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Identification of Wideband Sources Using Higher Order Statistics in noisy environment

S. Bourennane, A. Bendjama

Abstract— This paper deals with the localization of the wideband sources. We develop a new approach for estimating the wide band sources parameters. This method is based on the high order statistics of the recorded data in order to eliminate the Gaussian components from the signals received on the various hydrophones. In fact the noise of sea bottom is regarded as being Gaussian. Thanks to the coherent signal subspace algorithm based on the cumulant matrix of the received data instead of the cross-spectral matrix the wideband correlated sources are perfectly located in the very noisy environment. We demonstrate the performance of the proposed algorithm on the real data recorded during an underwater acoustics experiments.

Keywords**—**higher-order statistics, high resolution array processing techniques, localization of acoustics sources, wideband sources.

I. Introduction

CONSIDER an oceanic waveguide, composed by a water column, with sound velocity profile which may vary with range and depth, bounded above by a pressure release surface and below by smoothly range dependent seafloor. Introduce a point source and an array of N sensors, both submerged within the water column at the coordinates z_o and z_i $i=(1,...,N)$ respectively. Designate the source strength, at frequencies f_n $n=(1,...,M)$, by $\mathbf{s}(f_n)$. The complex sound pressure at z_k is constituted by the fields scattered by the objects in response to a source (plane wave) illumination plus additional noise

$$r(z_k,f_n)=\sum_{p=1}^{P}\alpha_p(z_k,z_p)s_p(f_n)\exp(-2i\pi f_n\tau_{pk}+n(z_k,f_n) \qquad (1)$$

where:

$s_p\alpha_p(z_k,z_p)$: Attenuation and spreading along ray p Associated to the pth object.

τ_{pk} : travel time hydrophone k along a ray p.

$n(z_k,f_n)$: Measurement noise for sensor k at the frequency f_n.

The noise terms are assumed to be spatially correlated, zero Mean Gaussian process with unknown cross-spectral matrix.

Let

$$\mathbf{r}(f_n)=[r(z_1,f_n),...,r(z_N,f_n)]^T$$

$$\mathbf{n}(f_n)=[n(z_1,f_n),...,n(z_N,f_n)]^T$$

And

$$\mathbf{s}(f_n)=[s_1(f_n),...,s_P(f_n)]^T$$

Then

$$\mathbf{r}(f_n)=\mathbf{A}(f_n)\mathbf{s}(f_n)+\mathbf{n}(f_n) \qquad for \quad n=1,...,M$$

(2)

Where

$$\mathbf{A}(f_n)=[a(f_n,\theta_1),...,a(f_n,\theta_P)]^T,$$

With

$$a(f_n,\theta_p)=\begin{bmatrix}\alpha_p(z_1,z_o)\exp(-2i\pi f_n\tau_{p1})\\ ....\\ \alpha_p(z_N,z_o)\exp(-2i\pi f_n\tau_{pN})\end{bmatrix}^T.$$

Given sources number P, the problem of interest is to estimate the DOAs of the sources in the region of interest from measurements. In addition to the model (1) we also assume that the signals are statistically independent. The higher order statistics of the received data are used to eliminate the Gaussian noise. The cumulant slice matrix is computed [1], [2]

$$\mathbf{C}_1(f_n)\overset{\Delta}{=}\mathrm{Cum}\left(r_1(f_n),r_1^*(f_n),\mathbf{r}(f_n),\mathbf{r}^+(f_n)\right) \qquad (3)$$
$$=\mathbf{A}(f_n)\mathbf{U}_s(f_n)\mathbf{A}^+(f_n)$$

Where $\mathbf{U}_s(f_n)$ is the diagonal kurtosis matrix, its ith element is defined as $\mathrm{Cum}\left(s(f_n),s^*(f_n),s(f_n),s^*(f_n)\right)$ with $i=(1,...,P)$. Then the $(P\times N)$ matrix denoted $\mathbf{H}(f_n)$ is formed in order to transform the received data with an aim of obtaining a perfect orthogonalization and eliminate the orthogonal noise component.

