Some results on Interval-valued fuzzy BG-algebras

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Abstract- In this note the notion of interval-valued fuzzy BG-algebras (briefly, i-v fuzzy BG-algebras), the level and strong level BG-subalgebra is introduced. Then we state and prove some theorems which determine the relationship between these notions and BG-subalgebras. The images and inverse images of i-v fuzzy BG-subalgebras are defined, and how the homomorphic images and inverse images of i-v fuzzy BG-subalgebras are studied.

Keywords- BG-algebra, fuzzy BG-subalgebra, intervalvalued fuzzy set, interval-valued fuzzy BG-subalgebra.

I. INTRODUCTION

In 1966, Y. Imai and K. Iseki [5] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [9] J. Neggers and H. S. Kim introduced the notion of d-algebras, which is generalization of BCK-algebras and investigated relation between d-algebras and BCK-algebras. Also they introduced the notion of d-algebras [8]. In [6] C. B. Kim, H. S. Kim introduced the notion of d-algebras which is a generalization of d-algebras. S. S. Ahn and H. D. Lee applied the fuzzy notions to d-algebras and introduced the notions of fuzzy d-algebras [1]. The concept of a fuzzy set, which was introduced in [11].

In [12], Zadeh made an extension of the concept of a fuzzy set by an interval-valued fuzzy set (i.e., a fuzzy set with an interval-valued membership function). This interval-valued fuzzy set is referred to as an i-v fuzzy set, also he constructed a method of approximate inference using his i-v fuzzy sets. Biswas [2], defined interval-valued fuzzy subgroups and S. M. Hong et. al. applied the notion of interval-valued fuzzy to BCI-algebras.

In the present paper, we using the notion of inteval-valued fuzzy set by Zadeh and introduced the concept of interval-valued fuzzy BG-subalgebras (briefly i-v fuzzy BG-subalgebras) of a BG-algebra, and study some of their properties. We prove that every BG-subalgebra of a BG-algebra X can be relized as an i-v level BG-subalgebra of an i-v fuzzy BG-sublagebra of X, then we obtain some related results which have been mentioned in the abstract.

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II. PRELIMINARY

Definition 2.1. [6] A BG-algebra is a non-empty set X with a consonant 0 and a binary operation * satisfying the following axioms:

(I) x * x = 0,

(II) x * 0 = x,

(III) (x * y) * (0 * y) = x,

for all $x, y \in X$.

For brevity we also call X a BG-algebra. In X we can define a binary relation \leq by $x \leq y$ if and only if x * y = 0.

Theorem 2.2. [6] In a BG-algebra X, we have the following properties:

(i) 0*(0*x) = x,

(ii) if x * y = 0, then x = y,

(iii) if 0 * x = 0 * y, then x = y,

(iv) (x * (0 * x)) * x = x,

For all $x, y \in X$.

A non-empty subset I of a BG-algebra X is called a subalgebra of X if $x*y \in I$ for any $x,y \in I$.

A mapping $f: X \longrightarrow Y$ of BG-algebras is called a BG-homomorphism if f(x * y) = f(x) * f(y) for all $x, y \in X$.

We now review some fuzzy logic concept (see [11]). Let X be a set. A fuzzy set A in X is characterized by a membership function $\mu_A: X \longrightarrow [0,1]$. Let f be a mapping from the set X to the set Y and let B be a fuzzy set in Y with membership function μ_B . The inverse image of B, denoted $f^{-1}(B)$, is the fuzzy set in X with membership function $\mu_{f^{-1}(B)}$ defined by $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ for all $x \in X$.

Conversely, let A be a fuzzy set in X with membership function μ_A . Then the image of A, denoted by f(A), is the fuzzy set in Y such that:

$$\mu_{f(A)}(y) = \begin{cases} \sup_{z \in f^{-1}(y)} \mu_A(z) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise} \end{cases}$$

A fuzzy set A in the BG-algebra X with the membership function μ_A is said to be have the sup property if for any subset $T\subseteq X$ there exists $x_0\in T$ such that $\mu_A(x_0)=\sup_{}\mu_A(t).$

An interval-valued fuzzy set (briefly, i-v fuzzy set) A defined on X is given by $A = \{(x, [\mu_A^L(x), \mu_A^U(x)])\}, \ \forall x \in X.$ Briefly, denoted by $A = [\mu_A^L, \mu_A^U]$ where μ_A^L and μ_A^U are any two fuzzy sets in X such that $\mu_A^L(x) \leq \mu_A^U(x)$ for all $x \in X$.

