

# Prediction of Dissolved Oxygen in Rivers Using a Wang-Mendel Method – Case Study of Au Sable River

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**Abstract**—Amount of dissolve oxygen in a river has a great direct affect on aquatic macroinvertebrates and this would influence on the region ecosystem indirectly. In this paper it is tried to predict dissolved oxygen in rivers by employing an easy Fuzzy Logic Modeling, Wang Mendel method. This model just uses previous records to estimate upcoming values. For this purpose daily and hourly records of eight stations in Au Sable watershed in Michigan, United States are employed for 12 years and 50 days period respectively. Calculations indicate that for long period prediction it is better to increase input intervals. But for filling missed data it is advisable to decrease the interval. Increasing partitioning of input and output features influence a little on accuracy but make the model too time consuming. Increment in number of input data also act like number of partitioning. Large amount of train data does not modify accuracy essentially, so, an optimum training length should be selected.

**Keywords**—Dissolved Oxygen, Au Sable, Fuzzy Logic Modeling, Wang Mendel

## I. INTRODUCTION

FORECASTING refers to a process by which the future behavior of a dynamical system is estimated based on our understanding and characterization of the system. If the dynamical system is not stable, the initial conditions become one of the most important parameters of the time series response, i.e. small differences in the start position can lead to a completely different time evolution. This is what is called sensitive dependence on initial conditions, and is associated with chaotic behavior [1,15] for the dynamical system.

More recently, soft computing [9] methodologies, such as neural networks, fuzzy logic, and genetic algorithms, have been applied to the problem of forecasting complex time series. These methods have shown clear advantages over the traditional statistical ones [11]. The main advantage of soft computing methodologies is that, we do not need to specify the structure of a model a priori, which is clearly needed in the classical regression analysis [2]. Also, soft computing models are non-linear in nature and they can approximate more easily complex dynamical systems, than simple linear statistical models. Of course, there are also disadvantages in using soft computing models instead of statistical ones. In classical regression models, we can use the information given by the parameters to understand the process. However, if the main

objective is to forecast as closely as possible the time series, then the use of soft computing methodologies for prediction is clearly justified.

The use of fuzzy set theory allows the user to include the unavoidable imprecision in the data. Fuzzy inference is the actual process of mapping from a given set of input variables to an output based on a set of fuzzy rules. The essence of the modeling is to identify fuzzy rules. Four fundamental units are necessary for the successful application of any fuzzy modeling approach. These are, namely, the fuzzification unit, the knowledge base (which is composed of the database and the rule base), the inference engine and defuzzification unit [16, 17]. The main problem with fuzzy logic modeling is related to the choice of the parameters. For this reason some methods such as ANFIS (Adaptive Network based Fuzzy Inference System), firstly proposed by Jang [18], Wang-Mendel [19] and etc. may be applied. Wang-Mendel is one of the easiest methods which lay in ad-hoc fuzzy logic modeling category. This technique is expressed in detail in section III.

Dissolved oxygen is one of the best indicators of the health of a water ecosystem. Dissolved oxygen can range from 0-18 parts per million (ppm), but most natural water systems require 5-6 parts per million to support a diverse population. Oxygen enters the water by direct absorption from the atmosphere or by plant photosynthesis. The oxygen is used by plants and animals for respiration and by the aerobic bacteria which consume oxygen during the process of decomposition. When organic matter such as animal waste or improperly treated wastewater enters a body of water, algae growth increases and the dissolved oxygen levels decrease as the plant material dies off and is decomposed through the action of the aerobic bacteria.

Decreases in the dissolved oxygen levels can cause changes in the types and numbers of aquatic macroinvertebrates which live in a water ecosystem. Species which cannot tolerate decreases in dissolved oxygen levels include mayfly nymphs, stonefly nymphs, caddisfly larvae and beetle larvae. As the dissolved oxygen levels decrease, these pollution-intolerant organisms are replaced by the pollution-tolerant worms and fly larvae.

Dissolved oxygen levels change and vary according to the time of day, the weather and the temperature. If yearly comparisons are made on dissolved oxygen levels, they should be done at the same time of day, during the same season and on a day with a temperature variation of only 10 degrees Celsius from the previous reading. A decrease in the dissolved oxygen levels is usually an indication of an influx of some type of organic pollutant.