$$\mathbf{H}(f_n)=\mathbf{\Lambda}_s^{-1/2}(f_n)\mathbf{V}_s^+(f_n) \qquad (4)$$

Where $\mathbf{\Lambda}_s(f_n)$ and $\mathbf{V}_s(f_n)$ are the P largest eigenvalues and the corresponding eigenvectors of the slice cumulant matrix $\mathbf{C}_1(f_n)$ respectively. The $(P\times 1)$ vector of the transformed received data

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S. Bourennane is with the Institute of Fresnel, UMR-CNRS 6133 group GSM, D.U de St-Jérôme, 13397 Marseille Cedex 20 FRANCE (phone: +330491288038; e-mail: bourennane@fresnel.fr).

A. Bendjama is with the Institute of Fresnel, UMR-CNRS 6133 group GSM, D.U de St-Jérôme, 13397 Marseille Cedex 20 FRANCE (phone: +330491288038; e-mail: bendjama@univ-corse.fr).

$$\begin{aligned}\mathbf{r}_t(f_n) &= \mathbf{H}(f_n)\mathbf{r}(f_n)\\ &= \mathbf{H}(f_n)\mathbf{A}(f_n)\mathbf{S}(f_n) + \mathbf{H}(f_n)\mathbf{n}(f_n)\end{aligned} \tag{5}$$

Afterwards, the corresponding $\left(P^2 \times P^2\right)$ cumulant matrix can be expressed as

$$\begin{aligned}\mathbf{C}_t(f_n) &= \mathrm{Cum}\left(r_t(f_n), r_t^+(f_n), \mathbf{r}_t(f_n), \mathbf{r}_t^+(f_n)\right)\\ &= \left((\mathbf{HA})(f_n) \otimes (\mathbf{HA})^*(f_n)\right)\mathbf{U}_s(f_n).\\ &\quad \left((\mathbf{HA})(f_n) \otimes (\mathbf{HA})^*(f_n)\right)^+\end{aligned} \tag{6}$$

Or

$$\mathbf{C}_t(f_n) = \underbrace{\left(\mathbf{B}(f_n) \otimes \mathbf{B}^*(f_n)\right)}_{=\mathbf{B}_\otimes(f_n)}\mathbf{U}_s(f_n)\left(\mathbf{B}(f_n) \otimes \mathbf{B}^*(f_n)\right)^+ \tag{7}$$

We can show that

$$\begin{aligned}\mathbf{C}_t(f_n) &= \left(\mathbf{H}(f_n) \otimes \mathbf{H}^*(f_n)\right)\left(\mathbf{A}(f_n) \otimes \mathbf{A}^*(f_n)\right)\mathbf{U}_s(f_n).\\ &\quad \left(\mathbf{A}(f_n) \otimes \mathbf{A}^*(f_n)\right)^+\left(\mathbf{H}(f_n) \otimes \mathbf{H}^*(f_n)\right)^+\\ &= \left(\mathbf{H}(f_n) \otimes \mathbf{H}^*(f_n)\right)\mathbf{A}_\otimes(f_n)\mathbf{U}_s(f_n).\\ &\quad \mathbf{A}_\otimes^+(f_n)\left(\mathbf{H}(f_n) \otimes \mathbf{H}^*(f_n)\right)^+\end{aligned} \tag{8}$$

We have [1]

$$\mathbf{B}_\otimes(f_n) = \left(\mathbf{H}(f_n) \otimes \mathbf{H}^*(f_n)\right)\mathbf{A}_\otimes(f_n) \tag{9}$$

Using the eigenvectors focusing operator defined as

$$\mathbf{T}(f_o, f_n) = \mathbf{V}_{ts}(f_o)\mathbf{V}_{ts}^+(f_n) \tag{10}$$

Where $\mathbf{V}_{ts}(.)$ are the eigenvectors of the largest eigenvalues of the cumulant matrix $\mathbf{C}_t(.)$.

