

Synthesis of Logic Circuits Using Fractional-Order Dynamic Fitness Functions

Cecília Reis, J. A. Tenreiro Machado and J. Boaventura Cunha

Abstract— This paper analyses the performance of a genetic algorithm using a new concept, namely a fractional-order dynamic fitness function, for the synthesis of combinational logic circuits. The experiments reveal superior results in terms of speed and convergence to achieve a solution.

Keywords— Circuit Design, Fractional-Order Systems, Genetic Algorithms, Logic Circuits.

I. INTRODUCTION

In the last decade genetic algorithms (GAs) have been applied in the design of electronic circuits, leading to a novel area of research called Evolutionary Electronics (EE) or Evolvable Hardware (EH) [1]. EE considers the concept for automatic design of electronic systems, employing search algorithms to develop good designs.

One decade ago Sushil and Rawlins [2] applied GAs to the combinational circuit design problem while John Koza [3] adopted genetic programming.

In the sequence of this work, Coello, Christiansen and Aguirre [4] presented a computer program that automatically generates high-quality circuit designs. Miller, Thompson and Fogarty [5] applied evolutionary algorithms for the design of arithmetic circuits. Kalganova, Miller and Lipnitskaya [6] proposed a new technique for designing multiple-valued circuits.

In order to solve complex systems, Torresen [7] proposed the method of increased complexity evolution. The idea is to evolve a system gradually as a kind of divide-and-conquer method. The evolved functions are the basic blocks adopted in further evolution of a larger and more complex system.

A major bottleneck in the evolutionary design of electronic circuits is the problem of scale. This refers to the very fast growth of the number of gates, used in the target circuit, as the number of inputs of the evolved logic function increases. This results in a huge search space that is difficult to explore even with evolutionary techniques. Another related obstacle is the time required to calculate the fitness value of a circuit [8]. A

possible method to solve this problem is to use building blocks either than simple gates.

The idea of using memory to achieve better fitness function performances was first introduced by Sano and Kita [9]. Their goal was the optimization of systems with randomly fluctuating fitness function and they developed a Genetic Algorithm with Memory-based Fitness Evaluation (MFEGA). The key ideas of the MFEGA are based on storing the sampled fitness values into memory as a search history, introducing a simple stochastic model of fitness values to be able to estimate fitness values of points of interest using the history for selection operation of the GA.

Following this line of research, and looking for better performance GAs, this paper proposes a GA for the design of combinational logic circuits using fractional-order dynamic fitness functions.

The area of Fractional Calculus (FC) deals with the operators of integration and differentiation to an arbitrary (including noninteger) order and is as old as the theory of classical differential calculus [11-12]. The theory of FC is a well-adapted tool to the modeling of many physical phenomena, allowing the description to take into account some peculiarities that classical integer-order models simply neglect. Nevertheless, the application of FC has been scarce until recently, but the advances on the theory of chaos motivated a renewed interest in this field.

Bearing these ideas in mind this article is organized as follows. Section 2 describes the adopted GA as well as the fractional-order dynamic fitness functions. Section 3 presents the simulation results and finally, section 4 outlines the main conclusions and addresses perspectives towards future developments.

II. THE ADOPTED GENETIC ALGORITHM

In this section we present the GA developed in the study, in terms of the circuit encoding as a chromosome, the genetic operators and the static and dynamic fitness functions.

A. Problem Definition

To design combinational logic circuits a GA strategy is adopted. The circuits are specified by a truth table and the goal is to implement a functional circuit with the least possible complexity. Two sets of logic gates have been defined, as shown in Table 1, being Gset a the simplest one (i.e., a RISC-like set) and Gset b a more complex gate set (i.e., a CISC-like set).

For each gate set the GA searches the solution space, based on a simulated evolution aiming the survival of the fittest strategy. In general, the best individuals of any population

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tend to reproduce and survive, thus improving successive generations. However, inferior individuals can, by chance, survive and also reproduce. In our case, the individuals are digital circuits, which can evolve until the solution is reached (in terms of functionality and complexity).

