

# A New Time Dependent, High Temperature Analytical Model for the Single-electron Box in Digital Applications

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**Abstract**—Several models have been introduced so far for single electron box, SEB, which all of them were restricted to DC response and or low temperature limit. In this paper we introduce a new time dependent, high temperature analytical model for SEB for the first time. DC behavior of the introduced model will be verified against SIMON software and its time behavior will be verified against a newly published paper regarding step response of SEB.

**Keywords**—Single electron box, SPICE, SIMON, Time dependent, Circuit model.

## I. INTRODUCTION

**S**INGLE electron devices are very interesting because of their very small size and ultra low power consumption.

They have been used in many digital [1-2] and analog [3] applications so far and are a candidate for replacing MOSFETs in future. Single electron devices have many varieties including Single Electron Tunneling Junction (SETJ), Single Electron Box (SEB), Single Electron Transistor (SET), etc, among all; the SEB is the simplest device and is the building block of all. A SEB consists of a small piece of metal, called the island, surrounded by an insulator material. There are two contacts too (see figure 1). The first contact, called the source, is very close to the island so that electrons can tunnel from the source through the insulator to the island; a tunnel junction. The other contact is not very close to the island and can affect the island voltage only with a capacitance effect; a capacitance junction. The electrical model therefore composed of a parallel RC branch for the tunneling junction and a capacitor as shown in figure 1.c. At low temperature, the SEB's main operation procedure is as follows. Assume that the gate-source voltage is equal to zero,  $V_{gs} = 0$  and there is no electron in the island at first. Now, if  $V_{gs}$  gradually increases, then the island-source voltage,  $V_{is}$ , is also increases. At a certain point, when the voltage of  $V_{is}$  is such that if an electron tunnels from the source to the island, then the tunneled electron causes a decrement in  $V_{is}$  twice of its initial value, in that point, an electron will tunnel to the island, hence, the island voltage flips, i.e., becomes negative as much as it was positive before the tunneling event. The island voltage is calculated from:

$$V_{is} = \frac{V_{gs} C_{gi}}{C_T} - \frac{Q}{C_T}, \quad (1)$$

where  $C_{gi}$  is the gate-island capacitor,  $Q$  is the charge of the island, and,  $C_T$  is the total capacitor;  $C_T = C_{gi} + C_{is}$ .

At the steady state and low temperature limit the island charge is equal to  $ne$  where  $n$  is the number of electrons and  $e$  is the unit charge. The threshold voltage,  $V_T$ , which is the  $V_{gs}$  voltage at the tunneling point is as follows:

$$V_T = \frac{e}{2C_{gi}}. \quad (2)$$

The basic theory of single electronics is the orthodox theory [4] and there are three main approaches for calculating single-electron devices including SEB; numerical methods, circuit models, and, analytical methods. The numerical methods are general methods which can simulate any single electron device including SEB. The circuit modeling has unique benefit that can be used when the device is in use in a circuit in conjunction with other circuit elements such as current sources, transistors, resistors, etc. To the best of our knowledge, there have been introduced two circuit models for SEB so far [3, 5], but no analytical model are introduced for the SEB yet. This is the goal of this paper to introduce a complete, i.e., time dependent and high temperature, analytical model for the SEB which is suitable for binary digital applications, i.e., when we work only with two states; zero and one.

Organization of rest of this paper is as follows. In Section II, we will introduce our analytical model. Section III discusses on some limiting cases and verifies the model by comparing its results with the previous results (whenever there exists a previous result). Section IV discusses on the limitation of the model, and, Section V devotes to conclusions.