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In this research it is tried to apply Wang Mendel method as an easy and efficient fuzzy logic modeling to predict dissolve oxygen in rivers. In order to check the method, records of eight stations in Au Sable watershed in Michigan, United States, is considered. Data corresponding to these stations represent various situations and may used for many purposes.

## II. SITE DESCRIPTION

The Au Sable River in Michigan runs approximately 208 km through the northern Lower Peninsula, through the towns of Grayling and Mio, and enters Lake Huron at Oscoda. It drops approximately 200 meters from its point-of-origin. It has a drainage basin of about 5,000 km<sup>2</sup> in north-central lower Michigan. The basin is approximately 145 km long and 16 to 48 km wide. The river basin is partially within the Huron National Forest and includes parts of Otsego, Montmorency, Crawford, Osco, Alcona, Roscommon, Ogemaw, and Iosco counties (Fig. 1). There are approximately 762 km of streams in the Au Sable River system. The mainstream includes 60 km

of impoundments. Table I is a list of streams within the watershed (a few small unnamed streams are not included).

### Physiography

The topography of the Au Sable river basin is rolling to flat. Maximum elevation above sea level is approximately 441 meter in the extreme western portion and the minimum elevation is approximately 183 meter on the extreme eastern end. The river basin has an approximate fall of 204 meter. The western half of the river basin is generally flat to slightly rolling and the eastern half is flat-broken only by stream channels.

Low swamps and marshes are common throughout the western half of the river basin, particularly in the river headwaters and margins (Fig. 2). The eastern half is drained and has relatively few lowland areas. The Au Sable's outstanding scenery is presented in dramatic fashion

TABLE I  
 AU SABLE STREAMS SYSTEM

Stream name	Length (km)	Stream name	Length (km)
Au Sable River (Mainstream)	208	Beaver Creek 5	8
Bradford Creek 5	8	Big Creek 4	6
Kolka Creek 8	13	Red Creek 2	3
East Branch Au Sable 17	27	West Branch Big Creek 14	23
Barker Creek 3	5	Hunt Creek 3	5
Wakely Creek 2	3	East Branch Big Creek 11	18
South Branch Au Sable 37	60	Lost Creek 8	13
Sauger Creek 2	3	Honeywell Creek 6	10
Douglas Creek 3	5	Wolf Creek 3	5
Thayer Creek 5	8	Cherry Creek 7	11
Hickey Creek 4	6	Loud Creek 2	3
Beaver Creek 10	16	Perry Creek 9	14
Robinson Creek 5	8	Couchy Creek 2	3
Hudson Creek 6	10	Comins Creek 4	6
East Creek 5	8	Glennie Creek 3	5
South Creek 2	3	Nine Mile Creek 3	5
Connors Creek 2	3	Blockhouse Creek 6	10
North Branch Au Sable 36	58	Wilbur Creek 5	8
Turtle Creek 4	6	Bamfield Creek 5	8
Chub Creek 5	8	Smith Creek 5	8
Big Creek 1	2	Stewart Creek 4	6
West Branch Big Creek 18	29	Hoppy Creek 3	5
Middle Branch Big Creek 9	14	South Branch Big Creek 7	11
East Branch Creek 14	23	Harper Creek 4	6
Wright Creek 7	11	Baker Creek 3	5
Whitewater Creek 2	3	Wildcat Creek 2	3
Sohn Creek 4	6	TOTAL	762

