

# Assessment of the Effect of Feed Plate Location on Interactions for a Binary Distillation Column

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**Abstract**—The paper considers the effect of feed plate location on the interactions in a seven plate binary distillation column. The mathematical model of the distillation column is deduced based on the equations of mass and energy balances for each stage, detailed model for both reboiler and condenser, and heat transfer equations. The Dynamic Relative Magnitude Criterion, DRMC is used to assess the interactions in different feed plate locations for a seven plate (Benzene-Toluene) binary distillation column ( the feed plate is originally at stage 4). The results show that whenever we go far from the optimum feed plate position, the level of interaction augments.

**Keywords**—Distillation column, assessment of interactions, feed plate location, DRMC.

## I. INTRODUCTION

**D**ISTILLATION is the most important industrial separation technology. Determining the number of stages required for the desired degree of separation and the location of the feed tray is merely the first steps in producing an overall distillation column design. In practice there are several factors that may affect the design specifications, and it can lead to some deviations that let the column no longer able to handle the separation task. The objective of this article is to assess the effect of deviations in locating the feed tray on the degree of interactions exist in a binary distillation column.

## II. DYNAMIC MODELING OF DISTILLATION COLUMN

The derivation of analytical expressions requires the assumptions of (Shinskey, 1979) [1]:

- Equilibrium stages.
- Constant relative volatility.
- Constant molar flows.

### A. Basic Process Equations

Total material balance on stage i:

$$dM_i/dt = L_{i+1} - L_i + V_{i-1} - V_i \quad (1)$$

Material balance for light component on each stage i:

$$\frac{d(M_i x_i)}{dt} = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i \quad (2)$$

Algebraic equations

The vapor composition  $y_i$  is related to the liquid composition  $x_i$  on the same stage through the algebraic vapor-liquid equilibrium

$$y_i = \frac{\alpha x_i}{1 + (\alpha - 1)x_i} \quad (3)$$

Where  $\alpha$  is the relative volatility. The above equations apply at all stages except in the top (condenser), feed stage and bottom (reboiler).

Feed stage,  $i = NF$

We assume the feed is mixed directly into the liquid at the feed stage

$$\frac{dM_i}{dt} = L_{i+1} - L_i + V_{i-1} + F \quad (4)$$

$$\frac{d(M_i x_i)}{dt} = L_{i+1} x_{i+1} + V_{i-1} y_{i-1} - L_i x_i - V_i y_i + F x_F \quad (5)$$

Total condenser  $i = NT$  ( $M_{NT} = M_D$ ,  $L_{NT} = L_T$ )

$$dM_i/dt = V_{i-1} - L_i - D \quad (6)$$

$$d(M_i x_i)/dt = V_{i-1} y_{i-1} - L_i x_i - D x_i \quad (7)$$

Reboiler,  $i = 1$  ( $M_i = M_B$ ,  $V_i = V_B = V$ )

$$dM_i/dt = L_{i+1} - V_i - B \quad (8)$$

$$d(M_i x_i)/dt = L_{i+1} x_{i+1} - V_i y_i - B x_i \quad (9)$$

Heat transfer

(i) Process side:  
Heat flux

$$q_0(t) = U_0 A_0 (T_w(t) - T_0(t)) \quad (10)$$

$$V_0(t) = \frac{q_0(t)}{\xi_0} \quad (11)$$

Bubble point

$$T_0(t) = f_0(x_0(t)) \quad (12)$$

(ii) Steam coil wall:

$$M_w C_p \frac{dT_w(t)}{dt} = q_c(t) - q_0(t) \quad (13)$$

(iii) Steam coil internals:

**Heat flux**

$$q_c(t) = U_c A_c (T_c(t) - T_w(t)) \quad (14)$$

**Steam condensation**

$$T_c(t) = f_c(P_c(t)) \quad (15)$$

**Condensate volumetric flow rate**

$$Q_c(t) = q_c(t) / \rho_c \xi_c \quad (16)$$

**Steam feed volumetric flow rate**

$$Q_{ss}(t) = C_v(t) \sqrt{(P_{ss}(t) - P_c(t)) / \rho_c} \quad (17)$$

**Control valve coefficient**

$$C_v(t) = C_{v_{max}} X_v(t) \quad (18)$$

**Coil capacitance**

$$V_c / (1.135 \bar{P}_c) \frac{dP_c}{dt} = Q_{ss}(t) - Q_c(t) \quad (19)$$

### B. The Linear Model

Linearization of the above equations around an operating state gives a linear model given in state space representation by:

$$\dot{x} = Ax + Bu \quad (20)$$

$$y = Cx \quad (21)$$

Where

$$x^T = (x_d, x_1, \dots, x_7, x_b, P_c, V_s) \quad u^T = (L_r, P_f, F, z_f, P_{ss}, X_v)$$

$$y^T = (x_7, x_b)$$

$A, B$  and  $C$  are given in [4].

