

Optimized Calculation of Hourly Price Forward Curve (HPFC)

Ahmed Abdolkhalig

Abstract—This paper examines many mathematical methods for molding the hourly price forward curve (HPFC); the model will be constructed by numerous regression methods, like polynomial regression, radial basic function neural networks & a furrier series. Examination the models goodness of fit will be done by means of statistical & graphical tools. The criteria for choosing the model will depend on minimize the Root Mean Squared Error (RMSE), using the correlation analysis approach for the regression analysis the optimal model will be distinct, which are robust against model misspecification. Learning & supervision technique employed to determine the form of the optimal parameters corresponding to each measure of overall loss. By using all the numerical methods that mentioned previously; the explicit expressions for the optimal model derived and the optimal designs will be implemented.

Keywords—Forward curve, furrier series, regression, radial basic function neural networks.

I. INTRODUCTION

THIS paper to illustrate a model for the valuation in any energy market, the quantitative analysis will be deeply emphasized to creates a model that reflect the market behavior, this model will be the hourly price forward curve (HPFC), which is the goal of this paper. The model provided will be constructed basically from the spot prices of any arbitrary energy market; the optimum model will be sought out for the valuation in the markets. Special attention will paid to Alberta electricity prices. Alberta's electricity market structure will reviewed followed with a description of the fundamental economic drivers of the spot prices just to understand the behavior of the prices in this market, then, the quantitative analysis is applied to create & implement the model (HPFC), but before that, a hint of the forward price curve is given.

II. HOURLY PRICE FORWARD CURVE

A. Definition of the "Hourly Forward Price Curve"?

An Hourly Price Forward Curve can be defined as (HPFC)

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as a list of prices as of today for the delivery of electricity for example at a series of different points of time in the future as shown in Fig. 1. These prices may be average prices for each hour period for a specific over a given period (e.g. day / week etc) as shown in Fig. 1, it represents the term-structure of forward prices in hourly resolution. The specific curve required depends on the purpose of the curve. For example, the looking to price a swap based on the average of prices over a week this is the forward curve would required [1].

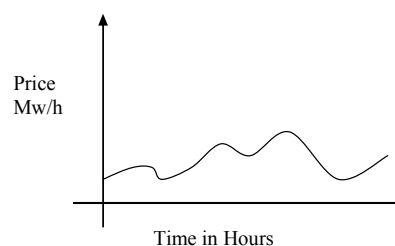


Fig. 1 A visionary plot of Hourly Price Forward Curve

B. Distinction between a Forward Curve and a Forecast

The forward price curve is used to mark trades to market. In liquid and deep markets such as interest rate, foreign currency, and widely traded physical commodities, forward curves are easily available and derived. But in young electricity markets, it is less clear what is meant by a forward curve [2]. Often this is derived from a schedule of spot price forecasts, so that the curve is used to predict the likely spot price of electricity several years in the future.

There are some traditional models [3,5,7,8] for practitioner to choose for modeling spot prices curve. Fischer Black and Myron Scholes published in 1973 their seminal paper on options pricing. The Black-Scholes option pricing model which was based on the extension of Brownian motion contributed to the explosive growth in trading of derivatives. The original Black-Scholes model allowed only for pricing options on a non-dividend paying stock [6]. Extensions to the Black-Scholes model, such as the Garman-Kohl Hagen and Black (1976) model, allowed for pricing comprehensive commodity options, and options on futures respectively.

Regardless of the strong assumptions that underlay this model, the simplicity of the Black's formula directly made it an accepted choice among practitioners. Unfortunately, Black's model makes several restrictive assumptions. In particular, it assumes that the evolution of futures prices can

be modeled using Geometric Brownian Motion (*GBM*). The use of Black's model may be inappropriate in cases where this assumption cannot be made. The Alberta Electricity market is a clear example of a market where the assumption of Geometric Brownian Motion to model spot prices in terms of hourly prices is unreasonable. The occurrence of plant outages at random intervals tends to generate discontinuous spikes in spot prices. Such spikes are inconsistent with a continuous time random walk, and must be explicitly accounted for in a model.

III. ANALYSIS OF ALBERTA ELECTRICITY MARKET PRICES DRIVERS

Today Alberta Power Pool Market organizes and operates the physical delivery of electricity in the province (Alberta, Canada) and the entire electricity purchased in the province [9] must pass through this market. All the generators offer to the pool to supply a specific amount of power for a particular duration at a specific price. At the same time, consumers put forward bids to buy electricity at or below an exacting price. The power pool after that uses these bids and offers to create a merit order. This merit order allows units to be dispatched from lowest price to highest price to serve load. Using the merit order, the system marginal price ("SMP") is determined, and the average of this price for each hour becomes the official pool price. The price for a given hour may range from \$0 to \$1000, the current price cap in Alberta.

A. Alberta Power Pool Price Equilibration

If the demand in Alberta Pool as shown in Fig. 2, it's tremendously insensitive even to large changes in price. This is because, the average consumer of electricity, the household, individuals give no thought to the current pool price when they turn on their appliances or cook dinner. The result that can be assumed at any point in time, demand is relatively inelastic, assuming the shape of a vertical line. Demand is served mostly by coal-fired, gas-fired, and hydro generation facilities. The order in which these units are dispatched depends on their variable cost of production and bidding behavior, while supply is currently a kinked curve, becoming very steep at high quantities.

From a purely physical point of view, supply and demand must always be in equilibrium. This is because electricity cannot easily be stored for later consumption. This characteristic of electricity markets makes it considerably different from other commodity markets like natural gas and crude oil, both of which can be stored and resold later when prices are higher, the primary consequence of not being able to store electricity in the Alberta electricity market is extreme price volatility when supply is constrained.

