About the Structural Stability of the Model of the Nonelectroneutral Current Sheath

V.V. Lyahov, and V.M. Neshchadim

Abstract—The structural stability of the model of a nonelectroneutral current sheath is investigated. The stationary model of a current sheath represents the system of four connected nonlinear differential first-order equations and thus they should manifest structural instability property, i.e. sensitivity to the infinitesimal changes of parameters and starting conditions. Domains of existence of the solutions of current sheath type are found. Those solutions of the current sheath type are realized only in some regions of seven-dimensional space of parameters of the problem. The phase volume of those regions is small in comparison with the whole phase volume of the definition range of those parameters. It is shown that the offered model of a nonelectroneutral current sheath is applicable for theoretical interpretation of the bifurcational current sheaths observed in the magnetosphere.

Keywords—Distribution function, electromagnetic field, magnetoactive plasma, nonelectroneutral current sheath, structural instability. bifurcational current sheath.

I. INTRODUCTION

THE report is devoted to the investigation of structural stability of the model of a nonelectroneutral current sheath offered by us in the paper [1] in which the kinetic theory of an equilibrium of the similar current sheaths formed at the arbitrary values of medium parameters was developed. We will give short highlights of the derivation of the model equations and the results of their solution.

As an equilibrium distribution function the following function of motion integrals was introduced:

$$f_{0\alpha} = \frac{m_\alpha}{2\pi\theta_{\alpha z}} n_0 (1+\alpha_\alpha)^{\frac{1}{2}} \exp\left[\frac{m_\alpha}{2\theta_{\alpha z}}(1+\alpha_\alpha)U_\alpha^2\right] \cdot$$
$$\exp\left[-\frac{W_\alpha}{\theta_{\alpha z}} - \frac{\alpha_\alpha P_{y\alpha}^2}{2\theta_{\alpha z}m_\alpha} + U_\alpha(1+\alpha_\alpha)\frac{P_{y\alpha}}{\theta_{\alpha z}}\right]. \quad (1)$$

where

$\alpha_\alpha = \dfrac{\theta_{\alpha z}}{\theta_{\alpha y}} - 1$ is a degree of the temperature anisotropy, (2)

$U_\alpha -$ is macroscopic velocity directed along the y-axis. (3)

Integrals of motion are the total energy and generalized momentum:

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$$W_\alpha = \frac{1}{2}m_\alpha(v_x^2 + v_y^2 + v_z^2) + e_\alpha\phi(z),$$
$$P_{y\alpha} = m_\alpha v_y + e_\alpha A_y(z). \quad (4)$$

Here, $\phi(z), A_y(z)$ are the electrical and magnetic potentials, and the problem is one-dimensional.

On the boundary z =0 where $\phi(0) = 0, A_y(0) = 0$ the distribution function (1) represents the anisotropic shifted Maxwell distribution:

$$f_{0\alpha} = \frac{m_\alpha}{2\pi\theta_{\alpha z}} n_0(1+\alpha_\alpha)^{\frac{1}{2}}\exp\left[-\frac{m_\alpha}{2\theta_{\alpha z}}[v_{\alpha z}^2 + (1+\alpha_\alpha)(v_{\alpha y}-U_\alpha)^2]\right]. \quad (5)$$

The distribution function offered is the generalization of Harris' distribution function [2] and includes the anisotropy of temperatures of plasma components along and across the sheath.

The density of particles and the current density for the equilibrium distribution function (1) were calculated and the equations for the electromagnetic potentials were derived:

$$\frac{d^2\psi}{d\xi^2} = \exp(\frac{\psi}{\gamma} - \frac{\omega_e}{\gamma}a - \frac{\alpha_e}{1+\alpha_e}\frac{\eta a^2}{2\gamma\mu}) - \exp(-\psi + \omega_i a - \frac{\alpha_i}{1+\alpha_i}\frac{\eta a^2}{2}), \quad (6)$$

$$\frac{d^2 a}{d\xi^2} = (\omega_e + \frac{\alpha_e}{1+\alpha_e}\frac{\eta a}{\mu})\exp(\frac{\psi}{\gamma} - \frac{\omega_e}{\gamma}a - \frac{\alpha_e}{1+\alpha_e}\frac{\eta a^2}{2\gamma\mu}) \cdot$$
$$-(\omega_i - \frac{\alpha_i}{1+\alpha_i}\eta a) \cdot \exp(-\psi + \omega_i a - \frac{\alpha_i}{1+\alpha_i}\frac{\eta a^2}{2}). \quad (7)$$