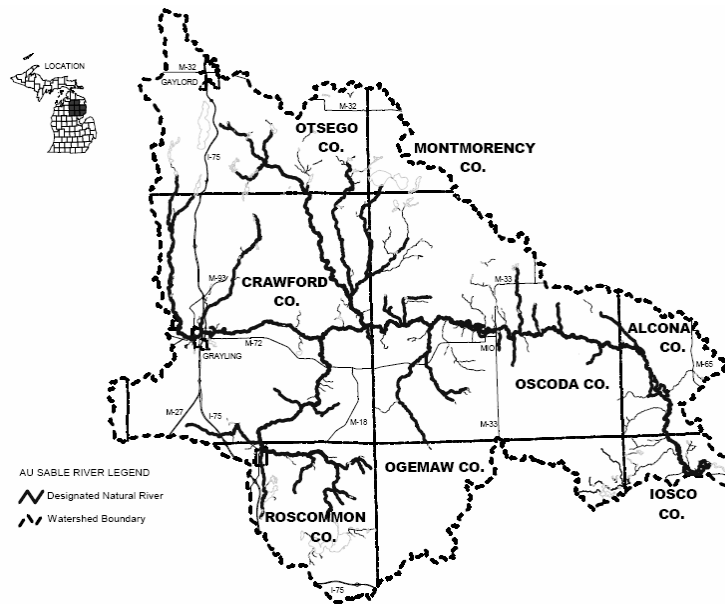


Fig. 1 The Au Sable River map [20]

by constantly changing topography. Each landform situation offers an attractive and varying display of geologic and vegetative conditions. High bluffs, lowland swamps, gentle slopes, river banks, upland plateaus and marshland often fluctuate over relatively short distances and provide background for the river's outstanding scenic resources. The watershed, like all others in the State of Michigan, shows the effects of glacial action. It lies in an area once covered by the Michigan Lake of the Pleistocene glacier and is characterized by glacial moraines and outwash plains. The basin is underlain by glacial drift up to several hundred meters deep with no outcroppings of bedrock material. The morainal areas are hilly with bold detached ridges. Outwash areas are relatively flat undulating plains except where cut by stream channels. The ancient lake bed area west of Oscoda is extremely flat and was covered during ancient glacial periods by the waters of Lake Huron. There are excellent examples of the effects of the ice, water, and wind on the landscape. Kettle lakes, oxbow lakes, eskers, drumlins, kames, terraces, sandblows, and deltas can be observed in the watershed.

#### Climate

The Au Sable River basin offers a climate typical of the state's "north country". The warm days and cool nights offer a pleasant haven for vacationers. The winters provide an excellent climate for skiers, snowmobiling, and other winter sports.

Weather data for the Au Sable basin indicates a record high of 44.5 Centigrade degrees with the record low of -44 Centigrade degrees, both recorded at Mio. A temperature of 38 is reached on an average of once in 10 years. At the other extreme, one can expect temperatures to fall below -18 an average of 25 days per year. The average yearly temperature for the basin is 6.2 Centigrade degrees. Precipitation is heaviest during the summer season averaging 63 percent of the

annual total during the six month period, April through September.

Heaviest rainfall for the basin is in September which shows an average of 86 millimeters. Lowest rainfall occurs in February with an average of 33 millimeters. Annual precipitation averages 719 for the 24 years of record. Summer skies tend to be generally free of cloud cover and westerly breezes are nearly constant. Winter skies are generally cloud covered and windy.

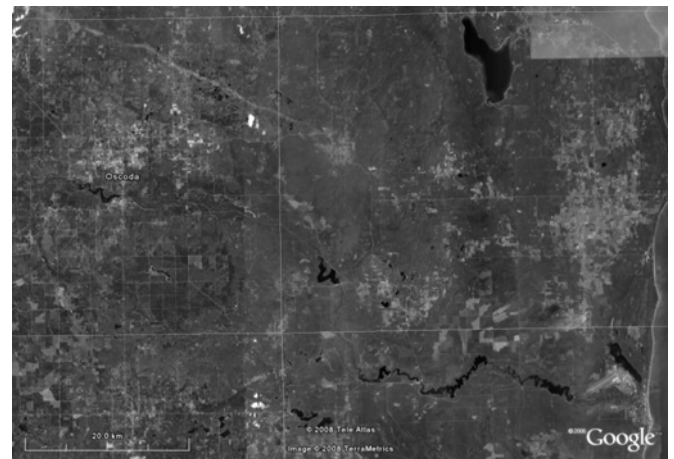


Fig. 2 Aerial picture from Au Sable watershed region

#### Water Quality

Highly stable water flows of very high quality water may be the single most significant trait of the Au Sable River. The coarse sand-gravel composition of the watershed allows rapid infiltration of water and tends to level precipitation into a steady groundwater contribution to stream flow. Water flows vary insignificantly throughout the season because most

inflow is from groundwater sources. This spring seepage is also an important factor to help maintain lower stream temperature during the summer months. However, river flow rates may respond to very rapid snowmelts and some sections will experience increases in water level and turbidity. High or dangerous water conditions are rare. The greatest river discharge occurs during April following snowmelt with an average discharge at Mio from 1961 to 1965 of 36.4 m<sup>3</sup>/s. The average discharge drops to 32.9 m<sup>3</sup>/s in May; 24.5 in June and 21.1 in July as compared to annual average of 28.0 m<sup>3</sup>/s.

