A numerical Modeling of Piping Phenomenon in Earth Dams

N. Zaki Alamdari, M. Banihashemi, A. Mirghasemi

***Abstract*—**To estimate the risks of dam failure phenomenon, it is necessary to understand this phenomenon and the involved governing factors. Overtopping and piping are the two main reasons of earth-dam failures. In the piping context, the piping is determined as a phenomenon which is occurred between two phases, the water liquid and the solid soil. In this investigation, the onset of piping and its development, as well as the movement of water in soil, are numerically approached. In this regard, a one-dimensional numerical model based on the mass-conserving finite-volume method is developed and applied in order to simulate the piping phenomenon in a continuous circular tunnel of given initial length and radius, located between upstream and downstream. The simulation result includes the time-variations of radius along the tunnel until the radius value reaches its critical and the piping phenomenon converts to overtopping.

***Keywords*—**Earth dam, dam break, piping, internal erosion

I. INTRODUCTION

OVERTOPPING and piping are the two most common failure procedures of hydraulic structures such as earth-dams, dykes and levees. Since the emphasis in dam safety has been always on floods and overtopping all related information are fairly well documented [8].However, the nature of earth dam failures shows the necessity of more understanding of piping as a significant concern of failures. Our data about previous failure of embankment dams shows that piping occurs for 43% of all embankment dam failures, 54% for dams constructed after 1950 [6]. Previous studies indicate different modes of piping failure which are defined by the mechanism causing the piping. The most common modes could be classified in 5 categories: 1) backwards erosion, 2) internal erosion, 3) tunneling, 4) suffusion, and 5) heave [6].

Some investigations show that four conditions must exist simultaneously for internal erosion and piping to occur [2]:

- Existence of a seepage flow path and a source of water
- Presence of erodible materials within the flow path and this material must be carried by the seepage flow
- An unprotected exit, from which the eroded material may escape

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For a pipe to form, the material being piped, or the material directly above, must be able to form and support roof for the pipe

When modeling internal erosion it can be convenient to start from a fluid mechanic point of view, which creates possibilities regarding the solution of the fluid flow through porous media, and the modeling can be performed in numerous ways as outlined by Bear (1972).

One possible way to model the flow through embankment dams is instead to solve for the momentum equations for two phases and then add an advection equation to deal with the motion of the surface formed between the phases.

This investigation is about the process of progression stage in piping erosion. In this stage a united long pipe forms in dam body and it starts to enlarge itself by a tangential force of water flow in it. Many experimental studies have been done to represent this process.

In most of these studies the test was based on the hole erosion with a constant pressure drop which has been proved to be a very precise method to quantify the rate of piping erosion in soil and determining the critical shear stress. But there are really rare attempts to model this phenomenon [1]. This investigation tries to present a numerical model for this case.

II. GOVERNING EQUATIONS

The purpose of this study is to investigate the erosion caused by piping. In this phenomenon the pipe which is developed from upstream to downstream of the body of dam starts to enlarge itself as a consequence of axial flow of water and the eroded particles carry by the water flow. The enlargement process of this pipe is described by a two phase - one dimensional model. The soil is taken to be homogeneous, water saturated.

We take Ω to denote the volume of the two-phase mixture and Γ to denote the fluid/soil interface. The mass conservation equations for the water/particle mixture and the eroded particle phase as well as the balance equation of momentum of the mixture within Ω can be written as follows the an Eulerian framework [4].

$$\frac{\partial p}{\partial t} + \vec{\nabla}.(\rho\vec{u}) = 0 \quad \frac{\partial pY}{\partial t} + \vec{\nabla}.(\rho\vec{u}Y) + \overrightarrow{\nabla}.\vec{J} = 0$$
$$\frac{\partial pu}{\partial t} + \vec{\nabla}.(\rho u \otimes u) = \nabla.T \tag{1}$$

In these equations, ρ is the density mixture, depending on the particle mass fraction Y,u is the mass-weighted average velocity, j is the mass diffusion flux of particles, and T is the Cauchy stress tensor in the mixture. As there is a process of erosion, a mass flux crosses the interface Γ.