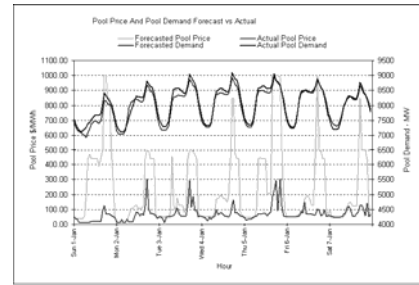


Fig. 2 Pool Price & Pool Demand Forecast vs. Actual

B. The changing in Alberta Electricity Prices (Volatility)

Fig. 3 illustrates an example of hourly prices, which are measured in dollars per megawatt hour (\$/MWH, Canadian dollars being the monetary unit here and in the remainder of this thesis), during the period Jan 1, 2000 to Oct 31, 2001. [6], a high changing in the prices during the seasonally seen. Here, electricity prices exhibits the most complicated cyclical patterns of all energy commodities, there are peaks reacting to heating and cooling needs.

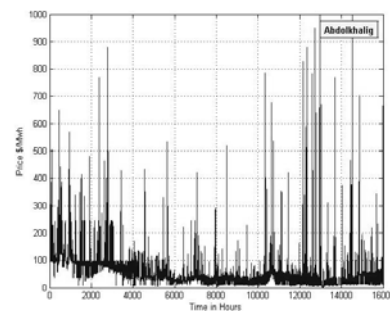


Fig. 3 A plot of hourly Alberta electricity prices from Jan 2000 to Oct 31, 2001

The spikiness of the dataset could be worth clearing in numeral values as follow: while the mean of the dataset is only \$56.472 (\$/Mwh), the dataset is characterized by occasional excursions to the \$500.00 and \$1000.00 level. These periods of high prices are usually followed by a return to a price of under \$100.00.

IV. MATHEMATICAL METHODS FOR MODELING HPFC

Modeling the *HPFC* in any Energy Market is necessary for knowing the prices behavior in the future. Regression Technique will be used for this to construct the optimum model. There are many Regression methods for modeling and each method can produce prediction with different accuracy. In the following sections, these methods will be discussed:

A. Polynomial Regression

The general equation of the Polynomial Regression [11] has the following form:

$$Y(t) = p_0 + p_1t + p_2t^2 + p_3t^3 + \dots + p_nt^n \quad (1)$$

Where p_0 an optional constant is term and p_1 through p_n are coefficients of increasing powers of t . The order of the

polynomial wished which fit the data must be specified.

- A linear equation in the form $Y(t) = p_0 + p_1x$ (2)
- A quadratic polynomial equation in the form $Y(t) = p_0 + p_1t + p_2t^2$ (3)
- A cubic polynomial equation in the form $Y(t) = p_0 + p_1t + p_2t^2 + p_3t^3$ (4)
- Higher (4th or 5th) order polynomials are useful for attempts to describe data points as fully as possible, but the terms generally cannot be meaningfully interpreted in any physical sense. Higher order terms can lead to odd and unreasonable results, especially beyond the range of the t values [12].

General polynomial model (Polynomial Models in One Variable) is $Y(t) = p_0 + \sum_{j=1}^n p_j t^j$ (5)

Assumed that it is possible that the data can be modeled by a quadratic polynomial function given in equation (4); $p = t \setminus y$ and the unknown coefficients p_0, p_1 and p_2 can be computed by doing a least squares fit, which minimizes the sum of the squares of the deviations of the data from the model. If there are six equations in three unknowns, then there are six data recorded assumed.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \\ 1 & t_4 & t_4^2 \\ 1 & t_5 & t_5^2 \\ 1 & t_6 & t_6^2 \end{bmatrix} \times \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} \quad (6)$$

The solution is found with the backslash operator ($p = t \setminus Y$).

Applying this to the data of Alberta Market Prices, the simulation of the hourly forward price curve by means of linear polynomial regression founded as shown in Fig. 4.

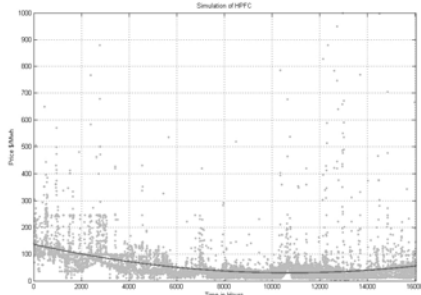


Fig. 4 Simulation of HPFC by using quadratic polynomial regression

B. Radial Basis Functions Neural Networks

Let's suppose that the approximation of a real valued function $y(x)$ by $Y(x)$ is given by the set of values $Y = (y_1, \dots, y_p)$ at the distinct points $X = (x_1, \dots, x_n) \subset R^d$.

This could be done by applying the second proposed method which is "RBF"

1) What is an RBF?

Let $y(x)$ to be a Radial Basis Function in the form:

$$Y(x) = \sum_{i=1}^p w_i G(|x - x_i|) + p(x) \quad (7)$$

$$p(x) = \sum_{j=1}^k a_j \gamma_j(x)$$

Where, $p(x)$ is a polynomial of degree at most k , w_i is a real-valued weight, $|\cdot|$ denotes the Euclidean norm, G is a basic Gaussian function, $G: R^+ \rightarrow R$, and $|x - x_i|$ is simply a distance -- how far x is from the point x_i . $p(x)$ is a basis in the space of polynomials of degree m , and m depends on G . Note that the most popular Gaussian RBF does not need $p(x)$, i.e., $p(x) = 0$. Thus, the approximating function is a weighted sum of RBF's $G(|x - x_i|)$.