$$\begin{aligned}\hat{\mathbf{C}}_t(f_o) &= \frac{1}{M}\sum_{n=1}^{M}\mathbf{T}(f_o, f_n)\mathbf{C}_t(f_n)\mathbf{T}^+(f_o, f_n)\\ &= \mathbf{V}_{ts}(f_o)\hat{\mathbf{\Lambda}}_{ts}(f_o)\mathbf{V}_{ts}^+(f_o)\end{aligned} \tag{11}$$

Where $\hat{\mathbf{\Lambda}}_{ts}(f_o) = \frac{1}{M}\sum_{n=1}^{M}\mathbf{\Lambda}_{ts}(f_n)$ is the arithmetic mean of the first largest eigenvalues of the cumulant matrix $\mathbf{C}_t(f_n)$. It is easy to show that

$$\hat{\mathbf{C}}_t(f_o) = \mathbf{V}_{ts}(f_o)\hat{\mathbf{\Lambda}}_{ts}(f_o)\mathbf{V}_{ts}^+(f_o) = \mathbf{B}_\otimes(f_o)\hat{\mathbf{U}}_s(f_o)\mathbf{B}_\otimes^+(f_o) \tag{12}$$

Where $\mathbf{B}_\otimes(f_o) \stackrel{\Delta}{=} \mathbf{T}(f_o, f_n)\mathbf{B}_\otimes(f_n)$

And $\hat{\mathbf{U}}_s(f_o) = \frac{1}{M}\sum_{n=1}^{M}\mathbf{U}_s(f_n)$. Multiplying both sides by $\mathbf{B}_\otimes^+(f_o)$, we get

$$\mathbf{B}_\otimes^+(f_o)\hat{\mathbf{C}}_t(f_o) = \mathbf{B}_\otimes^+(f_o)\mathbf{B}_\otimes(f_o)\hat{\mathbf{U}}_s(f_o)\mathbf{B}_\otimes^+(f_o) \tag{13}$$

Because the columns of $\mathbf{B}_\otimes(f_o)$ are orthogonal and the sources are decorrelated [1], $\mathbf{B}_\otimes^+(f_o)\mathbf{B}_\otimes(f_o)\hat{\mathbf{U}}_s(f_o)$ is a diagonal matrix which we will denote by $\mathbf{D}(f_o)$, so that we have

$$\mathbf{B}_\otimes^+(f_o)\hat{\mathbf{C}}_t(f_o) = \mathbf{D}(f_o)\mathbf{B}_\otimes^+(f_o) \tag{14}$$

Or

$$\hat{\mathbf{C}}_t(f_o)\mathbf{B}_\otimes^+(f_o) = \mathbf{D}(f_o)\mathbf{B}_\otimes(f_o) \tag{15}$$

This equation tells us that the columns of $\mathbf{B}_\otimes(f_o)$ are the left eigenvectors of $\hat{\mathbf{C}}_t(f_o)$ corresponding to the eigenvalues on the diagonal of the matrix $\mathbf{D}(f_o)$: however, since $\hat{\mathbf{C}}_t(f_o)$ is Hermitian, they are also the (right) eigenvectors of $\hat{\mathbf{C}}_t(f_o)$. Furthermore, the columns of $\mathbf{V}_{ts}(f_o)$ are also eigenvectors of $\hat{\mathbf{C}}_t(f_o)$ corresponding to the same (non-zero) eigenvectors of the diagonal matrix $\hat{\mathbf{U}}_s(f_o)$. Given that the eigenvalues of $\hat{\mathbf{C}}_t(f_o)$ are different, the orthonormal eigenvectors are unique up to phase term, this will likely be the case if the source kurtosis are different. Hence the difference between $\mathbf{V}_{ts}(f_o)$ and $\mathbf{B}_\otimes(f_o)$ is that the columns may be reordered and each column is multiplied by a complex constant.