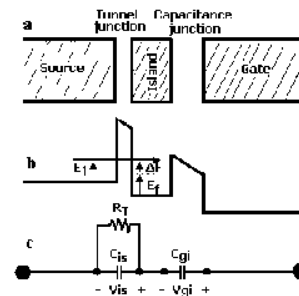


Fig. 1 (a) Scheme for the physical structure of an SEB. (b) Energy band diagram. (c) Equivalent circuit

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## II. THE NEW ANALYTICAL MODEL

As said before, the Basic theory regarding the single electron devices is the orthodox theory. In this theory we take state of the system to be equal to the number of electrons in all islands of the system. If we have only one island, which SEB is of this type, the theory after some simplifications and approximations [4] reduces to the following set of coupled differential equations.

$$\frac{\partial p_n}{\partial t} = \Gamma_{n+1,n} p_{n+1} + \Gamma_{n-1,n} p_{n-1} - \Gamma_{n,n+1} p_n - \Gamma_{n,n-1} p_n \quad (3)$$

where  $p_n$  is the probability the system being in the  $n$ th. state (i.e., there are  $n$  electrons in the island), and,  $\Gamma_{i,f}$  is the transition rate from an initial state  $i$  to a final state  $f$ :

$$\Gamma_{i,f} = \frac{-\Delta F}{e^2 R_T (1 - e^{\frac{\Delta F}{k_B T}})} \quad (4)$$

where  $R_T$  is the tunneling resistance,  $k_B$  is the Boltzmann constant,  $T$  is the absolute temperature, and,  $\Delta F$  is the difference in free energy:

$$\Delta F = \Delta(E_C - W), \quad (5)$$

Where  $E_C$  the stored energy in the two capacitors and,  $W$  is is the work done by the voltage source:

$$\Delta F = \frac{e^2 (1 + 2n\Delta n)}{2C_T} - e V_{gs} \frac{C_{gi}}{C_T} \Delta n, \quad (6)$$

where  $\Delta n$  is the difference in the state's number before and after the tunneling event. For digital applications it is enough to consider only  $n = 0$  and  $n = 1$  states. In this case we will have:

$$\frac{\partial p_1}{\partial t} = \Gamma_{0,1}(1 - p_1) - \Gamma_{1,0} p_1, \quad (7)$$

$$Q(t) = e p_1.$$

This simple first order differential equation has the following analytical solution:

$$p(t) = e^{-\int_0^t \Gamma_{0,1}(t') \Gamma_{1,0}(t') dt'} \left[ \int_0^t e^{\int_0^{t'} \Gamma_{0,1}(t'') \Gamma_{1,0}(t'') dt''} \Gamma_{0,1}(t') dt' \right] \quad (8)$$

In the following, we consider this analytical solution in some limiting cases and will verify it against the existing solutions.

## III. MODEL'S VERIFICATION

At the steady state, in which there are famous numerical methods, the solution will be:

$$\Gamma_{0,1}(1 - p_1) - \Gamma_{1,0} p_1 = 0$$

$$p_1 = \frac{\Gamma_{0,1}}{\Gamma_{0,1} + \Gamma_{1,0}} \quad (9)$$

We have compared our results with the results of SIMON [6] in this case. Figure 2 shows the results. As we see from the figure, there is a good agreement between the two results. In fact, our results, which comes from analytical solution is much

more accurate then the results of SIMON which comes from a numerical procedure.

In the low temperature limit we have:

$$\begin{cases} \Gamma_{0,1} = -\Delta F_{0,1} \\ \Gamma_{1,0} = 0 \end{cases} \text{ for } \Delta F_{0,1} < 0$$

$$\begin{cases} \Gamma_{0,1} = 0 \\ \Gamma_{1,0} = -\Delta F_{0,1} \end{cases} \text{ for } \Delta F_{0,1} > 0 \quad (10)$$

Therefore, at low temperature and steady state limit, we have:

$$\begin{cases} p_1 = 1 \\ p_0 = 0 \end{cases} \text{ for } \Delta F_{0,1} < 0$$

$$\begin{cases} p_1 = 0 \\ p_0 = 1 \end{cases} \text{ for } \Delta F_{0,1} > 0 \quad (11)$$

