 3.9192e+009 | 17.6018 |
| 810000 | 8.7334e+010 | 5.0442e+009 | 17.3139 |
| 1000000 | 1.0784e+011 | 6.3201e+009 | 17.0635 |

TABLE 4
 PRACTICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (1D-REAL VALUES INPUT MATRIX)

| Length of input matrix | Speed up ratio (n=400) | Speed up ratio (n=625) | Speed up ratio (n=900) |
|------------------------|------------------------|------------------------|------------------------|
| 10000 | 17.88 | 25.94 | 35.21 |
| 40000 | 17.19 | 25.11 | 34.43 |
| 90000 | 16.65 | 24.56 | 33.59 |
| 160000 | 16.14 | 24.14 | 33.05 |
| 250000 | 15.89 | 23.76 | 32.60 |
| 360000 | 15.58 | 23.23 | 32.27 |
| 490000 | 15.28 | 22.87 | 31.99 |
| 640000 | 14.08 | 22.54 | 31.78 |
| 810000 | 13.87 | 22.32 | 31.60 |
| 1000000 | 13.69 | 22.11 | 31.47 |

TABLE 5
 THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (2D-REAL VALUES INPUT MATRIX, n=20)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 3.1453e+008 | 4.2916e+007 | 7.3291 |
| 200x200 | 1.5706e+009 | 1.9610e+008 | 8.0091 |
| 300x300 | 3.7854e+009 | 4.7335e+008 | 7.9970 |
| 400x400 | 6.9590e+009 | 8.8203e+008 | 7.8898 |
| 500x500 | 1.1091e+010 | 1.4273e+009 | 7.7711 |
| 600x600 | 1.6183e+010 | 2.1130e+009 | 7.6585 |
| 700x700 | 2.2233e+010 | 2.9426e+009 | 7.5556 |
| 800x800 | 2.9242e+010 | 3.9186e+009 | 7.4623 |
| 900x900 | 3.7209e+010 | 5.0434e+009 | 7.3778 |
| 1000x1000 | 4.6136e+010 | 6.3191e+009 | 7.3010 |

TABLE 6
 THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (2D-REAL VALUES INPUT MATRIX, n=25)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 4.3285e+008 | 4.2909e+007 | 10.0877 |
| 200x200 | 2.3213e+009 | 1.9609e+008 | 11.8380 |
| 300x300 | 5.7086e+009 | 4.7334e+008 | 12.0602 |
| 400x400 | 1.0595e+010 | 8.8202e+008 | 12.0119 |
| 500x500 | 1.6980e+010 | 1.4273e+009 | 11.8966 |
| 600x600 | 2.4863e+010 | 2.1130e+009 | 11.7667 |
| 700x700 | 3.4246e+010 | 2.9425e+009 | 11.6381 |
| 800x800 | 4.5127e+010 | 3.9185e+009 | 11.5163 |
| 900x900 | 5.7507e+010 | 5.0434e+009 | 11.4025 |
| 1000x1000 | 7.1386e+010 | 6.3191e+009 | 11.2969 |

TABLE 7

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (2D-REAL VALUES INPUT MATRIX, n=30)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 5.4413e+008 | 4.2901e+007 | 12.6834 |
| 200x200 | 3.1563e+009 | 1.9608e+008 | 16.0966 |
| 300x300 | 7.9272e+009 | 4.7334e+008 | 16.7476 |
| 400x400 | 1.4857e+010 | 8.8201e+008 | 16.8444 |
| 500x500 | 2.3946e+010 | 1.4273e+009 | 16.7773 |
| 600x600 | 3.5193e+010 | 2.1130e+009 | 16.6552 |
| 700x700 | 4.8599e+010 | 2.9425e+009 | 16.5160 |
| 800x800 | 6.4164e+010 | 3.9185e+009 | 16.3745 |
| 900x900 | 8.1888e+010 | 5.0434e+009 | 16.2367 |
| 1000x1000 | 1.0177e+011 | 6.3191e+009 | 16.1052 |