The information we want is $\mathbf{A}_\otimes(f_o)$, which is given by (8)

$$\mathbf{A}_\otimes(f_o) = \left(\mathbf{H}(f_o) \otimes \mathbf{H}^*(f_o)\right)^\oplus \mathbf{B}_\otimes^+(f_o) \tag{16}$$

with $^\oplus$ denoting the pseudo-inverse of matrix. We do not have the matrix $\mathbf{B}_\otimes(f_o)$, but we have the matrix $\mathbf{V}_{ts}(f_o)$. Hence we can obtain a matrix $\mathbf{A}_\otimes^\bullet(f_o)$ such that

$$\mathbf{A}_\otimes^\bullet(f_o) = \left(\mathbf{H}(f_o) \otimes \mathbf{H}^*(f_o)\right)^\oplus \mathbf{V}_{ts}(f_o) \tag{17}$$

Furthermore, we obtain $\hat{\mathbf{A}}(f_o)$ by extracting out the first N rows and the first P columns of $\mathbf{A}_\otimes^\bullet(f_o)$. The estimate $\hat{\mathbf{A}}(f_o)$ will be permuted and scaled (column-wise) version of $\mathbf{A}(f_o)$.

This algorithm leads to the estimation of the transfer matrix without prior knowledge of the steering vector or the propagation model such as in the classical techniques. The information we want is, which is given by $\mathbf{A}_\otimes(f_o)$ (10)

$$\mathbf{A}_\otimes(f_o) = \left(\mathbf{H}(f_o) \otimes \mathbf{H}^*(f_o)\right)^\oplus \mathbf{B}_\otimes(f_o) \tag{18}$$

We do not have the matrix $\mathbf{B}_\otimes(f_o)$, but we have the matrix $\mathbf{V}_{ts}(f_o)$. Hence we can obtain a matrix $\mathbf{A}_\otimes(f_o)$ such that

$$\mathbf{A}_\otimes(f_o) = \left(\mathbf{H}(f_o) \otimes \mathbf{H}^*(f_o)\right)^\oplus \mathbf{V}_{ts}(f_o) \tag{19}$$

Furthermore, we obtain $\hat{\mathbf{A}}(f_o)$ by extracting out the first N rows and the first P columns of $\mathbf{A}_\otimes(f_o)$. The estimate will be permuted and scaled (column-wise) version of $\mathbf{A}(f_o)$.

This algorithm leads to the estimation of the transfer matrix of the different sources (objects) without prior knowledge of the propagation model (attenuation, absorption, variation of velocity...) such as in the classical techniques. Naturally one can use the high resolution algorithm to estimate the azimuths of the sources as MUSIC where we calculated the peak position of

$$Z(\theta) = \frac{1}{\mathbf{a}^+(\theta)\mathbf{V}_B\mathbf{V}_B^+\mathbf{a}(\theta)} \tag{20}$$

Where $\mathbf{V}_B$ is the noise subspace matrix spanned by the $(N-P)$ smallest eigenvectors. Afterwards the sediment is divided into several slices and by the use of estimated DOA and the known emitted signal the difference between the received and the calculated signals for each slice is evaluated.

II. Real Data and Results

The studied signals are recorded during an underwater acoustic experiment. The experimentation is carried out in an acoustic tank under conditions similar to those of a marine environment. The bottom of the tank is filled with sand. The experimental device is presented in Fig. 1. The emission is ensured by a transducer of the sonar type excited by amplitude pulses of approximately 300 volts and 0.2 μs of width. The acoustics wideband signal resulting is centered on the frequency of 500 KHz and its band-width varies from 300 to 700 KHz. In addition to the signal source a correlated Gaussian noise is emitted. The signal to noise ratio is varied. Our objective is to estimate the directions of arrival of the signals during the experimentation. The signals are received on a rectilinear antenna.