In addition to a stable flow, the water quality of the Au Sable River system is very high when compared to other rivers in the state. Using the standardized Water Quality Index, the Au Sable River at its mouth is shown to average 85.9. Water quality index consists of averaging numerical values from chemical, physical and biological parameters collected from monitoring stations on the river. Parameters used in establishing the WQI are: dissolved oxygen (D.O.), Fecal Coliforms, PH, Biochemical oxygen demand (BOD5), NO<sub>3</sub>-N (nitrogen), PO<sub>4</sub>-P (phosphates, temperature, turbidity and dissolved solids. Water quality of the Au Sable river system is protected for the following uses: (a) total body contact recreation; (b) agriculture; (c) industrial water supply; (d) navigation; and (e) public water supply. Most of the mainstream and tributaries, at least above Loud Dam are classed as cold water trout streams. Any designated stretches of the river system will be governed by the "nondegradation" rule of the Water Resources Commission's water quality standards.

### III. WANG AND MENDEL'S METHOD

The ad hoc data-driven RB generation process proposed by Wang and Mendel in [19] has been widely known because of its simplicity and good performance. It is based on working with an input-output data set  $E = \{e_1, \dots, e_p\}$  where  $e_l = (x_1^l; \dots; x_n^l; y^l)$ , representing the behavior of the problem being solved, using a previous definition of the data base composed of the input and output primary fuzzy partitions.

The generation of the rule base is put into effect by means of the following steps:

#### 1. Consider a Fuzzy Partition of the Input Variable Spaces

It may be obtained from the expert information (if it is available) or by a normalization process. If the latter is the case, perform a fuzzy partition of the input variable spaces dividing each universe of discourse into a number of equal or unequal partitions, select a kind of membership function and assign one fuzzy set to each subspace. In our case, we will work with symmetrical fuzzy partitions of triangular membership functions (Fig. 3).

#### 2. Generate a Candidate Linguistic Rule Set

This set will be formed by the rule best covering each example (input-output data pair) contained in  $E$ . Thus,  $p$  candidate linguistic rules will be obtained. The structure of these rules is obtained by taking a specific example, i.e., an  $n + 1$  dimensional real array ( $n$  input and 1 output values), and setting each one of the variables to the linguistic label (associated fuzzy set) best covering every array component.

#### 3. Give an Importance Degree to Each Rule

Let  $R_l : IF x_1 \text{ is } A_1 \text{ and } \dots \text{ and } x_n \text{ is } A_n \text{ THEN } y \text{ is } B$  be the linguistic rule generated from the example  $e_l, l = 1, \dots, p$ . The importance degree associated to it will be obtained as follows:

$$G(R_l) = \mu A_1(x_1^l) \cdot \dots \cdot \mu A_n(x_n^l) \cdot \mu B(y^l) \quad (1)$$

#### 4. Obtain a Final RB from the Candidate Linguistic Rule Set

To do so, the  $p$  candidate rules are first grouped in  $g$  different groups, each one of them composed of all the candidate rules presenting the same antecedent. We will note by  $R_{ij}$  the  $j$ -th rule in the  $i$ -th group. To compose the final rule base, the rule with the highest importance degree is chosen in each group  $i, i = 1, \dots, g$ . Hence,  $g$  will be both the number of different antecedent combinations in the candidate rule set and the number of linguistic rules in the final rule base generated.

The good behavior of the WM-method has been clearly demonstrated. However, sometimes the method does not perform as good as desired. It is due to a problem related to the way in which the rules are selected. One of the most interesting features of a Fuzzy Rule Base System (FRBS) is the interpolative reasoning it develops. This characteristic plays a key role in the high performance of FRBSs and is a consequence of the cooperation among the fuzzy rules composing the knowledge base. As it is known, the output obtained from a FRBS is not usually due to a single fuzzy rule but to the cooperative action of several fuzzy rules that have been fired because they match the system input to any degree. However, the operation mode followed by WM- method is to bracket the example data set into fuzzy subspaces (the antecedent combinations mentioned in step 4 of the algorithm) according to the covering degree, and to obtain afterwards the rule with the best performance in each subspace. Therefore, the global interaction among the rules of the knowledge base is not considered. This causes the finally obtained rule set, in spite of presenting a good local behavior, not to cooperate suitably. Moreover, the fact of locally processing these rules makes the method be more sensitive to noise.