Let $\overline{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)]$, for all $x \in X$ and let D[0,1] denotes the family of all closed sub-intervals of [0,1]. It is clear that if $\mu_A^L(x) = \mu_A^U(x) = c$, where $0 \le c \le 1$ then $\overline{\mu}_A(x) = [c,c]$ is in D[0,1]. Thus $\overline{\mu}_A(x) \in D[0,1]$,

World Academy of Science, Engineering and Technology International Journal of Mathematical and Computational Sciences Vol:1, No:5, 2007

for all $x \in X$. Therefore the i-v fuzzy set A is given by $A = \{(x, \overline{\mu}_A(x))\}, \forall x \in X \text{ where } \overline{\mu}_A : X \longrightarrow D[0, 1].$

Now we define refined minimum (briefly, rmin) and order " \leq " on elements $D_1=[a_1,b_1]$ and $D_2=[a_2,b_2]$ of D[0,1]

$$rmin(D_1, D_2) = [min\{a_1, a_2\}, min\{b_1, b_2\}]$$

$$D_1 \le D_2 \iff a_1 \le a_2 \land b_1 \le b_2$$

Similarly we can define > and =.

Definition 2.3. [1] Let μ be a fuzzy set in a BG-algebra. Then μ is called a fuzzy BG-subalgebra (BG-algebra) of X $\text{if } \mu(x*y) \geq \min\{\mu(x), \mu(y)\} \text{ for all } x,y \in X.$

Proposition 2.4. [3] Let f be a BG-homomorphism from X into Y and G be a fuzzy BG-subalgebra of Y with the membership function μ_G . Then the inverse image $f^{-1}(G)$ of G is a fuzzy BG-subalgebra of X.

Proposition 2.5. [3] Let f be a BG-homomorphism from Xonto Y and D be a fuzzy BG-subalgebra of X with the sup property. Then the image f(D) of D is a fuzzy BG-subalgebra

III. INTERVAL-VALUED FUZZY BG-ALGEBRA

From now on X is a BG-algebra, unless otherwise is stated.

Definition 3.1. An i-v fuzzy set A in X is called an interval-valued fuzzy BG-subalgebras (briefly i-v fuzzy BGsubalgebra) of X if:

$$\overline{\mu}_A(x*y) \ge rmin\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}$$

for all $x, y \in X$.

Example 3.2. Let $X = \{0, 1, 2, 3\}$ be a set with the following table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then X is a BG-algebra. Define $\overline{\mu}_A$ as:

$$\overline{\mu}_A(x) = \left\{ \begin{array}{ll} [0.3, 0.9] & \qquad & \text{if} \quad x \in \{0, 2\} \\ [0.1, 0.6] & \qquad & \text{Otherwise} \end{array} \right.$$

It is easy to check that A is an i-v fuzzy BG-subalgebra of

Lemma 3.3. If A is an i-v fuzzy BG-subalgebra of X, then for all $x \in X$

$$\overline{\mu}_A(0) \geq \overline{\mu}_A(x).$$

Proof. For all $x \in X$, we have

$$\begin{array}{lcl} \overline{\mu}_{A}(0) & = & \overline{\mu}_{A}(x*x) \geq rmin\{\overline{\mu}_{A}(x), \overline{\mu}_{A}(x)\} \\ & = & rmin\{[\mu_{A}^{L}(x), \mu_{A}^{U}(x)], [\mu_{A}^{L}(x), \mu_{A}^{U}(x)]\} \\ & = & [\mu_{A}^{L}(x), \mu_{A}^{U}(x)] = \overline{\mu}_{A}(x). \end{array}$$

Theorem 3.4. Let A be an i-v fuzzy BG-subalgebra of X. If there exists a sequence $\{x_n\}$ in X, such that $\lim_{n\to\infty} \overline{\mu}_A(x_n) =$ [1, 1] Then $\overline{\mu}_A(0) = [1, 1]$.