Table 1 Gate sets

Gate Set	Logic gates
Gset a	{AND,XOR,WIRE}
Gset b	{AND,OR,XOR,NOT,WIRE}

B. Circuit Encoding

In the GA scheme the circuits are encoded as a rectangular matrix A of logic cells as represented in figure 1.

Each cell is represented by three genes: $\langle input1 \rangle \langle input2 \rangle \langle gate\ type \rangle$, where $input1$ and $input2$ are one of the circuit inputs, if they are in the first column, or, one of the previous outputs, if they are in other columns. The $gate\ type$ is one of the elements adopted in the gate set. The chromosome is formed by as many triplets of this kind as the matrix size demands. For example, the chromosome that represents a 3×3 matrix is depicted in figure 2.

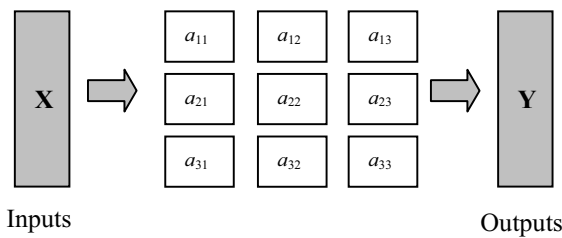


Figure 1: A 3×3 matrix A representing a circuit with input X and output Y.

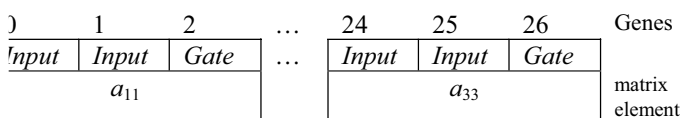


Figure 2: Chromosome for the 3×3 matrix of figure 1.

C. The Genetic Operators

The initial population of circuits (strings) is generated at random. The search is then carried out among this population. The population is evolved through the operators reproduction, crossover and mutation described in the sequel.

In what concern the reproduction operator, the successive generations of new strings are reproduced on the basis of their fitness function. In this case, it is used a tournament selection to select the strings from the old population, up to the new population.

For the crossover operator, the strings in the new population are grouped together into pairs at random. Single point crossover is then performed among pairs. The

crossover point is only allowed between cells to maintain the chromosome integrity.

The mutation operator changes the characteristics of a given cell in the matrix. Therefore, it modifies the gate type and the two inputs, meaning that a completely new cell can appear in the chromosome. Moreover, it is applied an elitist algorithm and, consequently, the best solutions are always kept for the next generation.

To run the GA we have to define the number of individuals to create the initial population P . This population is always the same size across the generations, until the solution is reached.

The crossover rate CR represents the percentage of the population P that reproduces in each generation. Likewise the mutation rate MR is the percentage of the population P that can mutate in each generation.

D. The Static and the Dynamic Fitness Functions

The goal of this study is to find new ways of evaluating the individuals of the population in order to achieve better performance GAs.

In this paper we propose two concepts for the fitness functions, namely the static fitness function F_s and the dynamic fitness function F_d .

The calculation of F_s in (1) is divided in two parts, f_1 and f_2 , where f_1 measures the functionality and f_2 measures the simplicity. In a first phase, we compare the output Y produced by the GA-generated circuit with the required values Y_R , according with the truth table, on a bit-per-bit basis. By other words, f_1 is incremented by *one* for each correct bit of the output until f_1 reaches the maximum value f_{10} , which occurs, when we have a functional circuit. Once the circuit is functional, in a second phase, the GA tries to generate circuits with the least number of gates. This means that the resulting circuit must have as much genes $\langle gate\ type \rangle \equiv \langle wire \rangle$ as possible. Therefore, the index f_2 , that measures the simplicity (the number of null operations), is increased by *one (zero)* for each *wire (gate)* of the generated circuit, yielding:

$$f_{10} = 2^{ni} \times no \quad (1a)$$

$$f_1 = f_1 + 1 \text{ if } \{\text{bit } i \text{ of } Y\} = \{\text{bit } i \text{ of } Y_R\}, i = 1, \dots, f_{10} \quad (1b)$$

$$f_2 = f_2 + 1 \text{ if } gate\ type = wire \quad (1c)$$

$$F_s = \begin{cases} f_1, & F_s < f_{10} \\ f_1 + f_2, & F_s \geq f_{10} \end{cases} \quad (1d)$$

where ni and no represent the number of inputs and outputs of the circuit.