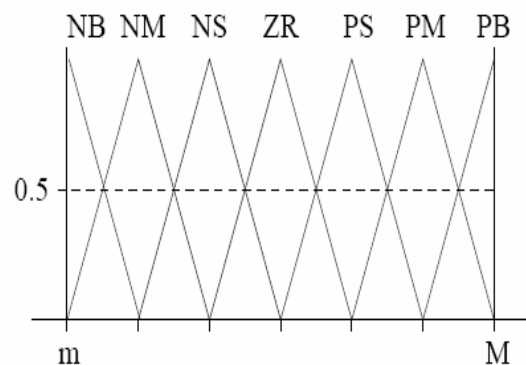


Fig. 3 Graphical representation of a uniform fuzzy partition

### IV. DATA, ANALYSES AND RESULTS

The dissolved oxygen record data studied herein were collected at eight US Geological Survey stations in Au Sable

watershed. Tables IIA, IIB and IIC present some characteristics of the rivers and statistics of daily and hourly data. Daily records are available at least for twelve years while hourly records are available for at least fifty days. Aerial pictures from eight stations are shown in Figs. 4a to 4e.

TABLE IIA  
 SOME CHARACTERISTICS OF THE STATIONS UNDER STUDY

Station No.	Location	Drainage Area (km <sup>2</sup> )	Datum of gag (m) above see level
04136000	Latitude 44°40'37", Longitude 84°17'33", NEAR RED OAK, Oscoda County, MI	2870	304.8
04136500	Latitude 44°39'36", Longitude 84°07'52", MIO, Oscoda County, MI	3525	283.3
04136900	Latitude 44°36'46", Longitude 83°50'16", NEAR MC KINLEY, Alcona County, MI	3919	253.0
04137005	Latitude 44°33'39", Longitude 83°48'10", NEAR CURTISVILLE, Alcona County, MI	4139	237.2
04137020	Latitude 44°27'48", Longitude 83°43'17", NEAR SOUTH BRANCH, Iosco County, MI	4374	-
04137025	Latitude 44°27'15", Longitude 83°40'28", NEAR GLENNIE, Iosco County, MI	4393	-
04137030	Latitude 44°28'22", Longitude 83°34'16", NEAR SIDTOWN, Iosco County, MI	4450	-
04137500	Latitude 44°26'11", Longitude 83°26'02", NEAR AU SABLE, Iosco County, MI	4504	178.0

The available time series are tried to be modeled using Wang-Mendel as one of fuzzy modeling techniques. As it was mentioned before, Wang Mendel before consists of plenty of parameters that affect on efficiency of modeling. Here it is attempted to investigate effect of each parameter.

Increase in input data interval reduces estimation approximation but for long periods, it can detect the variation trend intuitively. Figs. 5a to 5d show estimation of one and two-year periods with different learning periods for the first station. In Figs. 5a to 5c input data intervals are considered to be 30 days and 5 data are used to estimate the next values. In Fig. 5d, input data intervals are considered to be one day and 7 data are applied to estimate the next values. It can obviously observed that overall reduction and increment can be predicted by first three figures but in the forth one, even, the overall trend can not be predicted. It can be also observed from Figs. 5a to 5c that increasing in learning period in Wang Mendel technique has no significant effect in estimations, but it is strongly proposed that learning period consists at least two return periods, i.e., years.

In spite of overall trend, while estimation of just a number of values are considered, e.g. filling missed data, decreasing input data intervals are recommended. Fig. 6 shows estimation of next day dissolved oxygen using previous 7 days values with on day interval for daily records of the first station. In this case Root Mean Square Error become 0.47 mg/L while for 30 days interval the amount of RMSE becomes 0.82 which shows less accuracy.