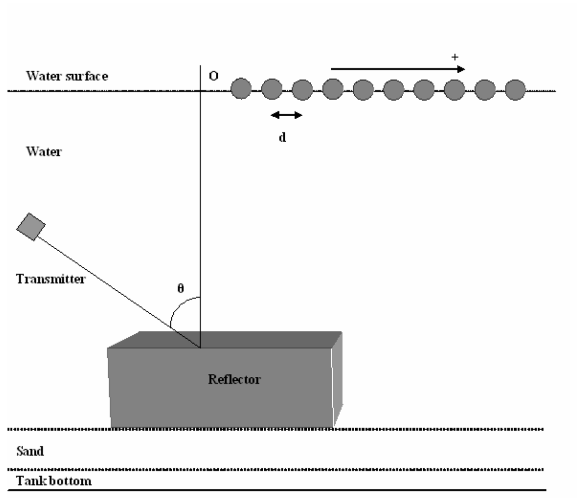

Fig. 1 Experimentation

The observed signals come from the various reflections on objects being in the tank. Generally the aim of acousticians is the detection, localization and identification of these objects. In this experiment we have recorded the reflected signals by a single receiver. The simulation of the reception antenna used in this type of experiment is ensured by a cylindrical hydrophone centered on 500 KHz, that one moves by regular step, in order to create a uniform linear antenna, of an initial position corresponding to the specular reflection of the first angle of emission to a final position given in order to ensure a correct reception for the greatest selected angle of emission. Concerning the data corresponding to this experiment, they simulate the response of an antenna made up of 40 hydrophones. Each received signal is sampled ($\Delta T = 0.2$ μs) on 2048 points. In this experiment the emission angle is $\theta = 30°$. Two paths are expected. The signals received on the sensors are visualized (Fig. 2): there are indeed two directions of arrival as shown on the Fig. 2. The expected angles are 56° and 64°.

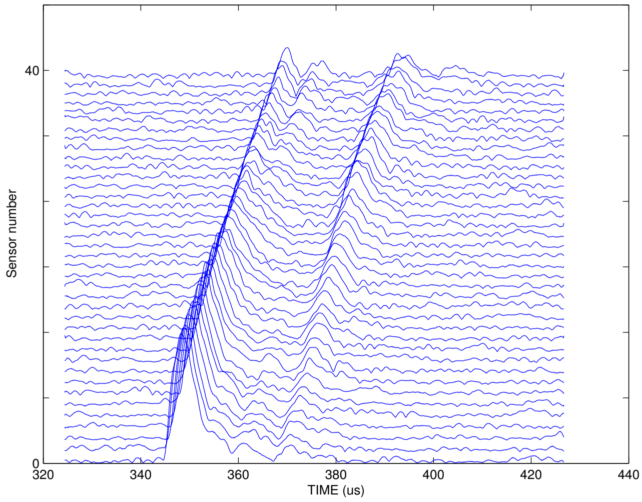

Fig. 2 Experiment and received signals

The results obtained by the MUSIC algorithm based on the cross-spectral matrix are shown Fig. 3 only one direction of arrival is estimated when two directions are expected.

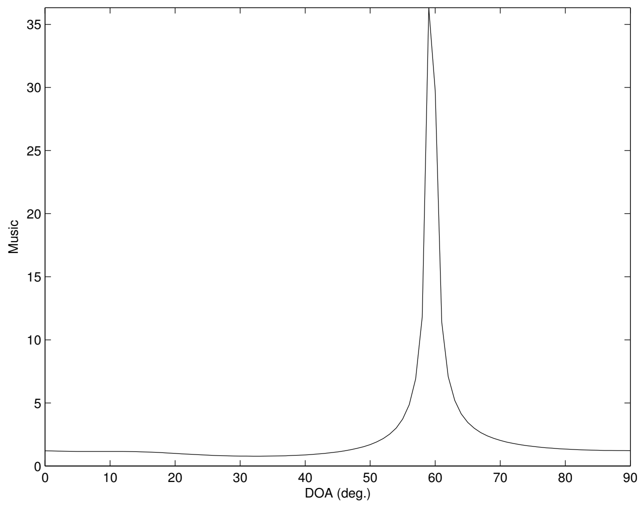

Fig. 3 Localization results with cross-spectral matrix

Fig. 4 shows the results obtained when our method is applied. The DOAs are perfectly estimated thanks to the cumulant matrix and the coherent signal subspace used in MUSIC algorithm.

