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Harmonics Elimination in Multilevel Inverter Using Linear Fuzzy Regression

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***Abstract*—**Multilevel inverters supplied from equal and constant dc sources almost don't exist in practical applications. The variation of the dc sources affects the values of the switching angles required for each specific harmonic profile, as well as increases the difficulty of the harmonic elimination's equations. This paper presents an extremely fast optimal solution of harmonic elimination of multilevel inverters with non-equal dc sources using Tanaka's fuzzy linear regression formulation. A set of mathematical equations describing the general output waveform of the multilevel inverter with non-equal dc sources is formulated. Fuzzy linear regression is then employed to compute the optimal solution set of switching angles.

***Keywords*—**Multilevel converters, harmonics, pulse width modulation (PWM), optimal control.

I. Introduction

MULTILEVEL inverter is considered as one of the most significant recent advances in power electronics. They have drawn tremendous interest in the field of high-voltage high-power applications such as laminators, mills, compressors, large induction motor drives, UPS systems, and static var compensation [1]. Its concept is based on producing small output voltage steps, resulting in better power quality. Despite the need for more power transistors, they operate at low voltage levels and also at low switching frequency so that the switching losses are also reduced. Some of the fundamental multilevel topologies include the diode-clamped, flying capacitor, and cascaded H-bridge structures [2]. Multilevel inverters are mostly supplied from dc sources obtained from fuel cells, ultra capacitors, ect. It is worth noting that in most of the reported work, it was assumed that the dc sources were all equal, which will probably not be the case in applications even if the sources are nominally equal.

The key issue in designing an effective multilevel inverter is to ensure that the total harmonic distortion (THD) of the output voltage waveform is within acceptable limits. Selective harmonic elimination pulse width modulation has been intensively studied in order to achieve low THD [3]. The output voltage waveform analysis using Fourier theory produces a set of non-linear transcendental equations. The solution of these equations, if exists, gives the switching angles required for certain fundamental component and selected harmonic profile. Iterative procedures such as *Newton-Raphson* method has been used to solve these sets of equations [4]. This method is derivative-dependent and may end in local optima, and a judicious choice of the initial values alone guarantees conversion. Another approach based on converting the transcendental equation into polynomial equations is presented in [5], where resultant theory is applied to determine the switching angles to eliminate specific harmonics. This approach, however, appears to be unattractive because as the number of inverter levels increases, so does the degree of the polynomials of the mathematical model. This is likely to lead to numerical difficulty and substantial computational burden as well.

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In this paper Tanaka's fuzzy linear regression formulation will employed to compute the optimal solution set of switching angles, after linearization, if it exists for each required harmonic profile. The switching angles are expressed as fuzzy numbers with a triangular membership function that has middle and spread value reflected on the estimated unknowns. The proposed fuzzy model is formulated as a linear optimization problem, where the objective is to minimize the sum of the spread of the states, subject to double inequality constraints. Theoretical studies for different case studies regarding the number of levels and harmonic profile will be carried out to assess the effectiveness and robustness of the proposed technique.

II. An Overview of Tanaka's Fuzzy Linear Regression

Fuzzy linear regression was introduced by Tanaka et. al [10] in 1982. The general form of Tanaka's formulation is given by:

$$\underset{\sim}{Y} = f(x) = A_0 + A_1 x_1 + A_2 x_2 + ... + A_n x_n = \boldsymbol{Ax} \qquad (1)$$

where $\underset{\sim}{Y}$ is output (dependant fuzzy variable), $\{x_1, x_2, ...x_n\}$ is a non fuzzy set of crisp independent parameters and $\{A_0, A_1, ..., A_n\}$ is a fuzzy set of symmetric members, unknowns, needs to be estimated. Each fuzzy element in that set may be represented by a symmetrical triangular membership function, shown in figure 1, defined by a middle and a spread values, p_i and c_i respectively. The middle is known as the model value and the spread denotes the fuzziness of that model value. The triangular membership function can be expressed as:

$$\mu_{\underset{\sim}{A}_i}(a_i) = \begin{cases} 1 - \dfrac{|\,p_i - a_i\,|}{c_i} & , \; p_i - c_i \le \; a_i \le p_i + c_i \\ 0 & , \quad otherwise \end{cases} \qquad (2)$$