2) Computation of the Coefficients (Weights w_i)

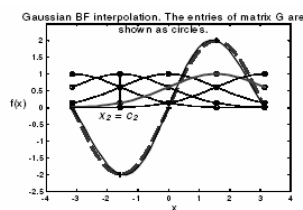
The coefficients are found by solving the linear system:

$$(G + \lambda I)w + \Gamma^T a = d \quad (8)$$

$$\Gamma w = 0$$

Where I is the identity matrix, and G is a so-called design matrix and $d = y = [y_1 \ y_2 \ \dots \ y_p]^T$. When, there is no polynomial term the solution follows the from $(G + \lambda I)w = d$

Look at this simple example for forming a Gaussian G matrix for clarifying and avoiding ambiguities.



$$G(x) = e^{-\|x\|^2}$$

$$f(x) = \sum_{i=1}^N c_i e^{-\|x - x_i\|^2}$$

$$G_{Gauss} = \begin{bmatrix} 1.00 & 0.61 & 0.14 & 0.01 & 0.00 \\ 0.61 & 1.00 & 0.61 & 0.14 & 0.01 \\ 0.14 & 0.61 & 1.00 & 0.61 & 0.14 \\ 0.01 & 0.14 & 0.61 & 1.00 & 0.61 \\ 0.00 & 0.01 & 0.14 & 0.61 & 1.00 \end{bmatrix}$$

3) Neural Networks' Interpretation of the RBF Approximation Scheme

A strict interpolating regularization network [4] for a one dimensional input x . The training data set comprises 5 examples as shown in Fig. 5. Centers correspond to the inputs, and all variances are equal. Bias shown is not mandatory and does not follow from the equations below:

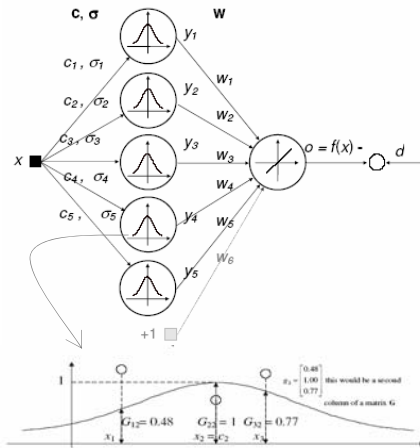


Fig. 5 Interpolating regularization network for a one dimensional input x

Thus, our model will be in terms of weighted and centered Gaussian in the form

$$y(x) = \sum_{i=1}^p w_i G(x - x_i)$$

Here x express the time.

$$y_1 = w_1 G(x_1 - x_1) + w_2 G(x_1 - x_2) + \dots + w_p G(x_1 - x_p)$$

$$y_2 = w_1 G(x_2 - x_1) + w_2 G(x_2 - x_2) + \dots + w_p G(x_2 - x_p)$$

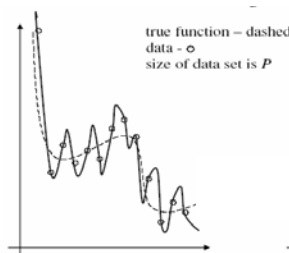
⋮

⋮

$$y_p = w_1 G(x_p - x_1) + w_2 G(x_p - x_2) + \dots + w_n G(x_p - x_p)$$

(9)

This can be represented graphically as follows:



INTERPOLATION

The solution is given by

$$y = Gw$$

$$w = G^{-1}y$$

$$y_a = Gw = y$$

(10)

This will be computationally expensive for $P \gg$, problem!

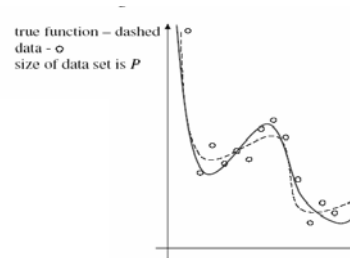
$$\text{So, instead of: } y(x) = \sum_{i=1}^p w_i G(x - x_i) \quad (11)$$

substitute by an approximation :

$$Y(x) = \sum_{i=1}^p w_i G(x - c_i) \quad (12)$$

Where $p \ll P$; the vectors c_i are called centers ($n \cong 10\% N$).

This can be represented again graphically after the APPROXIMATION as follows:



APPROXIMATION

Applying this to the data of Alberta Market Prices with sigma equals 900 and number of neurons equals 9, the simulation of the hourly forward price curve by means of RBF is shown in the Fig. 6 below.

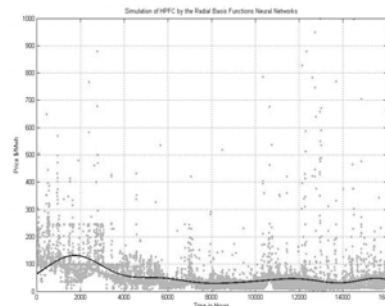


Fig. 6 Simulation of HPFC by the Radial Basis Functions Neural Networks

From Fig. 6, it seems that using Radial Basis Functions Neural Networks to simulate the HPFC is much better than the other model simulated by quadratic polynomial regression. Latter, the validation of all of these models and the model by means of Fourier series which will introduce in the next section will be checked.

C. A Fourier series (Trigonometric Polynomials)

Fourier series is a mathematical tool used for analyzing an arbitrary function by decomposing it into a weighted sum of much simpler sinusoidal component functions sometimes referred to as normal Fourier modes. The Fourier series expansion of is:

$$Y(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(w_n t) + b_n \sin(w_n t)] \quad (13)$$

Where, for any non-negative integer n ; $w_n = n \frac{2\pi}{T}$

Applying this to the data of Alberta Market Prices, the simulation of the hourly forward price curve by means of A Fourier series is shown in Fig. 7.