The concept of dynamic fitness function F_d results from an analogy with control systems where we have a variable to be controlled similarly with the GA case where we master the population through the fitness function. The simplest control system is the proportional algorithm; nevertheless, there can be other control algorithms, like the differential and the integral schemes. Therefore, applying the static fitness function corresponds to using a kind of proportional algorithm. If we want to implement a proportional-derivative or a proportional-integrative evolution the fitness

function needs a scheme of the type:

$$F_d = F_s + K D^\alpha [F_s] \quad (2)$$

where $-1 \leq \alpha \leq 1$ is the differential (integral) fractional-order for positive (negative) values of α and K is the 'gain' of the dynamical term.

The mathematical definition of a derivative of fractional order α has been the subject of several different approaches. For example, (3) represents the Grünwald-Letnikov definition of the fractional derivative of order α of the signal $x(t)$:

$$D^\alpha [x(t)] = \lim_{h \rightarrow 0} \left[\frac{1}{h^\alpha} \sum_{k=0}^{\infty} (-1)^k \frac{\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)} x(t-kh) \right] \quad (3)$$

where Γ is the gamma function and h is the time increment. This formulation [13] inspired a discrete-time calculation algorithm, based on the approximation of the time increment h through the sampling period T and a r -term truncated series yielding the equation:

$$D^\alpha [x(t)] \approx \frac{1}{T^\alpha} \sum_{k=0}^r \frac{(-1)^k \Gamma(\alpha+1)}{k! \Gamma(\alpha-k+1)} x(t-kT) \quad (4)$$

III. EXPERIMENTS AND SIMULATION RESULTS

Reliable execution and analysis of a GA usually requires a large number of simulations to provide a reasonable assurance that stochastic effects have been properly considered. Therefore, in this study are developed $n = 1000$ simulations for each case under analysis.

The experiments consist on running the GA to generate a typical combinational logic circuit, namely a 2-to-1 multiplexer (*M2-1*) and a 4-bit parity checker (*PC4*), using the fitness scheme described previously. The circuits are generated with the gate sets presented in Table 1 for $CR = 95\%$, $MR = 20\%$ and $P = 100$.

Having a superior GA performance means achieving solutions with a smaller number N of generations and a smaller standard deviation in order to reduce the stochastic nature of the algorithm.

The simulations investigate a differential scheme $\alpha = \{0, 0.25, 0.5, 0.75, 1\}$ in F_d for gains $10^{-3} \leq K \leq 10^2$. The implementation of the differential fractional order operator adopts (4) with a series truncation $r = 50$ terms.

Figures 5 and 6 show the average number of generations to achieve a solution $AV(N)$ and the standard deviation $SD(N)$ for the *M2-1* and *PC4* circuits, using the Gset a and Gset b, respectively.

We conclude that the F_d concept produces better results than the classical F_s . Moreover, the results reveal that, as it was expected from previous studies [10], the RISC-like set Gset a presents better performance than the CISC-like gate set Gset b for all values of (α, K) .