Therefore, since $A_i = (p_i, c_i)$, then equation (1) may be rewritten as:

$$\underset{\sim}{Y} = f(x) = (p_0, c_0) + (p_1, c_1)x_1 + ... + (p_n, c_n)x_n \qquad (3)$$

The membership function of output $\underset{\sim}{Y}$ may be given by:

$$\mu_{\underset{\sim}{Y}}(y) = \begin{cases} \max(\min([\mu_{\underset{\sim}{A}_i}(a_i)]) & , \{a \,|\, y = f(x, a)\} \ne \varnothing \\ 0 & , \quad otherwise \end{cases} \qquad (4)$$

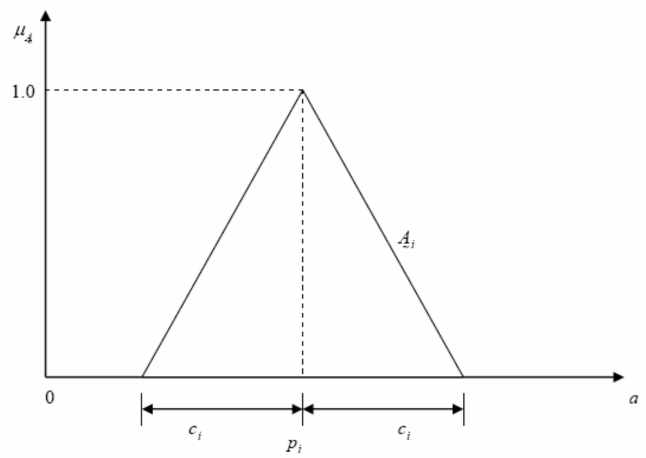

Fig. 1 Membership function of fuzzy coefficient $\underset{\sim}{A}$

Now, by substituting equation (3) in (4), the output membership function is given as:

$$\mu_{\underset{\sim}{Y}}(y) = \begin{cases} 1 - \dfrac{\left| y - \sum_{i=1}^{n} p_i x_i \right|}{\sum_{i=1}^{n} c_i |x_i|} & , x_i \ne 0 \\ 1 & , \; x_i = 0, \; y_i = 0 \\ 0 & , \; x_i = 0, \; y_i \ne 0 \end{cases} \qquad (5)$$

From regression point of view, equations (1-5) may be applied to m samples where the output can be either non-fuzzy, (certain or exact), in which no assumption of ambiguity is associated with the output or fuzzy (uncertain), where uncertainty in the out is involved due to human judgment or meters impression [11]. In this study both non fuzzy and fuzzy output will be considered.

The output membership function is depicted in Fig. 2.

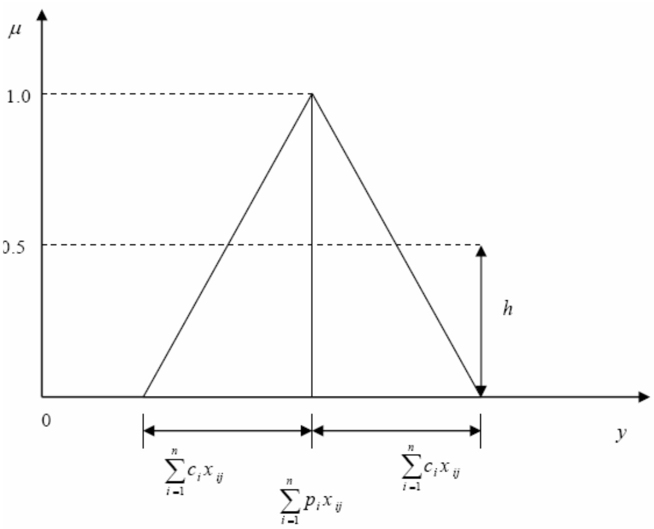

Fig. 2 Membership function of output

A. Non-fuzzy Output Model [10]

In this model, Tanaka converted regression model into a linear programming problem [10]. In this case the objective is to solve for the best parameters, i.e. A^*, such that the fuzzy output set is associated with a membership value grater than h as in ;

$$\mu_{\underset{\sim}{Y}_j}(y_j) \ge h, \qquad j = 1, ..., m \qquad (6)$$

where $h \in [0,1]$ is the degree of the fuzziness and is normally defined by the user.