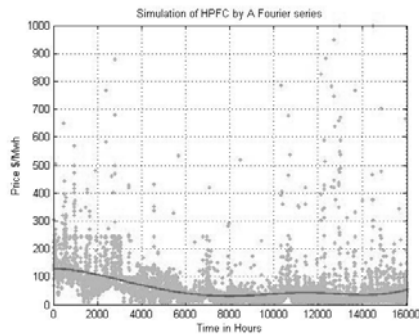


Fig. 7 Simulation of HPFC by the Fourier series

V. EVALUATING THE GOODNESS OF FIT OF THE MODELS

Once the data prices model implemented, the evaluation to test the goodness of fit should be done. This will be done by a visual examination measure of the curve displayed & by numerical measures; these will be valid for all. Means, the goodness of fit measures for both linear and nonlinear parametric fits should provide:

- Residuals
- Goodness of fit statistics

These measures could be grouped into two types: graphical and numerical. The residuals are graphical measures, while the goodness of fit statistics is numerical measures.

Generally speaking, graphical measures are more beneficial than numerical measures because they allow us to view the entire data set at once, and they can easily display a wide range of relationships between the model and the data. The numerical measures are more narrowly focused on a particular aspect of the data and often try to compress that information into a single number.

Also, it's possible that none of the fits by the model constructed can be considered the best one. In this case, it might be that it needed to select a different model. Conversely, it is also possible that all the goodness of fit measures indicate that a particular fit is the best one. However, the criteria that will follow are that; examine both the goodness of fit statistics and the graphical measures to check the minimum error should be done.

A. Some Definitions of Goodness of Fit Statistics

- The sum of squares due to error (*SSE*)
- r-square
- Adjusted r-square
- Root mean squared error (*RMSE*)

1) **Sum of Squares Due to Error:** in this statistic the total variation of the response values from the fit to the response values is measured. It is also called the summed square of residuals and is usually labeled as *SSE*.

$$SSE = \sum_{i=1}^n w_i (y_i - \bar{Y}_i)^2 \quad (14)$$

Where w_i are the weights. You can approximate the weights using an equation such as

$$w_i = \left(\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2 \right)^{-1} \quad (15)$$

A value of *SSE* closer to 0 indicates a better fit.

2) **r-Square:** this statistic measures how successful the fit is in clearing up the variation of the data. By other words, r-square is the square of the correlation between the response values and the predicted response values (spot & forward prices). It is also called the square of the multiple correlation coefficients and the coefficient of multiple determinations.

So, r-square can be defined as the ratio of the sum of squares of the regression (*SSR*) and the total sum of squares (*SST*). *SSR* is defined as:

$$SSR = \sum_{i=1}^n w_i (Y_i - \bar{y})^2 \quad (16)$$

SST is also called the sum of squares about the mean, it's define as

$$SST = \sum_{i=1}^n w_i (y_i - \bar{y})^2 \quad (17)$$

r-square is expressed as:

$$r - \text{square} = \frac{SSR}{SST} \quad (18)$$

3) **Degrees of Freedom Adjusted r-Square:** This statistic uses the r-square statistic defined above, and adjusts it based on the residual degrees of freedom. The residual degrees of freedom is defined as the number of response values n minus the number of fitted coefficients m estimated from the response.

$$\text{adjusted } r - \text{square} = 1 - \frac{SSE(n-1)}{SST(v)} \quad (19)$$

Where; $v = n - m$

The adjusted *r-square* statistic can take on any value less than or equal to 1, a value closer to 1 indicating a better fit.

4) **Root Mean Squared Error:** This statistic is also known as the fit standard error and the standard error of the regression

$$RMSE = s = \sqrt{MSE} \quad (20)$$

Where *MSE* is the mean square error or the residual mean square

$$MSE = \frac{SSE}{v} \quad (21)$$

A *RMSE* value closer to 0 indicates a better fit.

B. The Criteria for the Validation of the Models

There are many mathematical tools as mentioned before for the model validation, but the primary tool for most modeling applications is graphical residual analysis. Different types of

plots of the residuals from a fitted model are provided in the next sections to give information on the adequacy of different aspects of the model, but, as mentioned before, the criteria that will followed is that, both the goodness of fit statistics and the graphical measures should be examined to insure better fit .

The definition of the residuals from a fitted model is the differences between the responses observed at each recorded values of the explanatory variables and the corresponding prediction of the response computed using the regression model. Mathematically, the definition of the residual for the i^{th} observation in the data set is written ($e_i = y_i - Y_i$).

C. Comparison the Calculations of the Different Models

In the next sections, a first Comparison between the results of the different models will be created. The start will be with the analysis of the residuals curve result, and then go to analyze all the result of statistics measures. The different types of plots of the residuals from fitted models are as shown in Figs. 8-11.