TABLE 8

PRACTICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (2D-REAL VALUES INPUT MATRIX)

| Image size | Speed up ratio (n=20) | Speed up ratio (n=25) | Speed up ratio (n=30) |
|------------|-----------------------|-----------------------|-----------------------|
| 100x100 | 17.19 | 22.32 | 31.74 |
| 200x200 | 17.61 | 22.89 | 32.55 |
| 300x300 | 16.54 | 23.66 | 33.71 |
| 400x400 | 15.98 | 22.95 | 34.53 |
| 500x500 | 15.62 | 22.49 | 33.32 |
| 600x600 | 15.16 | 22.07 | 32.58 |
| 700x700 | 14.87 | 21.83 | 32.16 |
| 800x800 | 14.64 | 21.61 | 31.77 |
| 900x900 | 14.43 | 21.42 | 31.45 |
| 1000x1000 | 14.26 | 21.22 | 31.12 |

TABLE 9

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (1D-COMPLEX VALUES INPUT MATRIX, n=400)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 9.2111e+008 | 4.2926e+007 | 21.4586 |
| 200x200 | 3.7993e+009 | 1.9614e+008 | 19.3706 |
| 300x300 | 8.5963e+009 | 4.7344e+008 | 18.1571 |
| 400x400 | 1.5312e+010 | 8.8219e+008 | 17.3570 |
| 500x500 | 2.3947e+010 | 1.4275e+009 | 16.7750 |
| 600x600 | 3.4500e+010 | 2.1134e+009 | 16.3245 |
| 700x700 | 4.6972e+010 | 2.9430e+009 | 15.9604 |
| 800x800 | 3.9192e+009 | 6.1363e+010 | 15.6571 |
| 900x900 | 7.7673e+010 | 5.0442e+009 | 15.3985 |
| 1000x1000 | 9.5902e+010 | 6.3201e+009 | 15.1740 |

TABLE 10

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (1D-COMPLEX VALUES INPUT MATRIX, N=625)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 1.4058e+009 | 4.2919e+007 | 32.7558 |
| 200x200 | 5.9040e+009 | 1.9613e+008 | 30.1025 |
| 300x300 | 1.3401e+010 | 4.7343e+008 | 28.3061 |
| 400x400 | 2.3897e+010 | 8.8218e+008 | 27.0883 |
| 500x500 | 3.7391e+010 | 1.4275e+009 | 26.1934 |
| 600x600 | 5.3885e+010 | 2.1134e+009 | 25.4969 |
| 700x700 | 7.3377e+010 | 2.9430e+009 | 24.9324 |
| 800x800 | 9.5868e+010 | 3.9192e+009 | 24.4612 |
| 900x900 | 1.2136e+011 | 5.0442e+009 | 24.0590 |
| 1000x1000 | 1.4985e+011 | 6.3201e+009 | 23.7095 |

TABLE 11

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (1D-COMPLEX VALUES INPUT MATRIX, N=900)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 1.9653e+009 | 4.2911e+007 | 45.7993 |
| 200x200 | 8.4435e+009 | 1.9612e+008 | 43.0519 |
| 300x300 | 1.9240e+010 | 4.7343e+008 | 40.6410 |
| 400x400 | 3.4356e+010 | 8.8217e+008 | 38.9450 |
| 500x500 | 5.3791e+010 | 1.4275e+009 | 37.6817 |
| 600x600 | 7.7544e+010 | 2.1134e+009 | 36.6920 |
| 700x700 | 1.0562e+011 | 2.9430e+009 | 35.8870 |
| 800x800 | 1.3801e+011 | 3.9192e+009 | 35.2134 |
| 900x900 | 1.7472e+011 | 5.0442e+009 | 34.6375 |
| 1000x1000 | 2.1575e+011 | 6.3201e+009 | 34.1365 |

TABLE 12

PRACTICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (1D-COMPLEX VALUES INPUT MATRIX)

| Length of input matrix | Speed up ratio (n=400) | Speed up ratio (n=625) | Speed up ratio (n=900) |
|------------------------|------------------------|------------------------|------------------------|
| 10000 | 37.90 | 53.58 | 70.71 |
| 40000 | 36.82 | 52.89 | 69.43 |
| 90000 | 36.34 | 52.47 | 68.69 |
| 160000 | 35.94 | 51.88 | 68.05 |
| 250000 | 35.69 | 51.36 | 67.56 |
| 360000 | 35.28 | 51.02 | 67.15 |
| 490000 | 34.97 | 50.78 | 66.86 |
| 640000 | 34.67 | 50.56 | 66.58 |
| 810000 | 34.45 | 50.38 | 66.39 |
| 1000000 | 34.28 | 50.11 | 66.19 |