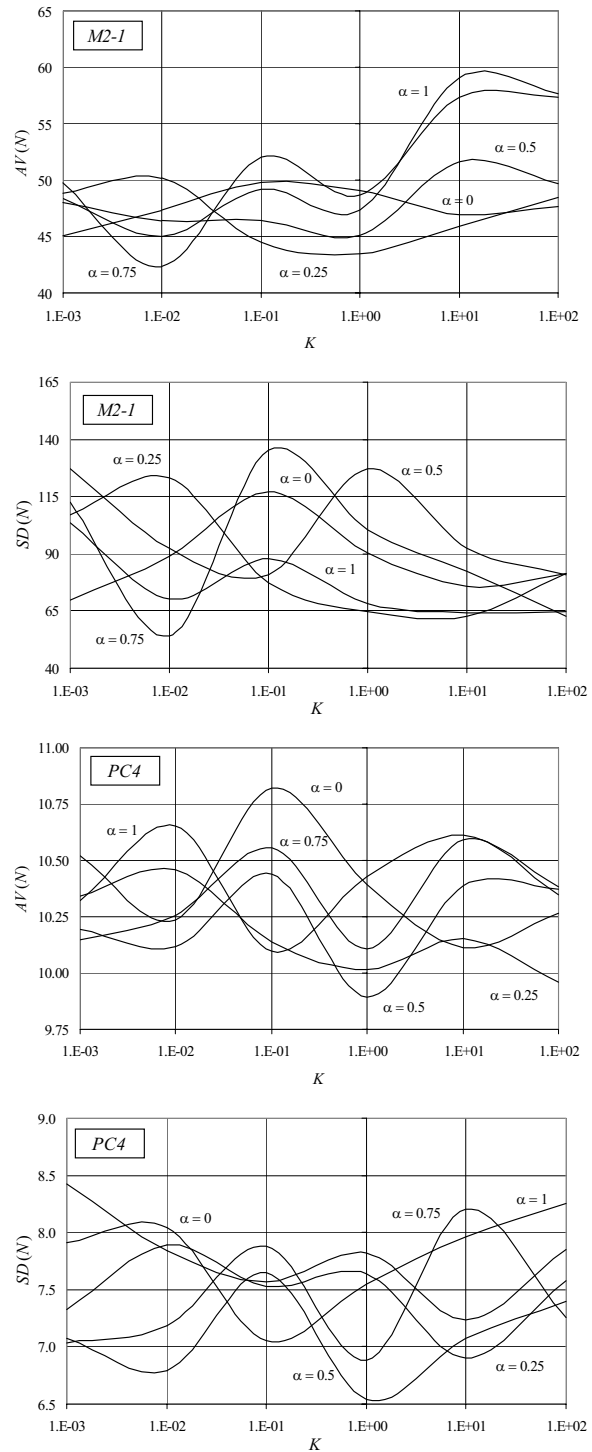


Figure 5: *M2-1* and *PC4* average number of generations to achieve a solution $AV(N)$ and standard deviation $SD(N)$ for integral and differential control schemes with Gset a.

IV. CONCLUSION

The new concept of fractional-order dynamic fitness function of the GA, demonstrates to be an important method that outperforms the traditional static fitness function approach. The tuning of the 'optimal' parameters (α, K) was established by numerical evaluation; therefore, future

research will address the problem of having a more systematic design method. These conclusions encourage further studies using not only a proportional-differential control scheme but also proportional-integral-derivative control schemes.

REFERENCES

- [1] Zebulum, R. S., Pacheco, M. A. and Vellasco, M. M., Evolutionary Electronics: Automatic Design of Electronic Circuits and Systems by Genetic Algorithms, CRC Press, 2001.
- [2] Louis, S.J. and Rawlins, G. J., "Designer Genetic Algorithms: Genetic Algorithms in Structure Design," in Proc. of the Fourth Int. Conference on Genetic Algorithms, 1991.
- [3] Koza, J. R., Genetic Programming. On the Programming of Computers by means of Natural Selection, MIT Press, 1992.
- [4] Coello, C. A., Christiansen, A. D. and Aguirre, A. H., "Using Genetic Algorithms to Design Combinational Logic Circuits", Intelligent Engineering through Artificial Neural Networks. Vol. 6, 1996, pp. 391-396.
- [5] Miller, J. F., Thompson, P. and Fogarty, T, Algorithms and Evolution Strategies in Engineering and Computer Science: Recent Advancements and Industrial Applications. Chapter 6, 1997, Wiley.
- [6] Kalganova, T., Miller, J. F. and Lipnitskaya, N., "Multiple_Valued Combinational Circuits Synthesised using Evolvable Hardware," in Proceedings of the 7th Workshop on Post-Binary Ultra Large Scale Integration Systems, 1998.
- [7] Torresen, J., "A Divide-and-Conquer Approach to Evolvable Hardware," in Proceedings of the Second International Conference on Evolvable Hardware. Vol. 1478, 1998, pp. 57-65.
- [8] Vassilev, V. K. and Miller, J. F., "Scalability Problems of Digital Circuit Evolution," in Proc. of the Second NASA/DOD Workshop on Evolvable Hardware, 2000, pp. 55-64.
- [9] Y. Sano and H. Kita, Optimization of Noisy Fitness Functions by means of Genetic Algorithms using History of Search with test of Estimation, Proceedings of CEC, 2002.
- [10] Cecilia Reis, J. A. Tenreiro Machado, and J. Boaventura Cunha, "Evolutionary Design of Combinational Logic Circuits", Journal of Advanced Computational Intelligence and Intelligent Informatics, Fuji Technology Press, Vol. 8, No. 5, pp. 507-513, Sep. 2004.
- [11] K. B. Oldham and J. Spanier. The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order. Academic Press, New York, 1974.
- [12] K. S. Miller and B. Ross. An Introduction to the Fractional Calculus and Fractional Differential Equations. John Wiley & Sons, New York, 1993.
- [13] J. A. Tenreiro Machado. Analysis and Design of Fractional-Order Digital Control Systems. *SAMS Journal Systems Analysis, Modelling, Simulation*, vol. 27: 107-122, 1997.