1) The Polynomial Regression Models

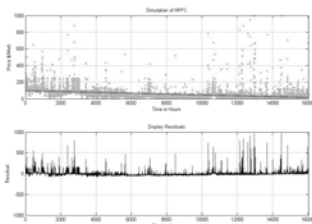


Fig. 8 Results of the model by using liner polynomial regression

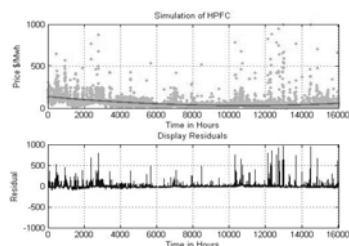


Fig. 9 Results of the model by using quadratic polynomial regression

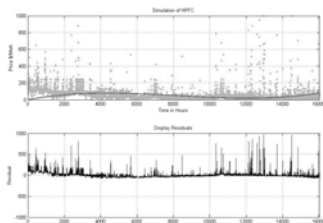


Fig. 10 Results of the model by using cubic polynomial regression

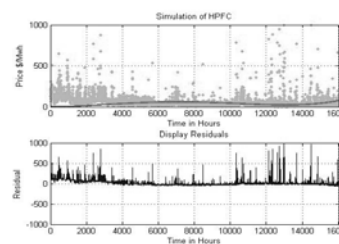


Fig. 11 Results of the model by using 4th degree polynomial regression

The quadratic model appears to fit the data, but, the residuals of all the models appear to be randomly distributed around zero. Therefore, a graphical evaluation of the fits does not reveal any obvious differences between the all equations. The numerical fit results for all the models fitted by the polynomial regression are shown below in the table:

TABLE I
 RESULTS OF FITS OF ALL THE POLYNOMIAL REGRESSION MODELS

Name of Regression	Tape	# Coefficient	SSE	RMSE	r-square
HPFC_LPR	Liner	2	1.0421e+004	53.4061	0.1609
HPFC_QPR	Quadratic	3	9.4098e+003	50.3881	0.2078
HPFC_CPR	cubic	4	1.2441e+004	62.3045	0.1960
HPFC_4th degree PR	4th	5	1.4257e+004	69.9207	0.1811
HPFC_5th degree PR	5th	6	1.5048e+004	72.9892	0.2270
HPFC_6th degree PR	6th	7	1.5496e+004	74.3881	0.2869

The goodness of fit statistics is shown. The statistics reveal a substantial difference between the equations, the second model in the table (2th degree polynomial regression) reveals the best results (minimum RMSE than all the others models), but, the r-square close to one than the first, third & forth. (But it's not better then fifth & sixth).

2) The Radial Basis Functions Neural Networks Models

Here, the residuals of the model for two parameters sigma and the number of the neurons are examined, the following plot represents this for sigma=900 & NN=10, Then; the two parameters are increased to sigma=1600 & NN=20, and observe the result as shown in Figs. 12, 13.

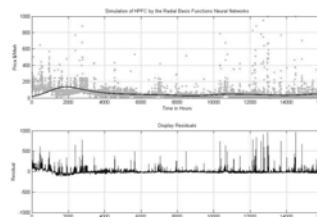


Fig. 12 Results of the model by using RBF with $\sigma=900$ & NN=9

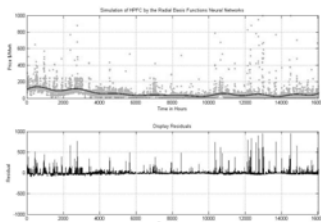


Fig. 13 Results of the model by using RBF with $\sigma=1600$ & $NN=20$

In the first plot, the models appear to fit the data poor. Looking into the residuals, it's appear to be almost randomly distributed around zero than the other model ($\sigma = 1600$ & $NN=20$), which seems more fit good for the data, since the residuals are more randomly distributed around zero.

The numerical fit results for the models fitted by the RBF are (RMSE= 48.9969) for the second model, this means, the goodness of fit statistics is, the statistics reveal a difference between this model and the entire last model.

The last one model (RBF regression model) reveals the best results (minimum RMSE than all the others models and better R-square close to one than other.

3) A Fourier Series (Trigonometric Polynomials)

Finally, coming to the Fourier series method, here, the examination of the method will be done under increase the number of the harmonics and looking to the best time period by changing it. The residuals will be checked, then again the numerical fit results for the models fitted by this method is checked too. The following plot; Fig. 14 represents this for number of harmonics=2 & time period equals to 16000, Then the two parameters must be increased to time period =20000 & $NH=5$, and the result is observed in Fig. 15.

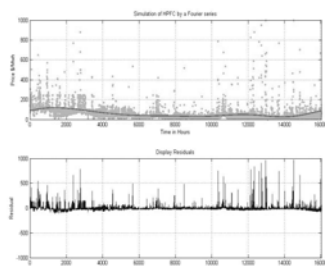


Fig. 14 Results of the model by using a Fourier series ($NH=2$ & $T=16000$)

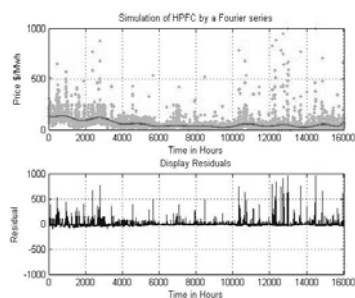


Fig. 15 Results of the model by using a Fourier series ($NH=5$ & $T=20000$)

VI. CORRELATION ANALYSIS

Correlation analysis is a statistical technique that evaluates the relationship between two variables; i.e., how closely they match each other in terms of their individual mathematical change. The question addressed is: if one variable (x) moves or a change in a certain direction does the second variable (y) also move or change in a similar or complementary direction?

A. The Coefficient of Correlation

The coefficient of correlation is a measure of the strength of the linear relationship between two variables and. It is computed [10] (for a sample of n measurements on x and y) as follows:

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} \quad (22)$$

$$SS_{xy} = \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}), \quad SS_{xx} = \sum_{i=1}^n (x_i - \bar{x})^2 \quad (23)$$

$$SS_{yy} = \sum_{i=1}^n (y_i - \bar{y})^2, \quad \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

B. The Coefficient of Determination

Another way to measure the contribution of x in predicting Y is to consider how much the errors of prediction of Y can be reduced by using the information provided by x .