### III. DYNAMIC RELATIVE MAGNITUDE CRITERION (DRMC)

The DRMC is a set of plots of magnitude/log frequency for its elements. The DRMC elements have been arranged into an array as diagonal and off-diagonal elements and interpreted as graphical representations, the diagonal elements are like the RGA relate open loop and closed loop behavior of the fully controlled system [3].

### A. The Construction of the DRMC Elements [4]

a) The diagonal elements

$$\delta_{ii}(s) = \frac{\left( \frac{y_i(s)}{u_i(s)} \right)_{\text{all loops are open}}}{\left( \frac{y_i(s)}{u_i(s)} \right)_{\text{all loops are closed except loop } i}} \quad (22)$$

b) The off-diagonal elements

$$\delta_{ij}(s) = \frac{\left( \frac{y_i(s)}{u_{jsp}(s)} \right)_{i \neq j}}{\left( \frac{y_k(s)}{u_{ksp}(s)} \right)_{k=csf}} \quad (23)$$

### B. Interpretation of DRMC Elements

The DRMC clearly expresses how the individual control loops respond to their own set-points through the diagonal elements and to other set points through the off-diagonal elements. From the definition of the criterion, the system interaction caused by the closed control loops, will be very weak for those pairs of variables with a relative magnitude of unity at loops resonant frequencies, as the magnitude of the diagonal elements of the DRMC between controlled variables  $y_i$  and manipulated variables  $u_i$  departs from unity, more interaction must be expected.

The diagonal elements of the DRMC carry information about how a single loop will respond to changes in its own set point. However they don't supply any useful indication about the direction and magnitude of dynamic interaction with other loops.

The off-diagonal elements  $\delta_{ji}$  express how much the  $j$ th loop is excited relative to the response of the  $i$ th loop when a set point is made in the  $i$ th loop. The  $\delta_{ji}$  for the range of frequencies where a system works (i.e. the loop resonant frequencies) should be much smaller than unity for the rejection of true interaction or disturbance between loops.

### IV. THE OPTIMAL FEED LOCATION

The feed-stage location is that location which, with a given set of other operating specifications, will result in the widest separation between  $x_D$  (the top product) and  $x_B$  (the bottom product) for a given number of stages. Or, if the number of stages is not specified, the optimum feed location is the one that requires the lowest number of stages to accomplish a specified separation between  $x_D$  and  $x_B$ . The optimum feed location can be estimated in the design stage using graphical or analytical methods.

### A. Graphical Method for Locating the Feed Tray

The optimal feed stage location is at the intersection of the two operating lines in the McCabe-Thiele diagram [5].

### B. Shortcut Formula for Estimating the Feed Location

There exist several simple shortcut formulas for estimating the feed point location. One may use Skogestad's approximate formula [6], (This formula is an approximation of Kremser's formula [5]).

Where,  $N$  is the total number of stages in the column,  $N_T$  the number of stages in the top section, and  $N_B$  the number of stages in the bottom section,  $y_F$  and  $x_F$  are the compositions in the feed stage and are obtained by solving the following two equations:

$$z = qx_F + (1-q)y_F \quad (26)$$

$$y_F = \frac{\alpha x_F}{1 + (\alpha - 1)x_F} \quad (27)$$

For  $q = 1$  (liquid feed) we find  $x_F = z$  and for  $q = 0$  (vapor liquid) we find  $y_F = z$  (in the other cases we must solve a second order equation). Where  $q$  is the fraction of liquid in the feed.