TABLE IIB  
 STATISTICS OF DAILY RECORDS OF DISSOLVED OXYGEN, TEMPERATURE AND DISCHARGE FROM THE STATIONS

Station No.	Maximum DO from daily records (mg/L)	Minimum DO from daily records (mg/L)	Mean DO from daily records (mg/L)	Standard deviation of DO from daily records (mg/L)	Mean temperature from daily records (°C)	Mean discharge from daily records (m <sup>3</sup> /sec)
04136000	14.8	7	10.78	1.78	8.9	21.1
04136500	14.3	6.3	10.24	1.93	9.8	26.2
04136900	15.3	6.1	10.34	1.86	10.3	30.6
04137005	13.6	4.3	10.01	2.12	9.9	32.1
04137020	14.0	5.3	10.20	2.28	10.5	-
04137025	14.5	4.6	10.14	2.29	10.5	-
04137030	14.5	4.8	9.94	2.37	11.2	-
04137500	13.5	5.7	9.94	2.18	11.0	37.0

TABLE IIC  
 STATISTICS OF DAILY RECORDS OF DISSOLVED OXYGEN, TEMPERATURE AND DISCHARGE FROM THE STATIONS

Station No.	Maximum DO from hourly records (mg/L)	Minimum DO from hourly records (mg/L)	Mean DO from hourly records (mg/L)	Standard deviation of DO from hourly records (mg/L)	Mean temperature from hourly records (°C)	Mean discharge from hourly records (m <sup>3</sup> /sec)
04136000	13.6	11.5	12.64	0.50	2.6	20.7
04136500	13.4	11.7	12.61	0.36	1.8	28.1
04136900	14.1	7.9	11.07	1.08	2.2	40.2
04137005	13.5	11.1	12.05	0.57	1.6	35.0
04137020	13.8	12.0	12.93	0.48	1.2	-
04137025	13.3	11.6	12.39	0.24	1.1	-
04137030	13.7	11.0	13.03	0.55	0.9	-
04137500	13.4	10.6	11.5	0.59	0.9	36.6