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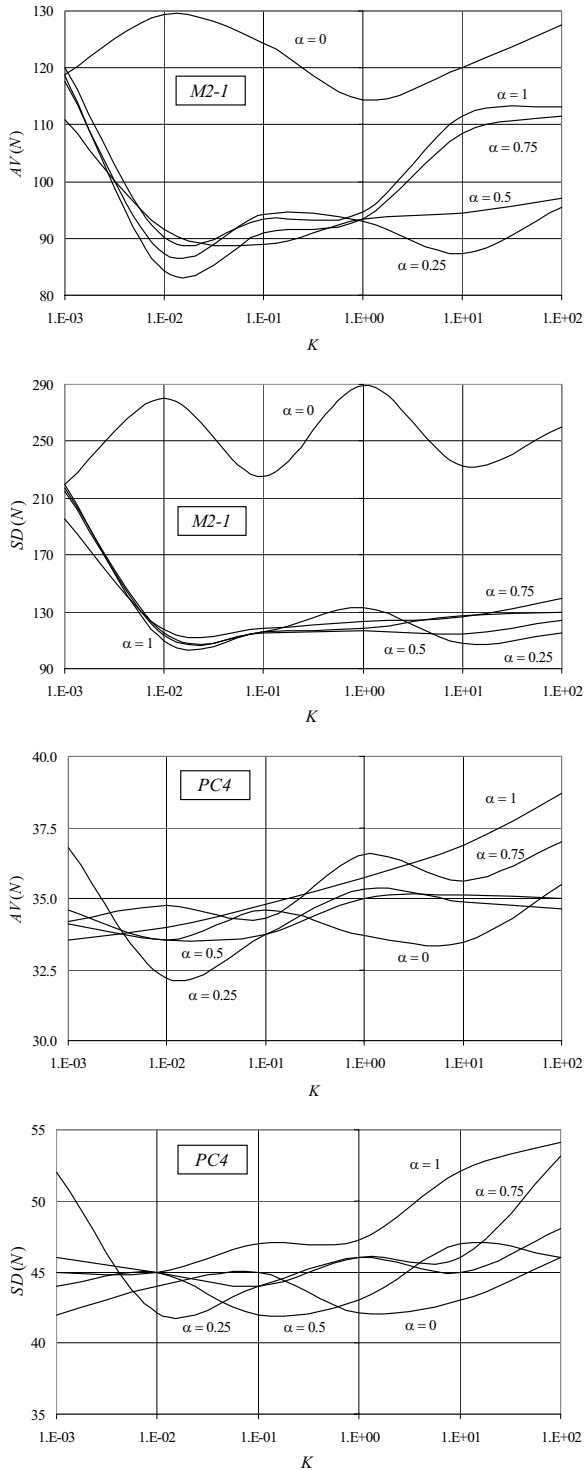


Figure 6: M2-1 and PC4 average number of generations to achieve a solution AV(N) and standard deviation SD(N) for integral and differential control schemes with Gset b.