At the low temperature limit, when the applied  $V_{gs}$  is a square pulse, there is a simpler form as well:

$$p_1(t) = \begin{cases} 1 - e^{-\frac{(V_{gs} - V_T) C_{gi}}{q C_T R_T} t} & p_1(0) = 0 \\ e^{-\frac{(V_{gs} - V_T) C_{gi}}{q C_T R_T} t} & p_1(0) = 1 \end{cases} \quad (12)$$

There is a work [7] regarding the pulse of a SEB and we can compare above result with the results of that work. Figure 3 shows both results. Three square pulses with different amplitudes are applied to the SEB and the time dependent responses are calculated from equation 12 in conjunction with the previous results from numerical solution. Again we see that there is a good agreement between the two results. There is not any other work regarding the SEB that we compare our results with and we conclude that our analytical model is correct. In the following section we discuss on the limitation and shortcoming of the model.

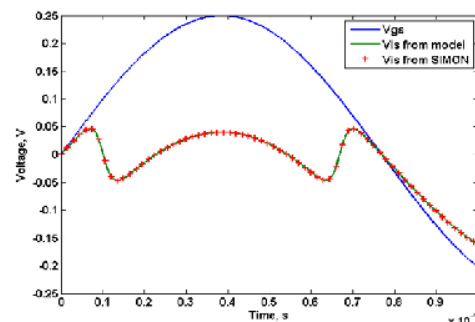


Fig. 2 Our model's result in comparison with the SIMON's results. The SEB specifications are  $C_{gi} = 0.8\text{aF}$ ,  $C_{is} = 0.2\text{aF}$ ,  $R_T = 1\text{M}\Omega$ ,  $T = 100\text{K}$

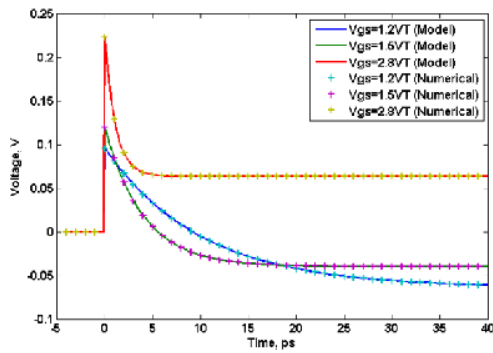


Fig. 3 Response of the SEB to some pulses with different amplitudes calculated from analytical model(solid lines) and numerical model (Ref. [7]). The SEB specifications are same as those in figure 2 and  $T = 0$

#### IV. MODEL'S LIMITATIONS

The limitations of the model comes from the fact that we ignore all terms of the master equation (Equ.3) other than the first two terms, i.e.,  $n = 0$  and  $n = 1$  terms. This leads to a limitation on the range of the voltages we can apply to the device and a limitation on the range of temperature even we work with the permitted  $V_{gs}$  voltage. Limitation on the voltage range is simple:

$$-V_T < V_{gs} < 3V_T \quad (13)$$

but, limitation on temperature range is a function of the concerning bit error, BE.

$$BE = e^{-\frac{e \left( \frac{|V_{gs}| C_{gi}}{2C_T} \right)}{K_B T}} \quad (14)$$

where  $K_B$  is the Boltzmann constant and  $T$  is the absolute temperature. For example if  $C_T = 1\text{aF}$  and we consider a BE equal to  $1e-4$ , then at  $T = 10\text{K}$ , we should take a distance equal to about 10% from the  $V_T$  edge. However, we see that in digital applications neither of these two limitations are important and the model can be used successfully.

#### V. CONCLUSIONS

This paper introduced an analytical model for the SEB for using in binary digital applications. Then three limiting cases distinguished and formulated. They were the steady state high temperature, steady state low temperature, and, low temperature transient state but with constant  $V_{gs}$ , i.e., pulse response. The model then was verified by comparison of its results with the results of SIMON (for steady state high temperature case) and numerical results (for pulse response). The comparisons proved the accuracy of our analytical model.

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