Therefore, with equation (6) as a condition, the main objective is to find the fuzzy coefficients that minimize the spread of all fuzzy output for all data set. Note that the fuzziness in the output is due to fuzziness assumed in the system structure A^*. Thus, given non-fuzzy data (y_i, x_i), the fuzzy parameters $\boldsymbol{A}^* = (\boldsymbol{p}, \boldsymbol{c})$ may be solve for by the linear programming formulation as:

$$F_{non-fuzzy} = \min(\sum_{j=1}^{m} \sum_{i=1}^{n} c_i x_{ij}) \qquad (7)$$

Subject to:

$$y_j \ge \sum_{i=1}^{n} p_i x_{ij} - (1-h) \sum_{i=1}^{n} c_i x_{ij} \qquad (8)$$

$$y_j \le \sum_{i=1}^{n} p_i x_{ij} + (1-h) \sum_{i=1}^{n} c_i x_{ij} \qquad (9)$$

Note that from in (8) and (9), $\sum_{i=1}^{n} p_i x_{ij}$, defines the middle value and $\sum_{i=1}^{n} c_i x_{ij}$ defines the sympatric spread to the left, constraint (8), and to the right, constraint (9), as illustrated in Fig. 2. As can be seen from the Fig. 2, as the degree of fuzziness, h, increases the spread, c_i, increases and therefore the uncertainty associated with the p_i would increase [12]. The prove and detailed derivation may be found in [10, 13].

III. Proposed Problem Formulation

Assuming that the non-equal dc sources are known, and taking into consideration the characteristics of the inverter waveform, from its odd nature, half- and quarter-wave symmetry, Fourier series expansion of the stepped output voltage waveform of the multilevel inverter with non-equal dc sources can be expressed as [10]:

$$V_o(\omega t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{dc}}{n\pi}\{V_1\cos(n\theta_1) + V_2\cos(n\theta_2) + \cdots$$
$$\cdots + V_s\cos(n\theta_s)\}\sin(n\omega t) \tag{10}$$

Where the product V_iV_{dc} is the value of the i^{th} dc source. Equation (10) has s variables (θ_1, θ_2, θ_3,……, θ_s), where $0 \le \theta_1 < \theta_2 < \cdots < \theta_s \le \pi/2$, and a set of solutions is obtainable by equating *s-1* harmonics to zero and assigning a specific value to the fundamental component, as given below:

$$\left.\begin{array}{l} V_1\cos(\theta_1) + V_2\cos(\theta_2) + \cdots + V_s\cos(\theta_s) = m \\ V_1\cos(3\theta_1) + V_2\cos(3\theta_2) + \cdots + V_s\cos(3\theta_s) = 0 \\ V_1\cos(5\theta_1) + V_2\cos(5\theta_2) + \cdots + V_s\cos(5\theta_s) = 0 \\ \vdots \\ V_1\cos(n\theta_1) + V_2\cos(n\theta_2) + \cdots + V_s\cos(n\theta_s) = 0 \end{array}\right\} \tag{11}$$

Where $m=V_1/(4V_{dc}/\pi)$, and the modulation index m_a is given by $m_a=m/s$. An objective function is then needed for the optimization procedure, which is selected as a measure of effectiveness of eliminating selected order of harmonics while maintaining the fundamental component at pre-specified value. Therefore, this objective function to be minimized is defined as:

$$F(\theta_1, \theta_2, \cdots, \theta_3) = \left[\sum_{n=1}^{s} V_1\cos(\theta_n) - m\right]^2 +$$
$$+\left[\sum_{n=1}^{s} V_2\cos(3\theta_n)\right]^2 + \cdots + \left[\sum_{n=1}^{s} V_2\cos((2s-1)\theta_s)\right]^2 \tag{12}$$

The optimal switching angles are obtained by minimizing (12) subject to the constraint $0 \le \theta_1 < \theta_2 < \cdots < \theta_s \le \pi/2$, and consequently the required harmonic profile is achieved.