The sample coefficient of determination is developed from the relationship between two kinds of variation: the variation of the Y values in a data set around:

- The fitted regression line.
- Their own mean.

The term variation in both cases is used in its usual statistical sense to mean “the sum of a group of squared deviations”.

The first variation is the variation of Y values around the regression line, i.e., around their predicted values. This variation is the sum of squares for error (SSE) of the regression model

$$SSE = \sum_{i=1}^n (y_i - Y_i)^2 \quad (24)$$

The second variation is the variation of y values around their own mean

$$SS_{yy} = \sum_{i=1}^n (y_i - \bar{y})^2 \quad (25)$$

It is easy to verify that

$$r^2 = \frac{SS_{yy} - SSE}{SS_{yy}} = 1 - \frac{SSE}{SS_{yy}} \quad (26)$$

Where; r is the coefficient of correlation, defined in the equation (22).

VII. PRACTICAL ANALYSIS OF THE COEFFICIENT OF DETERMINATION, r^2

Now, by applying equations (22-26) given in previous sections to all the models constructed and by applying it on Alberta Market prices, the results of each one as follows:

A. Correlation Analysis of the result of Polynomial Regression Model:

Defined in equation (1) that the general Polynomial Regression equation has the following form:

$$Y(t) = p_0 + p_1t + p_2t^2 + p_3t^3 + \dots + p_nt^n \quad (27)$$

The correlation analysis of the liner polynomial, quadratic polynomial, cubic polynomial, 4th order polynomial and the 5th order polynomial will be calculated. 5th order will be considered enough to give permissible error).

1) Results of the Model by Using Liner Polynomial Regression

The liner polynomial is in the form $Y(t) = p_0 + p_1t$

Coefficients:

$$= 96.8761, \quad = -0.0050$$

Goodness of fit:

$$\text{SSE: } 1.0421e+004$$

$$\text{r-square: } 0.1609$$

$$\text{RMSE: } 53.4061$$

2) Results of the Model by Using Quadratic Polynomial

Quadratic polynomial is in the form $Y(t) = p_0 + p_1t + p_2t^2$

Coefficients:

$$= 136.4662, \quad = -0.0199, \quad = 0.0000$$

Goodness of fit:

$$\text{SSE: } 9.4098e+003$$

$$\text{r-square: } 0.2078$$

$$\text{RMSE: } 50.3$$

Regression summary

- Quadratic term clearly very important
- Fairly impressive r-square
- Decrease in RMSE to 94.18

3) Results of the Model by Using Cubic Polynomial

Cubic polynomial is in the form

$$Y(t) = p_0 + p_1t + p_2t^2 + p_3t^3$$

Coefficients:

$$= 0.0000, \quad = 0.0409, \quad = -0.0000, \quad = 0.0000$$

Goodness of fit:

$$\text{SSE: } 1.2441e+004$$

$$\text{r-square: } 0.1960$$

$$\text{RMSE: } 62.3045$$

Regression summary

- Cubic term clearly adds to fit.
- High correlation between estimates for linear & cubic
- Impressive increase in RMSE to 80.7%

4) Results of the Model by using 4th Degree Polynomial

4th degree polynomial is in the form

$$Y(t) = p_0 + p_1t + p_2t^2 + p_3t^3 + p_4t^4$$

Coefficients:

$$\begin{aligned} &= 1.0e-005, & &= 1.0e-005, \\ &= 1.0e-005 * 0.4864, & &= 1.0e-005 * -0.0001, \\ &= 1.0e-005 \end{aligned}$$

Goodness of fit:

$$\text{SSE: } 1.4257e+004$$

$$\text{r-square: } 0.1811$$

$$\text{RMSE: } 69.9207$$

Regression summary

- Adding 4th power doesn't improve fit at all.
- Impressive increase in RMSE to 80.7%

5) Results of the Model by using 5th Degree Polynomial

5th degree polynomial is in the form

$$Y(t) = p_0 + p_1t + p_2t^2 + p_3t^3 + p_4t^4 + p_5t^5$$

Coefficients:

$$\begin{aligned} &= 1.0e-009 * 0, & &= 1.0e-009 * 0, \\ &= 1.0e-009 * 0 & &= 1.0e-009 * 0.4747 \\ &= 1.0e-009 * -0.0001 & &= 1.0e-009 * 0.0000 \end{aligned}$$

Goodness of fit:

$$\text{SSE: } 1.5048e+004$$

$$\text{r-square: } 0.2270$$

$$\text{RMSE: } 72.9892$$

Regression summary

- Adding 5th power doesn't improve fit at all.
- Impressive increase in RMSE to 89.1%

B. Correlation Analysis of the RBF Neural Networks Models

The general form of our hourly price forward curve (HPFC) modeled by RBF for number of neurons NN will be as in the next form:

$$Y(t) = w_1 \cdot \exp(-((t - c_1) / \sigma)^2) + w_2 \cdot \exp(-((t - c_2) / \sigma)^2) + \dots + w_{NN} \cdot \exp(-((t - c_{NN}) / \sigma)^2) \quad (28)$$

1) Results of the model by using RBF with $\sigma=1601$ & $NN=10$

The model will be in the form

$$Y(t) = w_1 \cdot \exp(-((t - c_1) / \sigma)^2) + w_2 \cdot \exp(-((t - c_2) / \sigma)^2) + \dots + w_{10} \cdot \exp(-((t - c_{10}) / \sigma)^2) \quad (29)$$

Goodness of fit:

$$\text{SSE: } 241.4264$$

$$\text{r-square: } 0.7576$$

$$\text{RMSE: } 50.9351$$

Regression summary

- Impressive increase in r-square 264 % than 6th degree polynomial.
- Residuals still show need for more Neural.