As the boundary conditions we have chosen:

$$a(\xi = 0) = a'(\xi = 0) = 0;$$
$$\psi(\xi = 0) = \psi'(\xi = 0) = 0. \quad (8)$$

We introduced the following dimensionless quantities:

$$\mu = \frac{m}{M}, \gamma = \frac{\theta_{ze}}{\theta_{zi}}, \eta = \frac{\theta_{zi}}{Mc^2}, \psi = \frac{e}{\theta_{zi}}\phi,$$

$$a = \frac{ec}{\theta_{zi}}A_y, \xi^2 = \frac{e^2 n_0}{\theta_{zi}\varepsilon_0}z^2, \omega_i = \frac{U_i}{c}, \omega_e = \frac{U_e}{c}. \quad (9)$$

The numerical solution of the problem (6)−(8) was realized using Maple 14 Package. The solution for two current sheaths

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was obtained. Figs. 1, 2, and 3 show some of the obtained performances for one of the solutions. Fig. 1 shows a magnetic field profile, 2 - a profile of the ion current, 3 - a profile of an electron current for the parameter values:

$$\alpha_i = 1.0 \cdot 10^{-3}; \alpha_e = 5.5 \cdot 10^{-8}; \theta_i = 1.6 \cdot 10^{-15} J;$$
$$\gamma = 1 \cdot 10^{-1}; \eta = 1.07 \cdot 10^{-5}; \omega_e = -0.234 \cdot 10^{-3}; \quad (10)$$
$$\omega_i = 0.234 \cdot 10^{-2}; \mu = 0.00055; n_0 = 0.16 \cdot 10^{6} m^{-3}.$$
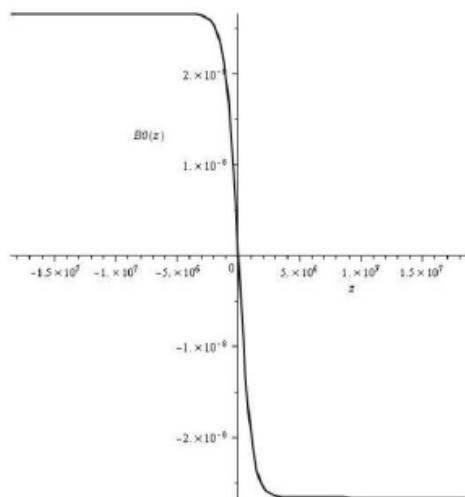

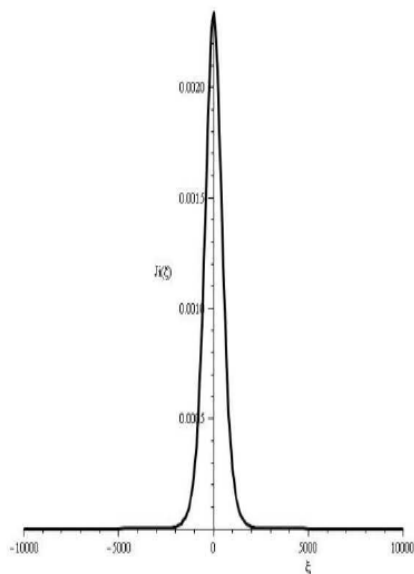Fig. 1 Magnetic field profile
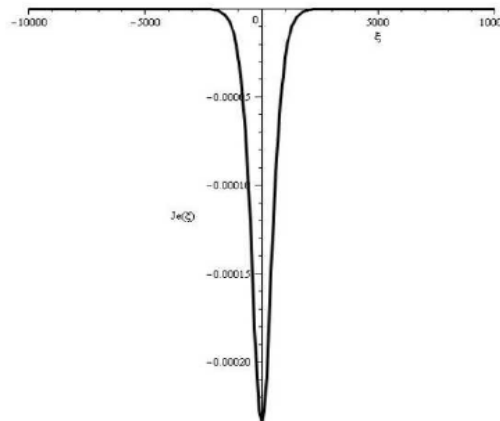

Fig. 2 Profile of the ion current

Fig. 3 Profile of the electron current

The magnetic field profiles are given in the dimensional form, and the remaining profiles are dimensionless. The profiles for the second sheath are qualitatively similar to those given above

II. STATEMENT OF THE PROBLEM

The equations (6) and (7) represent the system of 4 connected nonlinear differential first-order equations and thus they should manifest structural instability property, i.e. sensitivity to the infinitesimal changes of parameters and starting conditions. While similar Harris's equations [2] are linear and, hence, are structurally stable. We will investigate a structural stability of the system (6) and (7) and find the existence domains of solutions of current sheath type.