Proof. By Lemma 3.3, we have $\overline{\mu}_A(0) \geq \overline{\mu}_A(x)$, for all $x \in X$, thus $\overline{\mu}_A(0) \ge \overline{\mu}_A(x_n)$, for every positive integer n. Consider

$$[1,1] \ge \overline{\mu}_A(0) \ge \lim_{n \to \infty} \overline{\mu}_A(x_n) = [1,1].$$

Hence $\overline{\mu}_A(0) = [1, 1].$

Theorem 3.5. An i-v fuzzy set $A = [\mu_A^L, \mu_A^U]$ in X is an i-v fuzzy BG-subalgebra of X if and only if μ_A^L and μ_A^U are fuzzy BG-subalgebra of X.

Proof. Let μ_A^L and μ_A^U are fuzzy BG-subalgebra of X and $x, y \in X$, consider

$$\begin{array}{lcl} \overline{\mu}_A(x*y) & = & [\overline{\mu}_A(x*y), \overline{\mu}_A(x*y)] \\ & \geq & [\min\{\mu_A^L(x), \mu_A^L(y)\}, \min\{\mu_A^U(x), \mu_A^U(y)\}] \\ & = & r\min\{[\mu_A^L(x), \mu_A^U(x)], [\mu_A^L(y), \mu_A^U(y)]\} \\ & = & r\min\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}. \end{array}$$

This completes the proof.

Conversely, suppose that A is an i-v fuzzyBG-subalgebras of X. For any $x, y \in X$ we have

of
$$A$$
 . For any $x,y \in A$ we have
$$[\mu_A^L(x * y), \mu_A^U(x * y)] = \overline{\mu}_A(x * y)$$
 $\geq rmin\{\overline{\mu}_A(x), \overline{\mu}_A(y)\} = rmin\{[\mu_A^L(x), \mu_A^U(x)], [\mu_A^L(y), \mu_A^U(y)]\} = [min\{\mu_A^L(x), \mu_A^L(y)\}, min\{\mu_A^U(x), \mu_A^U(y)\}].$ Therefore $\mu_A^L(x * y) \geq min\{\mu_A^L(x), \mu_A^L(y)\}$ and $\mu_A^U(x * y) \geq min\{\mu_A^U(x), \mu_A^U(y)\}$, hence we get that μ_A^L and $\mu_A^U(x * y) \geq min\{\mu_A^U(x), \mu_A^U(y)\}$, hence we get that $\mu_A^U(x) = rmin\{\mu_A^U(x), \mu_A^U(y)\}$.

fuzzyBG-subalgebras of X.

Theorem 3.6. Let A_1 and A_2 are i-v fuzzy BG-subalgebras of X. Then $A_1 \cap A_2$ is an i-v fuzzy BG-subalgebras of X.

Corollary 3.7. Let $\{A_i|i\in\Lambda\}$ be a family of i-v fuzzy BG-subalgebras of X. Then $\bigcap A_i$ is also an i-v fuzzy BG-

subalgebras of X.

Definition 3.8. Let A be an i-v fuzzy set in X and $[\delta_1, \delta_2] \in$ D[0,1]. Then the i-v level BG-subalgebra $U(A; [\delta_1, \delta_2])$ of A and strong i-v BG-subalgebra $U(A; >, [\delta_1, \delta_2])$ of X are defined as following:

$$U(A; [\delta_1, \delta_2]) := \{ x \in X \mid \overline{\mu}_A(x) \ge [\delta_1, \delta_2] \},$$

World Academy of Science, Engineering and Technology International Journal of Mathematical and Computational Sciences Vol:1, No:5, 2007

$$U(A; >, [\delta_1, \delta_2]) := \{x \in X \mid \overline{\mu}_A(x) > [\delta_1, \delta_2]\}.$$