TABLE 13

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (2D-COMPLEX VALUES INPUT MATRIX, N=20)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 6.2946e+008 | 4.2916e+007 | 14.6674 |
| 200x200 | 3.1431e+009 | 1.9610e+008 | 16.0281 |
| 300x300 | 7.5755e+009 | 4.7335e+008 | 16.0040 |
| 400x400 | 1.3927e+010 | 8.8203e+008 | 15.7894 |
| 500x500 | 2.2197e+010 | 1.4273e+009 | 15.5519 |
| 600x600 | 3.2386e+010 | 2.1130e+009 | 15.3266 |
| 700x700 | 4.4493e+010 | 2.9426e+009 | 15.1206 |
| 800x800 | 5.8520e+010 | 3.9186e+009 | 14.9340 |
| 900x900 | 7.4465e+010 | 5.0434e+009 | 14.7649 |
| 1000x1000 | 9.2329e+010 | 6.3191e+009 | 14.6110 |

TABLE 14

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (2D-COMPLEX VALUES INPUT MATRIX, N=25)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 8.6605e+008 | 4.2909e+007 | 20.1836 |
| 200x200 | 4.6445e+009 | 1.9609e+008 | 23.6856 |
| 300x300 | 1.1422e+010 | 4.7334e+008 | 24.1301 |
| 400x400 | 2.1198e+010 | 8.8202e+008 | 24.0333 |
| 500x500 | 3.3973e+010 | 1.4273e+009 | 23.8028 |
| 600x600 | 4.9746e+010 | 2.1130e+009 | 23.5427 |
| 700x700 | 6.8519e+010 | 2.9425e+009 | 23.2856 |
| 800x800 | 9.0290e+010 | 3.9185e+009 | 23.0418 |
| 900x900 | 1.1506e+011 | 5.0434e+009 | 22.8142 |
| 1000x1000 | 1.4283e+011 | 6.3191e+009 | 22.6027 |

TABLE 15

THE THEORETICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (2D-COMPLEX VALUES INPUT MATRIX, N=30)

| Length of input matrix | Number of computation steps required for classic complex valued neural networks | Number of computation steps required for fast complex valued neural networks | Speed up ratio |
|------------------------|---|--|----------------|
| 100x100 | 1.0886e+009 | 4.2901e+007 | 25.3738 |
| 200x200 | 6.3143e+009 | 1.9608e+008 | 32.2021 |
| 300x300 | 1.5859e+010 | 4.7334e+008 | 33.5045 |
| 400x400 | 2.9722e+010 | 8.8201e+008 | 33.6981 |
| 500x500 | 4.7904e+010 | 1.4273e+009 | 33.5640 |
| 600x600 | 7.0405e+010 | 2.1130e+009 | 33.3197 |
| 700x700 | 9.7225e+010 | 2.9425e+009 | 33.0412 |
| 800x800 | 1.2836e+011 | 3.9185e+009 | 32.7581 |
| 900x900 | 1.6382e+011 | 5.0434e+009 | 32.4825 |
| 1000x1000 | 2.0360e+011 | 6.3191e+009 | 32.2193 |

TABLE 16
PRACTICAL SPEED UP RATIO FOR COMPLEX VALUED TIME DELAY NEURAL NETWORKS (2D-COMPLEX VALUES INPUT MATRIX)

| Image size | Speed up ratio (n=20) | Speed up ratio (n=25) | Speed up ratio (n=30) |
|------------|-----------------------|-----------------------|-----------------------|
| 100x100 | 38.33 | 46.99 | 62.88 |
| 200x200 | 39.17 | 47.79 | 63.77 |
| 300x300 | 38.44 | 48.86 | 64.83 |
| 400x400 | 37.92 | 47.23 | 65.99 |
| 500x500 | 37.32 | 46.89 | 64.89 |
| 600x600 | 36.96 | 46.48 | 64.01 |
| 700x700 | 36.67 | 46.08 | 63.31 |
| 800x800 | 36.38 | 45.78 | 62.64 |
| 900x900 | 36.11 | 45.54 | 61.95 |
| 1000x1000 | 35.86 | 45.36 | 61.40 |