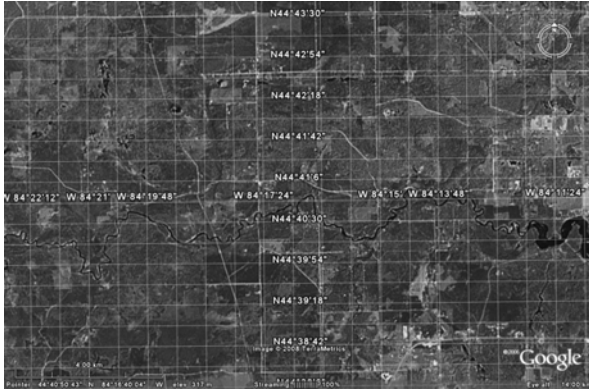


Fig. 4a Aerial picture from station 1



Fig. 4b Aerial picture from station 2

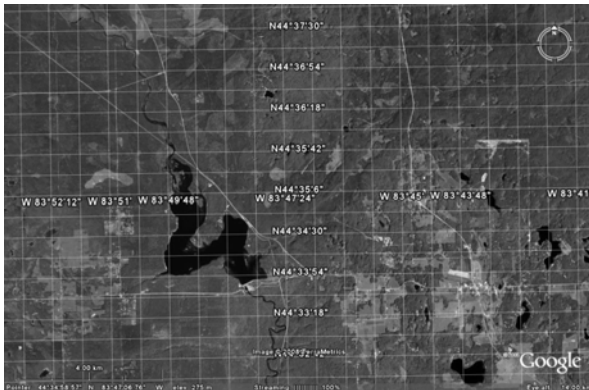


Fig. 4c Aerial picture from stations 3 and 4

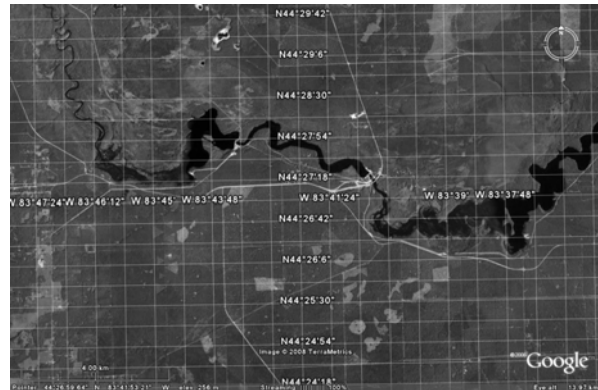


Fig. 4d Aerial picture from stations 5 and 6

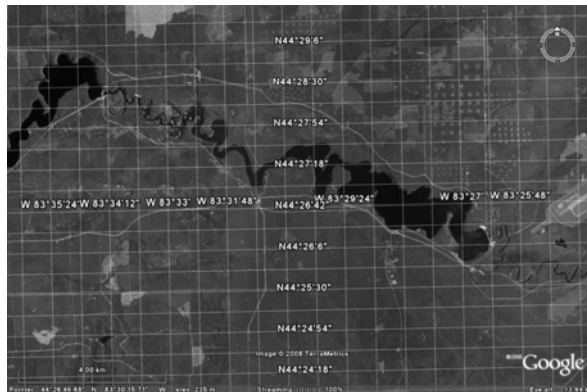


Fig. 4e Aerial picture from stations 7 and 8

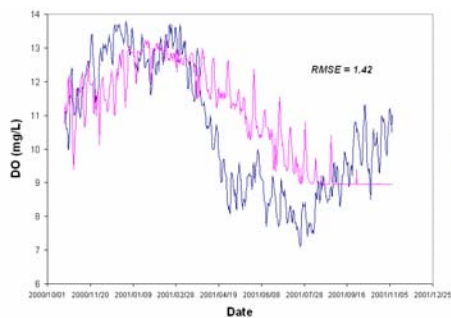


Fig. 5a Comparison of estimated and measured values with 3 years learning period and 30-days interval input

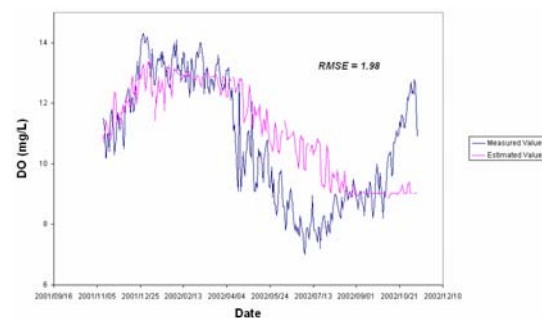


Fig. 5b Comparison of estimated and measured values with 4 years learning period and 30-days interval input

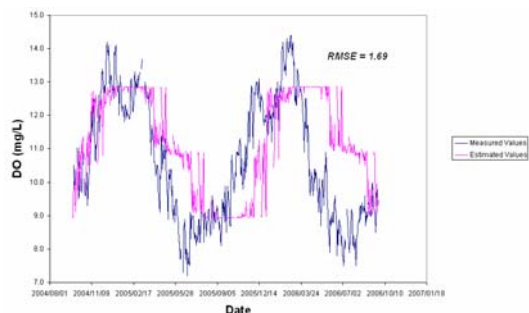


Fig. 5c Comparison of estimated and measured values with 6 years learning period and 30-days interval input