Theorem 3.9. Let A be an i-v fuzzy BG-subalgebra of X and B be closure of image of μ_A . Then the following condition are equivalent:

- (i) A is an i-v fuzzy BG-subalgebra of X.
- (ii) For all $[\delta_1, \delta_2] \in Im(\mu_A)$, the nonempty level subset $U(A; [\delta_1, \delta_2])$ of A is a BG-subalgebra of X.
- (iii) For all $[\delta_1,\delta_2]\in Im(\mu_A)\setminus B$, the nonempty strong level subset $U(A;>,[\delta_1,\delta_2])$ of A is a BG-subalgebra of X.
- (iv) For all $[\delta_1, \delta_2] \in D[0, 1]$, the nonempty strong level subset $U(A; >, [\delta_1, \delta_2])$ of A is a BG-subalgebra of X.
- (v) For all $[\delta_1, \delta_2] \in D[0, 1]$, the nonempty level subset $U(A; [\delta_1, \delta_2])$ of A is a BG-subalgebra of X.

Proof. (i \longrightarrow iv) Let A be an i-v fuzzy BG-subalgebra of X, $[\delta_1,\delta_2]\in D[0,1]$ and $x,y\in U(A;<,[\delta_1,\delta_2])$, then we have $\overline{\mu}_A(x*y)\geq rmin\{\overline{\mu}_A(x),\overline{\mu}_A(y)\}>rmin\{[\delta_1,\delta_2],[\delta_1,\delta_2]\}=[\delta_1,\delta_2]$ thus $x*y\in U(A;>,[\delta_1,\delta_2])$. Hence $U(A;>,[\delta_1,\delta_2])$ is a BG-subalgebra of X. (iv \longrightarrow iii) It is clear.

 $\begin{array}{ll} \text{(iii} \,\longrightarrow\, \text{ii) Let} \,\, [\delta_1,\delta_2] \,\in\, Im(\mu_A). \,\, \text{Then} \,\, U(A;[\delta_1,\delta_2]) \\ \text{is a nonempty. Since} \,\, U(A;[\delta_1,\delta_2]) \,=\, \bigcap_{[\delta_1,\delta_2]>[\alpha_1,\alpha_2]} U(A;>) \end{array}$

, $[\delta_1, \delta_2]$), where $[\alpha_1, \alpha_2] \in Im(\mu_A) \setminus B$. Then by (iii) and Corollary 3.8, $U(A; [\delta_1, \delta_2])$ is a BG-subalgebra of X.

(ii \longrightarrow v) Let $[\delta_1,\delta_2] \in D[0,1]$ and $U(A;[\delta_1,\delta_2])$ be nonempty. Suppose $x,y \in U(A;[\delta_1,\delta_2])$. Let $[\beta_1,\beta_2] = min\{\mu_A(x),\mu_A(y)\}$, it is clear that $[\beta_1,\beta_2] = min\{\mu_A(x),\mu_A(y)\} \geq \{[\delta_1,\delta_2],[\delta_1,\delta_2]\} = [\delta_1,\delta_2]$. Thus $x,y \in U(A;[\beta_1,\beta_2])$ and $[\beta_1,\beta_2] \in Im(\mu_A)$, by (ii) $U(A;[\beta_1,\beta_2])$ is a BG-subalgebra of X, hence $x * y \in U(A;[\beta_1,\beta_2])$. Then we have $\overline{\mu}_A(x * y) \geq rmin\{\mu_A(x),\mu_A(y)\} \geq \{[\beta_1,\beta_2],[\beta_1,\beta_2]\} = [\beta_1,\beta_2] \geq [\delta_1,\delta_2]$. Therefore $x * y \in U(A;[\delta_1,\delta_2])$. Then $U(A;[\delta_1,\delta_2])$ is a BG-subalgebra of X.