C. Correlation Analysis of a Fourier Series Models

The general form of our hourly price forward curve (HPFC) modeled by Fourier series will be as in the next form:

$$Y(t) = a_0 + a_1 \cdot \cos(t * w) + b_1 * \sin(t * w) + a_2 * \cos(2 * t * w) + b_2 * \sin(2 * t * w) + \dots + a_N * \cos(N * t * w) + b_N * \sin(N * t * w) \quad (30)$$

1) Results of the model by using a Fourier series (NH=2 & T=16000)

$$Y(t) = a_0 + a_1 \cdot \cos(t * w) + b_1 * \sin(t * w) + a_2 * \cos(2 * t * w) + b_2 * \sin(2 * t * w) \quad (31)$$

Coefficients:

- = 56.4365
- = 25.0413
- = 21.5590
- = 1.07770
- = 20.1436

Goodness of fit:

- SSE: 9.6825e+003
- r-square: 0.1894
- RMSE: 51.4456

VIII. OPTIMIZATION (MINIMIZING THE LOSS)

Any engineer is interested in determining optimal settings of the model Factors (parameters); that is, to determine for each parameter a level of permissible error that optimizes the model response error. The loss or the error (RMSE) provides an objective measure of predictive error for a specific choice of model parameters. Thus; the goal is to find the values of the model parameters that minimize the error.

The problem of finding minimum error can be solved by an iterative numerical technique called gradient descent. It works as follows:

1. Choose some (random) initial values for the model parameters.
2. Calculate the error function with respect to each model parameter.
3. The model parameters change so that a short distance in the direction of the greatest rate of decrease of the error is recorded.
4. Steps 2 and 3 will repeated until error gets close to zero.

A. Optimization for HPFC Modeled by using a Fourier Series

To optimize the model by changing the time period T until getting the smallest error. The time will change from zero to double of the time period that prices recorded. This change must occur until guaranteed that there is minimum error. The minimum error was recorded at T=25300 as shown in Fig. 16, results show that, there is decrease in the error to minimum value of 48.4623 at number of harmonics equals 20 and increase in the correlation coefficients, means better curve simulation Fig. 17.

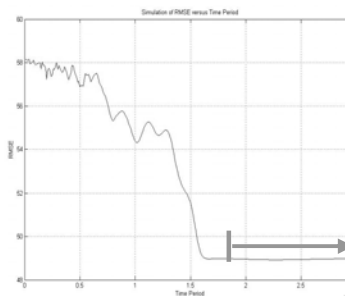


Fig. 16 Plot of the error RMSE versus time period

Results after optimization:

RMSE = 48.4623, r_square= 0.2439

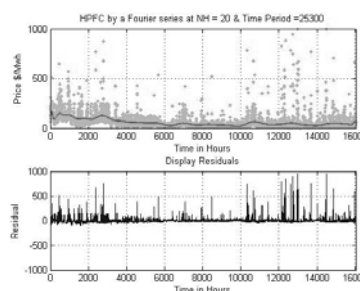


Fig. 17 Plot of the HPFC after optimizing the model by using (NH=20 & T=25300)

B. Optimization for HPFC modeled by RBF Neural Networks Models

In the optimization of the models constructed by RBF regression, models fit to data collected using response surface designs is used. How the optimal region to run a process does is determined? The answer is the optimization will be done in an iterative process again. So, the optimization will be done here by changing both Sigma & number of Neurons until the target output is hit, which is minimize process output error, after repeating the iteration number of times until the goal is hit which is finding values of the parameters to minimize the error of the process. The error is found minimum by using Mat lab code when sigma=1001 & NN=19, the number of iterative was 20*20 =400 .The following plot (Fig. 18) show the change in the error with changing the parameters sigma & NN.

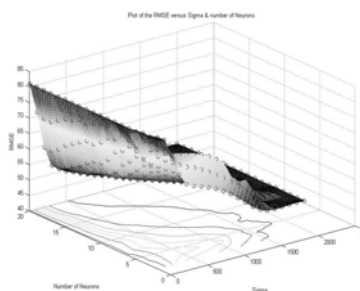


Fig. 18 Plot of the error RMSE versus NN & sigma

Now; this value of the parameter T; which give almost the optimal model corresponding to the various choices of overall measure of loss, the other parameters "Sigma & number of Neurons" must be changed again to determine the coefficients "Weights" which gives the optimum model.

The plot of the HPFC with parameters sigma=801 & NN=39 is shown in (fig.19).Even though, the error could be again minimized if the number of the neurons is increased, but this really computationally expensive (for number of neurons equal to 60,it take a time of six hours on our computer!).So, if the time needed to make better optimization is vital, then a sophisticated machine to perform this task is needed.

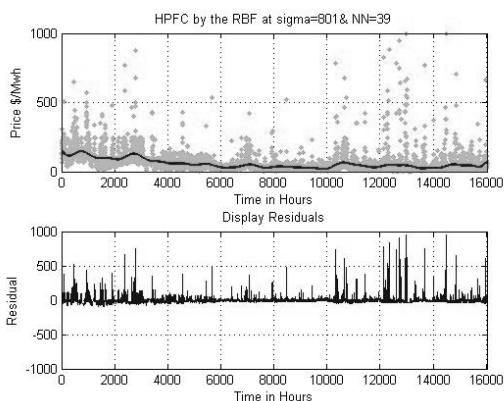


Fig. 19 Plot of the HPFC after optimizing the model

Here, modeling the HPFC by RBF regression will consist of twenty weighting coefficients, these coefficients & the optimum corresponding parameters will form the equation for prediction the forward prices in the future.