We will touch one more problem. Cluster's observations [3]–[6] recently have confirmed the previous ISEE [7] and Geotail [8] observations showing that the magnetosphere current sheath can have a bifurcation structure with two off-center current peaks. Theoretically the kinetic model of a thin current sheath developed [9] has been modified to explain splitting of the current [10] In the paper [11] it is shown that such structure can be described by means of the kinetic tangential equilibrium with a distribution function representing the sum of the infinite number of elementary functions parameterized by means of the vector potential.

Let's show that the equations (6) and (7) derived based on the offered distribution function (1) can also simulate such bifurcation current sheaths.

III. THE RESULTS OF THE INVESTIGATION

The Cauchy problem (6)−(8) naturally has the solution in the vicinity of a point of laying down the side conditions at all values of parameters. The solution of the current sheath type on the interval exists only at some combinations of parameters. As one would expect, the nonlinear system (6)−(7) manifests the structural instability: at some critical values of parameters the solution is bifurcated, i.e. passes from one type to another; for example, it passes from the solution of the current sheath type to a trivial solution. It has

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appeared that the solution of the system (6)–(7) exists only in some points of seven-dimensional space of the values of parameters $\alpha_e, \alpha_i, \beta, \eta, \omega_e, \omega_i, \mu$. So, among the 4096 calculated solutions with different parameters defined on the grid nodes such as:

$$10^{-6} < \alpha_e < 10^2, 10^{-6} < \alpha_i < 10^2,$$
$$10^{-2} < \beta < 1, 10^{-6} < \eta < 10^{-4}, \qquad (11)$$
$$10^{-6} < \omega_e < 10^{-2}, 10^{-6} < \omega_i < 10^{-2}, \mu = 0.00055,$$

only 318 solutions correspond to a current sheath. Each of the intervals was divided into 3 equal parts. The solution exists not only in those nodal points of space of parameters, but also in some their neighborhood. In the remaining region of space of parameters $\alpha_e, \alpha_i, \beta, \eta, \omega_e, \omega_i, \mu$ the equations (6) and (7) have no solutions of the current sheath type on the interval. The intervals of parameters (11) correspond to the magnetospheric values. Thus parameters $\alpha_e, \alpha_i, \omega_e, \omega_i$ are free parameters of the model.

Figs. 4 to 6 show the typical solutions. Fig. 4 shows magnetic field profile, Fig. 5 - an ion current profile, Fig. 6 - an electron current profile for the following values of the parameters:

$$\alpha_i = 1.0 \cdot 10^{-6}; \alpha_e = 667; \theta_i = 1.6 \cdot 10^{-15} J; \gamma = 1.0;$$
$$\eta = 1.0 \cdot 10^{-4}; \omega_e = -3.0 \cdot 10^{-3}; \omega_i = 1.0 \cdot 10^{-2}; \qquad (12)$$
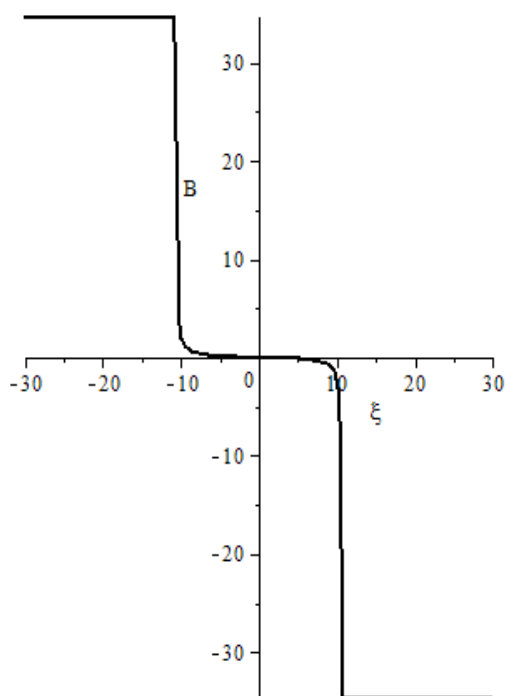$$\mu = 0.00055, n_0 = 0.16 \cdot 10^6 m^{-3}.$$
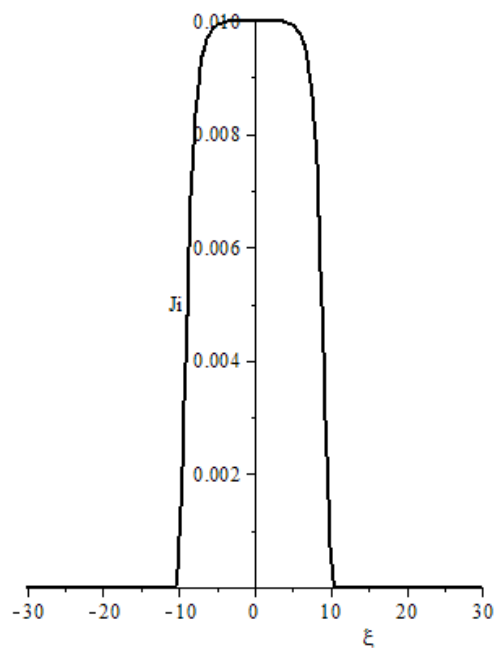