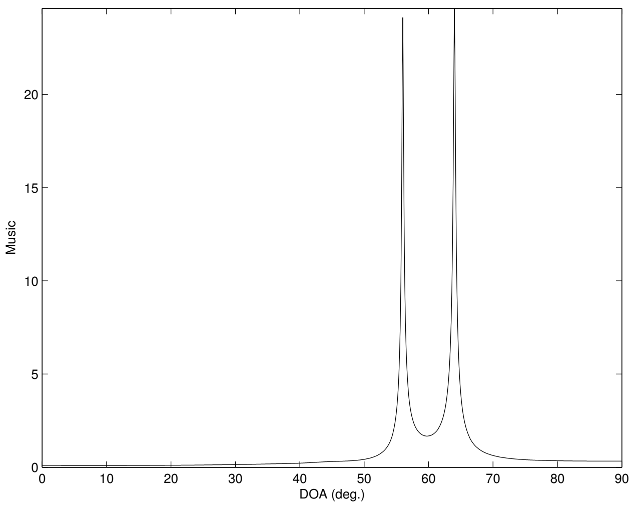

Fig. 4 Localization results with cumulant matrix

A second experiment is considered for the near field sources localisation. In this section we have compared the analytical
Cramér-Rao Bound (CRB) results obtained in [6] with the error variances of the DOAs estimated with the proposed method (PM) and the cross-spectral matrix method (CM). For this purpose, our algorithm was run 500 times at each signal to noise ratio. Fig. 5 shows the comparison of analytical *CRB* calculated for two sources. The symbols ':' (dotted) and '-.' (dashdot) show the error variances for (PM) method and (CM) method, respectively.

The CRB values given Fig. 5 are obtained by averaging the CRB over the sources. The superiority of the PM over the CM algorithm is clearly seen at low SNR with different noise variances. The effectiveness of the coherent signal subspace algorithm based on the cumulant matrix to localize the totally correlated sources is confirmed even in the presence of unknown spatially correlated noise.

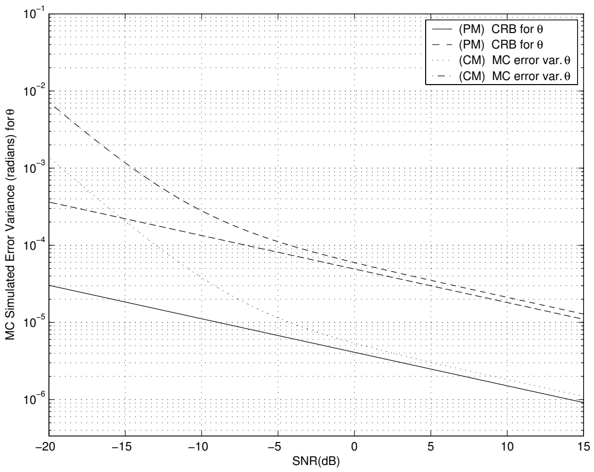

Fig. 5 Comparison of analytical *CRB* (CM) & (PM) methods

For this case, since there are two propagation paths in the medium we observe that the simulated error variances approach the lower limit provided from the Cramér-Rao bounds as the signal to noise ratio increases. This is expected by the results reported in [6] which state that MUSIC is an efficient estimator for large *SNR* values. The results presented here, therefore, reveals the fact that at high signal to noise ratios MUSIC is an efficient estimator for near field sources localization problems, too.

III. Conclusion

In this paper, we proposed a new approach to localize the sources by using the high resolution methods based on the higher order statistics and in particular the four order cumulant matrices. The proposed algorithm shows its effectiveness which it estimates correctly and blindly the transfer matrix which leads to the DOAs of the sources even the noise is unknown and spatially correlated.

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Salah Bourennane received his PhD degree from Institut National Polytechnique de Grenoble, France, in 1990 in the signal processing. Currently, he is Professor at the Ecole Généraliste d'Ingénieur de Marseille, France. His research interests are in the statistical signal processing, array processing, image processing, multidimensional signal processing and performances analysis.

A. Bendjama received the Ph.D. degree in physics from Corsica University, France, in 2001. He currently is with the institute of Fresnel, Marseille, France, where he works in the GSM group. His general interests are in the areas of statistical signal processing, estimation, and detection theory. In particular, his research interests include Gaussian signal detection, localization, estimation and tracking using higher order statistics, with applications to radar, digital communications and acoustic field.