***Results after optimization:**

$$r_square = 0.2425, RMSE = 48.5848$$

The results show that, there is decrease in the error to RMSE= 48.9644 at parameters of sigma=1001 and NN=19,also the correlation coefficient come closer to one then before , means better curve simulation.

IX. IMPLEMENTATION

In this section, the optimal model derived in the last sections is implemented. The implementation modeled by RBF Neural Networks Models will apply to EEX market for data reading of more than five years. However, this implementation is intuitively can be applied to any other Market Data, but, keeping in mind that for the markets that have high changing prices between the seasons of the year, the regression for period that give parameters which draw curve should be run to deeply express the prices without care about the time needed for modeling, that is, the period of the iterative should be expanded as possible to get permissible error.

The following plot shows the behavior of the prices of EEX market in the period of (from June 2001 until June 2006):

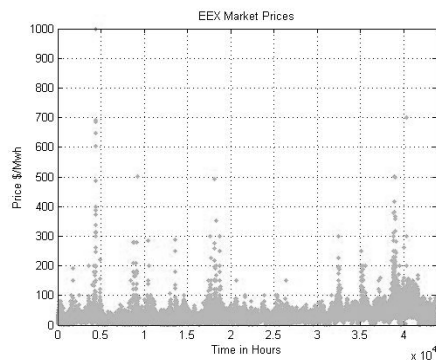


Fig. 20 Plot of EEX Market Pries

The prices with average equal 32.98 & standard deviation equal 25.57, which are means the pries are changing between 24.54 & 41.41, which implies low volatility than Alberta Market Prices. So, the regression could be repeated with less number of the iterative for getting the permissible error.alternatively, if the time is not vital , high number of iteration can be run , but, the error resulting from the model will not be significantly differ than if low number is run!

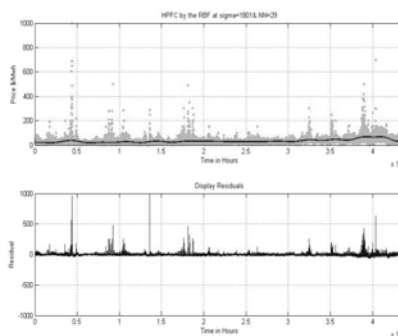


Fig. 21 Plot of the optimized HPFC after optimizing of EEX Market

As it's clear from the plot, the curve fit wills the prices along the five years period with smallest permissible error. The following are the results of the model:

Name of Regression	Tap e	# Coefficient t	SSE	RMSE	r-square
HPFC_RBF	RBF	29	3.5548e+004	22.4084	0.2492

Finally, the optimized HPFC for EEX Market under these criteria could be written e as follows:

$$Y(t) = w_1 \exp^{(-0.5*(t-c_1)/sigma)^2} + w_2 \exp^{(-0.5*(t-c_2)/sigma)^2} + \dots w_{29} \exp^{(-0.5*(t-c_{29})/sigma)^2}$$

w1= 0.0057	w11= 0.0164	w21= 0.0326
w2= -0.0116	w12= -0.0091	w22= -0.0400
w3= 0.0219	w13= 0.0082	w23= 0.0554
w4= -0.0247	w14= -0.0041	w24= -0.0650
w5= 0.0278	w15= 0.0048	w25= 0.1072
w6= -0.0277	w16= -0.0026	w26= -0.0429
w7= 0.0307	w17= 0.0053	w27= 0.3744
w8= -0.0292	w18= -0.0063	w28= 0.1289
w9= 0.0284	w19= 0.0140	w29= 0.8280
w10= -0.0212	w20= -0.0203	w30= -1.0714

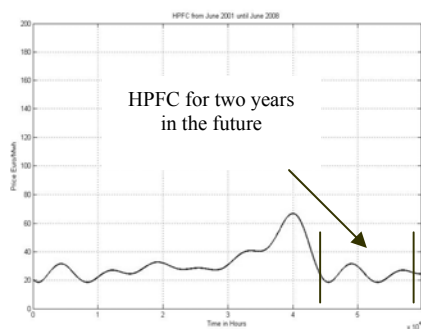


Fig. 22 Optimized HPFC of EEX Market with time of two years in the future

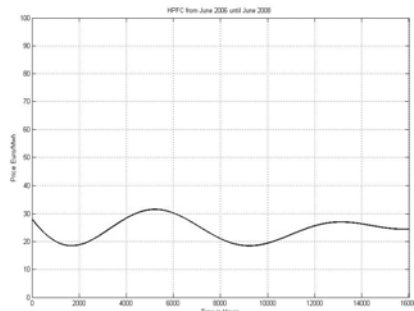


Fig. 23 Optimized HPFC of EEX Market for time of two years in the future

X. CONCLUSION

In this paper, the optimized method for modeling the Hourly Price Forward Curve (*HPFC*) using the regression technique is implemented. This implementation is approached using three methods (polynomial regression, *RBF* neural network regression & using a Fourier series (Trigonometric Polynomials Regression)). The comparisons among these three methods using the *RMSE* (Root Mean Square Error) as overall measure of error have made. In comparing these three methods, the maximum loss values are observed close to fifty before optimization. For the method one, the degree of the polynomial containing of the loss values, then it is not wide that is, the variation of loss is high. For all the methods, it is observed that the loss is not monotonic for $Y(t)$ as a function of the degree for method one and not monotonic for the other $Y(t)$ which is defined in the other methods.

From a computational point of view, Method 1 is simple. Method 2 is more complicated compared to the method 3. Method 2 is computationally harder than the other two

methods because we apply the optimization technique to solve for N naturals, (where, $N \gg 1$) in two parameters.

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