Fig. 4 Magnetic field profile
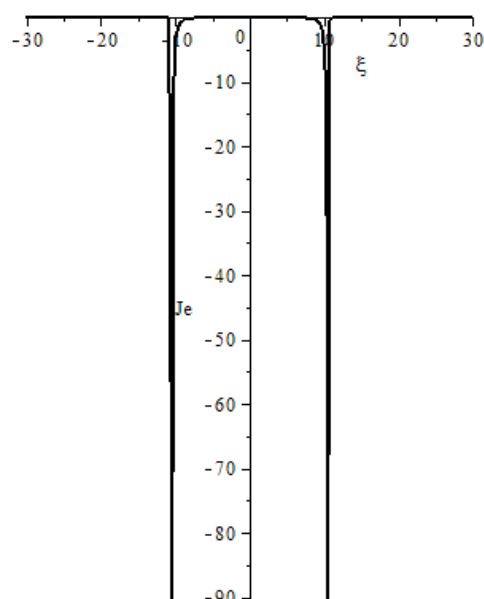

Fig. 5 Ion current profile

Fig. 6 Electron current profile

All the quantities are presented in the dimensionless form. In Fig. 4 the magnetic field reaches $5 \cdot 10^{-8}$ Tl in the dimensional form.

It appears that at the some sets of parameters, for example, such as (10), the ion and electron currents have one maximum, and at the other sets, for example, such as (12), the electron current has two maximums. The majority of solutions obtained for (11) have two-humped (i.e. double-peak) structure for electron current.

Certainly, by means of grid (11) only part of solutions of current sheath type was finded out. Increasing of the number

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of grid nodes will make it possible to find some more solutions. The following statement still remain unchanged: solution points on each of seven axes of parameter space represent a countable set while the values of parameters represent a set of continuum power. Solutions in the neighborhood of those points of the countable set are structurally stable, i.e. an infinitesimal change of the solution corresponds to an infinitesimal change of the value of parameter. However, at the great disturbances of parameter exceeding some threshold this solution breaks.

IV. Conclusion

The equations (6) and (7) derived based on the offered distribution function (1) can simulate the bifurcation current sheaths observed in the magnetosphere.

The solution of the nonlinear problem (6)-(8) is structurally unstable. Solutions of the current sheath type are realized only in some regions of seven-dimensional space of parameters $\alpha_e, \alpha_i, \beta, \eta, \omega_e, \omega_i, \mu$. The phase volume of those regions is small in comparison with the whole phase volume of the definition range of those parameters.

Thus, the stationary current sheath is realized only in those areas of the outer space where parameters $\alpha_e, \alpha_i, \beta, \eta, \omega_e, \omega_i, \mu$ have completely defined ("permissible") values.

These conclusions are based on the application of the specific equilibrium distribution function (1) to the current sheath problem. But this function is a generalization of the widely known distribution Harris' function for the case of the anisotropic temperatures and is the exact solution of the corresponding stationary kinetic equation describing the collisionless magnetoactive plasma.

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