Let us take n to denote the normal unit vector of Γ oriented outwards from the flow, and vΓ to denote the normal velocity of Γ. The jump equations over Γ are [4]:

$$[\![\rho(v_\Gamma - u).n]\!] = 0$$
$$[\![\rho Y(v_\Gamma - u).n]\!] = [\![j.n]\!]$$
$$[\![\rho u(v_\Gamma - u).n]\!] = -[\![T.n]\!] \quad (2)$$

Where [a] = $a_b - a_{soil}$ is the jump of any physical variable a across the interface, and a_b and a_{soil} stands for the limiting value of a on the flow and soil sides of the interface.

By simplifying the equation of conservation of mass and conservation of momentum governed on the mixture in the control volume (which is the including volume of two phase mixture) and jump equation to denote the fluid/solid interface (which guaranty the equality of exhausting eroded particles from medium and internals to the medium) this equations has suggested by Lachouette et al [4]:

Let us take a cylinder Ω with current radius R(x, t) (initial value R0), and length L (Fig. 1) u(x, t) to denote the average longitudinal velocity, p(x, t) to denote the pressure and φ(x, t) to denote the average volume fraction of the solid phase in the mixture, where x denotes the axial coordinate and a denotes the mean value of any quantity a across a section. The following system is finally obtained:

Total mass jump equation:

$$\frac{\partial R}{\partial t} = \frac{\dot{m}}{\rho_{soil}}\left(1 + \left(\frac{R}{x}\right)^2\right)^{1/2} \quad (3)$$

Total mass balance equation:

$$\frac{\partial \bar{u}}{\partial x} + 2\frac{\bar{u}}{R}\frac{\partial R}{\partial x} = 0 \quad (4)$$

Sediment mass balance equation:

$$\frac{\partial \bar{\varphi}}{\partial t} + \bar{u}\frac{\partial \bar{\varphi}}{\partial x} = \frac{2\dot{m}}{R\rho_{soil}}(\varphi_{soil} - \bar{\varphi}) \quad (5)$$

Axial momentum balance equation:

$$\bar{\rho}\left(\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x}\right) = \frac{2}{R}(\tau_b - \bar{u}\dot{m}) - \frac{\partial P}{\partial x} \quad (6)$$

The variable mixture density is:

$$\rho = \varphi(\rho^p - \rho^w) + \rho^w \quad (7)$$

Where ρ^p and ρ^w are the (constant) soil particle and water densities.

1- Soil density is:

$$\rho_{soil} = (\rho^p - \rho^w)\varphi_{soil} + \rho^w$$

Where:

$$\varphi_{soil} = 1 - n$$

And φ_{soil} is the compactly of the soil while n is the porosity. Erosion laws dealing with soil surface erosion by a tangential flow are often written as threshold laws such as:

$$\dot{m} = \begin{cases} k_{er}(|\tau_b| - \tau_c) & \text{if } |\tau_b| > \tau_c \\ 0 & \text{otherwise} \end{cases}$$

$\dot{m}$ is the total flux of eroded material (consisting of both particles and water) crossing the interface.

τ_b is the tangential shear stress at the interface

τ_c is the critical shear stress

and $k_{er}(\frac{s}{m})$ is the coefficient of soil erosion.

III. Numerical Results

Equations number 1, 2, 3 and 4 are solved by first order finite volumes in a repetitive operation to get to a convergence.

In every stage of time solving the problem is described step by step as below:

1- Solving the advection of u equation
2- Solving the equation of conservation of momentum and continuity and obtaining the new pressure and velocity
3- Solving the total mass jump equation and obtaining new radius
4- Solving the advection of Φ equation
5- Solving the equation of conservation of solid particles (Sediment mass balance equation) and obtaining new densities
6- Obtaining the average of quantities based on time and repeat the operation until getting to convergence
7- Going to the next time step
8- Continuing the same operation to get to the critical radius or decrease the water level in reservoir until the pressure is not enough for erosion any more

Due to the entrance of pure water the boundary condition at the arrival of pipe is φ=0. The pressure at the beginning of pipe is calculated by knowing the head of water in every step of time. And the exit pressure in every step of time is zero.