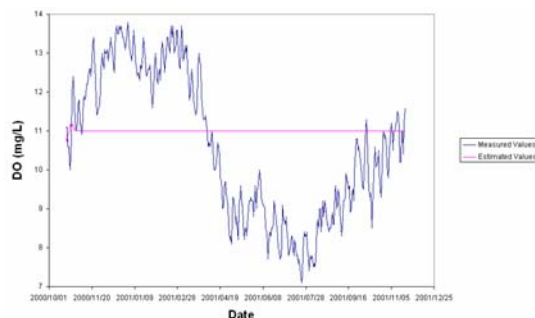


Fig. 5d Comparison of estimated and measured values with 1-day interval input

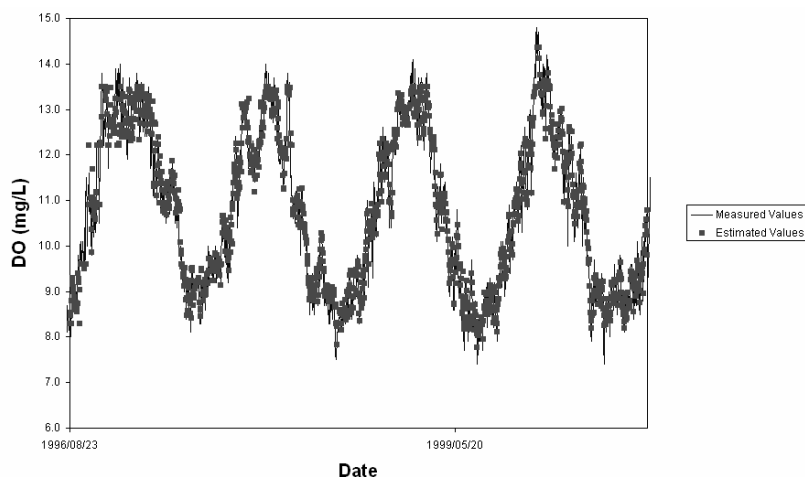


Fig. 6 Estimation of any next day value of dissolved oxygen using 7 one-day interval pervious data

Effect of number of output partitioning and input features and other parameters of Wang Mendel method are shown in Tables IIIA and IIIB. It can be observed that increasing number of features twice just make the accuracy 10 percent. This makes the period of model run about 10 times.

#### V. CONCLUDING REMARKS

Wang Mendel technique is used in this research to model dissolve oxygen in eight stations in Au Sable river. In order to increase accuracy of estimation some parameters in WM model should be tuned. Imposing this model to the records we conclude:

- a. To model a long period such as a year, it is better to use long intervals to cover a significant period. Number of input data also may help us in this purpose. For example 6 input data with 30-days

interval is suitable, since it cover a half year and is enough to detect the trend

- b. In hourly forecasting or short period prediction, the upcoming values do not relate to far records so, it is convenient to use a limited number of input data with close intervals in the model.
- c. Very low and very large number of partitioning in input and output features arise some problems. Low number of partitioning may result in rough and inaccurate outcome. Very large number of partitioning increase the run period of the model considerably and do not guarantee efficiency of the model necessarily. 5 divisions, seems to be suitable value for input and output features partitioning.

TABLE IIIA  
 ROOT MEAN SQUARE ERROR (RMSE) FOR DAILY AND HOURLY ESTIMATION USING 5 PRIOR RECORDS

Station No.	Daily estimation				Hourly estimation			
	1-day interval		30-day intervals		1-hour interval		12 hours intervals	
	n*=5	n=9	n=5	n=9	n=5	n=9	n=5	n=9
04136000	0.47	0.42	0.82	0.81	0.14	0.14	0.14	0.15
04136500	0.43	0.40	0.76	0.64	0.13	0.11	0.14	0.13
04136900	0.45	0.38	0.79	0.78	0.11	0.12	0.12	0.11
04137005	0.39	0.35	0.73	0.71	0.12	0.10	0.11	0.12
04137020	0.42	0.36	0.76	0.73	0.15	0.16	0.13	0.15
04137025	0.38	0.33	0.72	0.57	0.13	0.14	0.14	0.13
04137030	0.36	0.29	0.71	0.55	0.16	0.15	0.15	0.17
04137500	0.37	0.35	0.71	0.56	0.15	0.16	0.15	0.13

\* Number of input and output feature partitioning

TABLE IIIB  
ROOT MEAN SQUARE ERROR (RMSE) FOR DAILY AND HOURLY ESTIMATION USING 12 PRIOR RECORDS

Station No.	Daily estimation				Hourly estimation			
	1-day interval		30-day intervals		1-hour interval		12 hours intervals	
	n=5	n=9	n=5	n=9	n=5	n=9	n=5	n=9
04136000	0.38	0.36	0.53	0.61	0.11	0.12	0.16	0.13
04136500	0.38	0.38	0.50	0.37	0.12	0.10	0.14	0.13
04136900	0.39	0.30	0.52	0.69	0.07	0.10	0.11	0.11
04137005	0.38	0.29	0.49	0.48	0.11	0.09	0.11	0.12
04137020	0.41	0.28	0.63	0.68	0.11	0.12	0.11	0.14
04137025	0.29	0.29	0.64	0.36	0.11	0.10	0.13	0.13
04137030	0.32	0.22	0.45	0.40	0.13	0.10	0.16	0.19
04137500	0.37	0.27	0.60	0.44	0.14	0.12	0.15	0.11

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