IV. Implementation of Case Studies

The generalized transcendental equations of multilevel inverter are solved using the described fuzzy linear regression algorithm. The proposed technique has been applied to one case in order to confirm its ruggedness. The simulation results are obtained accordingly using Matlab [17]. It is assumed that the level of the non-equal dc sources can be measured, and Vdc has a nominal value of 1 p.u. and so does V1, while the following sources will acquire different given values less than 1 p.u. For each inverter topology with a specific number of levels, a large number of solution sets can be obtained according to the values of m and the dc sources V1, V2, V3,…ect.

A. Case 1: 5-level inverter; V_1=1 p.u., V_2=0.9 p.u.

The proposed technique is applied to minimize the defined cost function for the above stated case. The CPU execution time required for convergence is 1.27 sec.

The fuzzy linear regression algorithm is used to find the switching angles for the abovementioned case. However, the solution exists for a limited range of *m*, where $0.84 \le m \le 1.59$. Despite this is a natural phenomenon of multilevel inverters even with equal dc sources, the obtained range of *m* is wider than that obtained from conventional techniques. Fig. 3 illustrates the variation of the switching angles θ_1 and θ_2 versus *m*. As an example, an operating point when *m*=1.5 was chosen which sets the fundamental output voltage, V_f, to be 1.9 p.u. (s =2, m=1.5, V_{dc}=1 p.u., $V_f = (4mV_{dc}/\pi)$ =1.9 p.u.). For this point, the optimum values of the switching angles are: θ_1=9.815° and θ_2=55.122°. Fig. 4 shows the inverter output voltage and the corresponding harmonic spectrum at the abovementioned operating point. It is clear that the targeted 3rd harmonic is eliminated and the fundamental component is equal to 1.9 p.u. as desired.

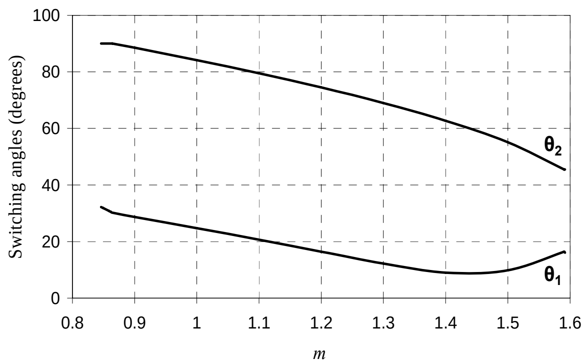

Fig. 3 Solutions for 2 angles versus *m* for Case 1

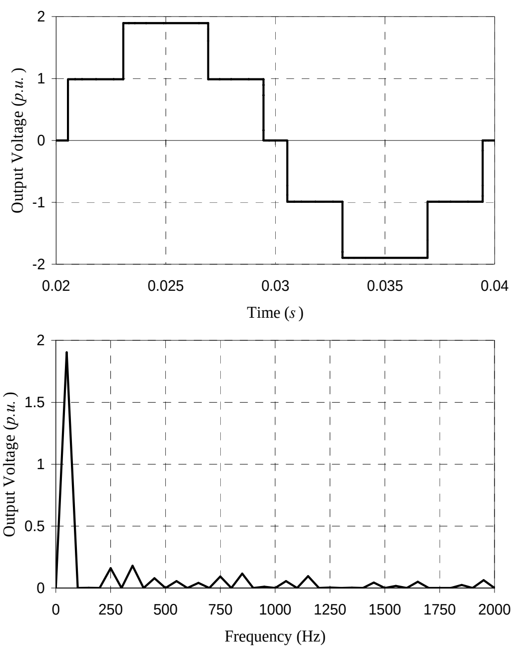

Fig. 4 Output voltage and Corresponding FFT of case 1 at m=1.5

V. Conclusion

Harmonic elimination of multilevel inverters with non-equal dc voltage sources using a fuzzy linear regression algorithm has been presented. It overcomes the complicated computations associated with conventional iterative techniques, and the large number of parameters required for GA. It also reduces both the computational burden and running time, and ensures the accuracy and quality of the calculated angles. This method was found superior to conventional techniques that my fail to converge if higher levels with non-equal dc sources are sought. In order to prove the feasibility and effectiveness of the proposed algorithm, it is applied to different study cases regarding the number of inverter levels and targeted harmonics to be eliminated. For each case defined by the values of the dc sources and the required harmonic profile, optimal solution can be found over a definite range of modulation index. Further testing and experimental verification is required to further assess the proposed method.

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