### V. THE ASSESSMENT OF THE EFFECT OF FEED PLATE LOCATION ON INTERACTIONS

Figs. 1, 2, 3, 4 and 5 show the DRMC diagonal and off-diagonal elements for the cases where the feed plate is plate 4, 5, 7, 3, and 1 respectively. The resonant frequencies for the two loops  $\omega_r$  (rad / min) for the above cases are listed in Table 1, where  $\omega_{r1}$  the resonant frequency for the top loop and  $\omega_{r2}$  the resonant frequency for the bottom loop.

TABLE I  
 THE FREQUENCIES WHERE THE SYSTEM WORKS

The feed plate location	$\omega_{r1}$	$\omega_{r2}$
Plate 4	0.221	0.06
Plate 5	0.070	0.03
Plate 7	0.300	0.04
Plate 3	0.010	0.01
Plate 1	0.01	0.01

### VI. CONCLUSION

From the above figures for DRMC diagonal and off diagonal elements at the resonant frequencies, we end up with the graphs shown in Figs. 6 and 7 that represent the distance of diagonal elements from unity for the studied cases. From Figs. 6 and 7 we deduce that the location that gives a system with small degree of interaction is plate 4 and whenever we go far from this optimum feed location the degree of interactions gets larger.

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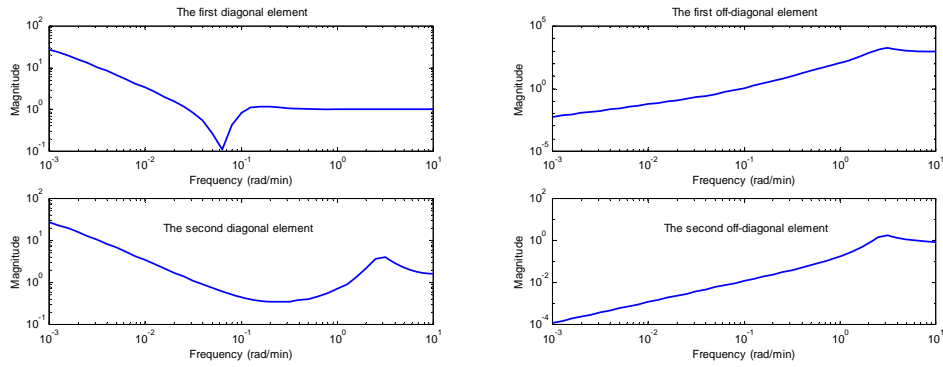


Fig. 1 DRMC elements ( $x_F = 4$ )

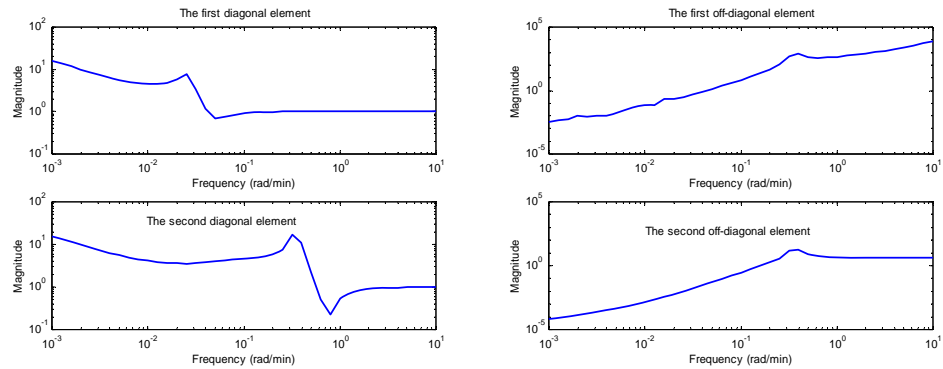


Fig. 2 DRMC elements ( $x_F = 5$ )

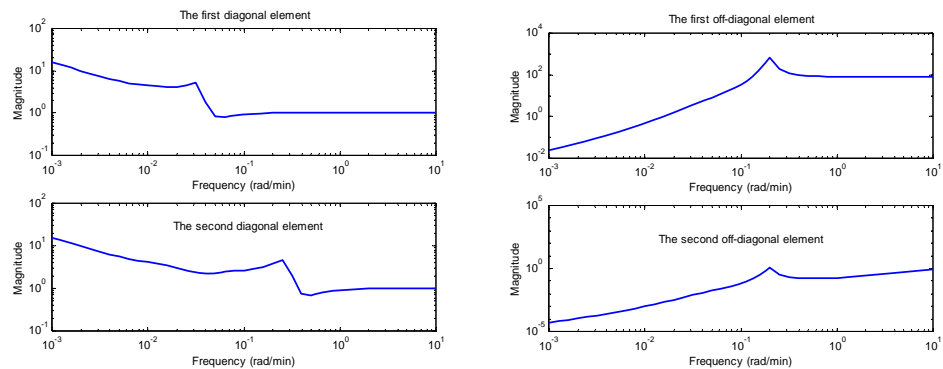


Fig. 3 DRMC elements ( $x_F = 7$ )