Initial conditions include defining the initial radius and velocity in the beginning which will be assumed constant along the pipe only in the first step. Obviously the initial density at the whole pipe considered to be zero.

The results of Teton dam is used to verifying the model and it has been also attempted to simulate the enlargement of pipe in the body of this dam. The enlargement process continues until getting to the critical radius which is computed by this formula [5]:

$$R_{crit} = \frac{2c}{\acute{\rho}(tan\emptyset + 2)} + \frac{tan\emptyset}{(tan\emptyset + 2)}(H_{dam} - H_p) \quad (8)$$

Where:

H_{dam} is elevation of the top of the dam (m)
C is cohesion of the dam material (kPa)
H_p is elevation of the pipe center-line (m)
Φ is friction angle of the dam material
And $\acute{\rho}$ is submerged density of dam material (kg/ m^3).

Teton dam was constructed in 1972. According to a report by Ray, et al. (1976), in 5 June 1976 a large leak caused by piping occurred 40 m below crest near the right abutment the failure started as a piping failure about 10:00 am and slowly increased the rate of outflow until about 12:00 noon when the portion of the dam above the piping hole collapsed [3].

TABLE I
CHARACTERISTICS OF TETON DAM [3], [7]

Parameter	Dimension	Value
Hdam	m	90
Hu	m	80
Hp	m	49

Φ	Degree	40
N	-	0.2
Length of pipe	M	200
Pp	Kg/m2	2500
Ker	s/m	0.002
τc	Pa	20
Surface area	m2	7889700
c	kPa	49
Rcr	m	15
R0	m	0.25
Δt	sec	0.002
Δx	m	2

The variation of radius according to the time is shown in the Fig. 1. This figure shows that the radius in a specified point in 2 hours after the start of piping is approximately equal to the radius calculated by equation 8. It could be predicted that the rate of erosion increases according to the radius which itself is caused by increasing the velocity and consequently the shear stress.

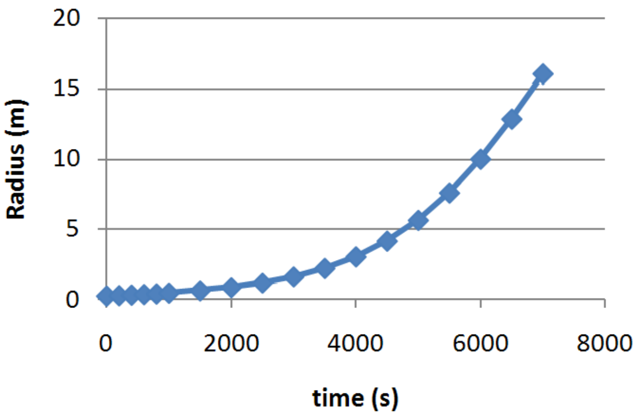

Fig. 1 the variation of radius according to time

IV. Conclusion

Since dam breaking is always accompanied by disasters in downstream, studying the process of crack propagation is so important to predict the time of failure. Unfortunately in most investigations in this field all concentrations are on overtopping and piping is usually neglected. The purpose of this study is to simulate the erosion in the formed pipe in the earth dam body affected by a flow parallel to the surface. For this purpose the mass conservation and momentum conservation equations in the control volume and jump equation to denote the fluid/solid interface is solved by using finite volume method. The erosion of pipe continues until to get to the critical radius. It's when the top of pipe collapse and the breaking continues like the overtopping. The reasons of Teton dam were used to verify the model which proved that the results stand in an acceptable range. Since the coefficient of erosion could be used as an indicator to alert the remaining time to breaching, it is the first important parameter for water retaining structures. The point is to be noted that in this case visual detection of the piping event and reporting is required. The coefficient of erosion can be calculated by doing the Hole Erosion Test in laboratory. The second important parameter is the maximum pipe diameter prior to roof collapse and to breaching. It can serve as an indicator of the peak flow.

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