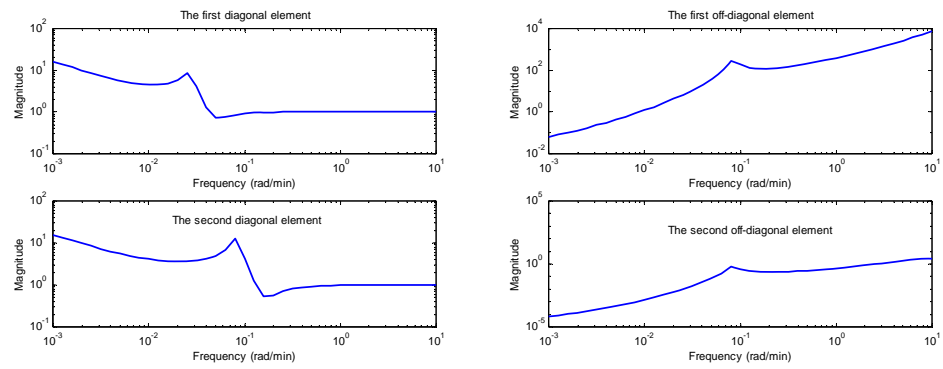


Fig. 4 DRMC elements ( $x_F = 3$ )

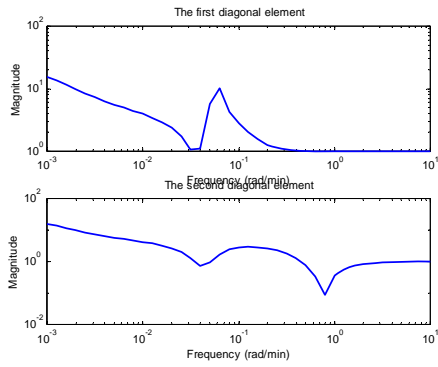


Fig. 5 DRMC elements ( $x_F = 1$ )

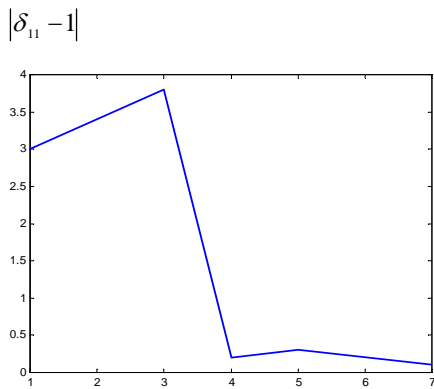
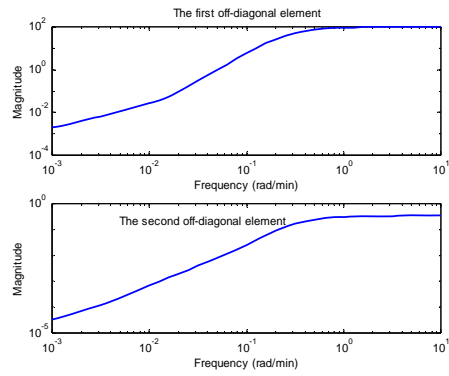


Fig. 6 The distance between the first diagonal elements and unity with respect to feed plate location

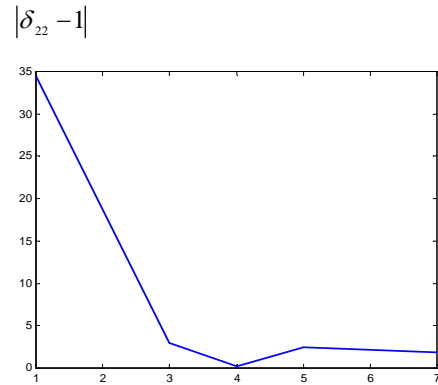


Fig. 7 The distance between the second Diagonal elements and unity with respect to feed plate location