 $(\mathbf{v} \longrightarrow \mathbf{i})$ Assume that the nonempty set $U(A; [\delta_1, \delta_2])$ is a BG-subalgebra of X, for every $[\delta_1, \delta_2] \in D[0, 1]$. In contrary, let $x_0, y_0 \in X$ be such that

$$\overline{\mu}_A(x_0 * y_0) < rmin\{\overline{\mu}_A(x_0), \overline{\mu}_A(y_0)\}.$$

Let $\overline{\mu}_A(x_0)=[\gamma_1,\gamma_2],$ $\overline{\mu}_A(y_0)=[\gamma_3,\gamma_4]$ and $\overline{\mu}_A(x_0*y_0)=[\delta_1,\delta_2].$ Then

 $[\delta_1,\delta_2] < rmin\{[\gamma_1,\gamma_2],[\gamma_3,\gamma_4]\} = [min\{\gamma_1,\gamma_3],min\{\gamma_2,\gamma_4\}]. \\ \text{the set}$

So $\delta_1 < min\{\gamma_1, \gamma_3\}$ and $\delta_2 < min\{\gamma_2, \gamma_4\}$.

Consider

$$[\lambda_1, \lambda_2] = \frac{1}{2} \overline{\mu}_A(x_0 * y_0) + rmin\{\overline{\mu}_A(x_0), \overline{\mu}_A(y_0)\}$$

We get that

$$\begin{aligned} [\lambda_1, \lambda_2] &= \frac{1}{2} ([\delta_1, \delta_2] + \min\{\gamma_1, \gamma_3\}, \min\{\gamma_2, \gamma_4\}]) \\ &= [\frac{1}{2} (\delta_1 + \min\{\gamma_1, \gamma_3\}), \frac{1}{2} (\delta_2 + \min\{\gamma_2, \gamma_4\})] \end{aligned}$$

Therefore

$$\min\{\gamma_1, \gamma_3\} > \lambda_1 = \frac{1}{2}(\delta_1 + \min\{\gamma_1, \gamma_3\}) > \delta_1$$

$$\min\{\gamma_2, \gamma_4\} > \lambda_2 = \frac{1}{2}(\delta_2 + \min\{\gamma_2, \gamma_4\}) > \delta_2$$

Hence

 $[\min\{\gamma_1, \gamma_3\}, \min\{\gamma_2, \gamma_4\}] > [\lambda_1, \lambda_2] > [\delta_1, \delta_2] = \overline{\mu}_A(x_0 * y_0)$

so that $x_0 * y_0 \notin U(A; [\delta_1, \delta_2])$ which is a contradiction, since

$$\overline{\mu}_{A}(x_{0}) = [\gamma_{1}, \gamma_{2}] \ge [\min\{\gamma_{1}, \gamma_{3}\}, \min\{\gamma_{2}, \gamma_{4}\}] > [\lambda_{1}, \lambda_{2}]$$
$$\overline{\mu}_{A}(y_{0}) = [\gamma_{3}, \gamma_{4}] \ge [\min\{\gamma_{1}, \gamma_{3}\}, \min\{\gamma_{2}, \gamma_{4}\}] > [\lambda_{1}, \lambda_{2}]$$

imply that $x_0,y_0\in U(A;[\delta_1,\delta_2])$. Thus $\overline{\mu}_A(x*y)\geq rmin\{\overline{\mu}_A(x),\overline{\mu}_A(y)\}$ for all $x,y\in X$. Which completes the proof.

Theorem 3.10. Each BG-subalgebra of X is an i-v level BG-subalgebra of an i-v fuzzy BG-subalgebra of X.

Proof. Let Y be a BG-subalgebra of X, and A be an i-v fuzzy set on X defined by

$$\overline{\mu}_A(x) = \left\{ \begin{array}{ll} [\alpha_1, \alpha_2] & \quad \text{if} \quad x \in Y \\ [0, 0] & \quad \text{Otherwise} \end{array} \right.$$

where $\alpha_1, \alpha_2 \in [0, 1]$ with $\alpha_1 <, \alpha_2$. It is clear that $U(A; [\alpha_1, \alpha_2]) = Y$. Let $x, y \in X$. We consider the following cases:

case 1) If $x, y \in Y$, then $x * y \in Y$ therefore $\overline{\mu}_A(x * y) = [\alpha_1, \alpha_2] = rmin\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = rmin\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}.$ case 2) If $x, y \notin Y$, then $\overline{\mu}_A(x) = [0, 0] = \overline{\mu}_A(y)$

case 2) If $x,y \notin Y$, then $\overline{\mu}_A(x) = [0,0] = \overline{\mu}_A(y)$ and so $\overline{\mu}_A(x*y) \geq [0,0] = rmin\{[0,0],[0,0]\} = rmin\{\overline{\mu}_A(x),\overline{\mu}_A(y)\}.$

case 3) If $x\in Y$ and $y\not\in Y$, then $\overline{\mu}_A(x)=[\alpha_1,\alpha_2]$ and $\overline{\mu}_A(y)=[0,0]$. Thus $\overline{\mu}_A(x*y)\geq [0,0]=rmin\{[\alpha_1,\alpha_2],[0,0]\}=rmin\{\overline{\mu}_A(x),\overline{\mu}_A(y)\}$.

case 4) If $y \in Y$ and $x \notin Y$, then by the same argument as in case 3, we can conclude that $\overline{\mu}_A(x*y) \geq rmin\{\overline{\mu}_A(x), \overline{\mu}_A(y)\}.$

Therefore A is an i-v fuzzy BG-subalgebra of X.

Theorem 3.11. If A is an i-v fuzzy BG-subalgebra of X, then the set

$$X_{\overline{\mu}_A} := \{ x \in X \mid \overline{\mu}_A(x) = \overline{\mu}_A(0) \}$$

is a BG-subalgebra of X.

Definition 3.12. [2] Let f be a mapping from the set X into a set Y. Let B be an i-v fuzzy set in Y. Then the inverse image of B, denoted by $f^{-1}[B]$, is the i-v fuzzy set in X with the

membership function given by $\overline{\mu}_{f^{-1}[B]}(x) = \overline{\mu}_B(f(x)),$ for all $x \in X$.

Lemma 3.13. [2] Let f be a mapping from the set X into a set Y. Let $m = [m^L, m^U]$ and $n = [n^L, n^U]$ be i-v fuzzy sets in X and Y respectively. Then

 $\begin{array}{l} \text{(i) } f^{-1}(n) = [f^{-1}(n^L), f^{-1}(n^U)], \\ \text{(ii) } f(m) = [f(m^L), f(m^U)], \end{array}$

Proposition 3.14. Let f be a BG-homomorphism from X into Y and G be an i-v fuzzy BG-subalgebra of Y with the membership function μ_G . Then the inverse image $f^{-1}[G]$ of G is an i-v fuzzy BG-subalgebra of X.

Proof. Since $B = [\mu_B^L, \mu_B^U]$ is an i-v fuzzy BG-subalgebra of Y, by Theorem 3.5, we get that μ_B^L and μ_B^U are fuzzy BG-subalgebra of Y. By Proposition 2.4, $f^{-1}[\mu_B^L]$ and $f^{-1}[\mu_B^U]$ are fuzzy BG-subalgebra of X, by above lemma and Theorem 3.5, we can conclude that $f^{-1}(B) = [f^{-1}(\mu_B^L), f^{-1}(\mu_B^U)]$ is an i-v fuzzy BG-subalgebra of X.

Definition 3.15. [2] Let f be a mapping from the set X into a set Y, and A be an i-v fuzzy set in X with membership function μ_A . Then the image of A, denoted by f[A], is the i-v fuzzy set in Y with membership function defined by:

$$\overline{\mu}_{f[A]}(y) = \left\{ \begin{array}{ll} rsup_{z \in f^{-1}(y)}\overline{\mu}_A(z) & \text{if } f^{-1}(y) \neq \emptyset \\ [0,0] & \text{otherwise} \end{array} \right.$$

Where $f^{-1}(y) = \{x \mid f(x) = y\}.$

Theorem 3.16. Let f be a BG-homomorphism from X onto Y. If A is an i-v fuzzy BG-subalgebra of X, then the image f[A] of A is an i-v fuzzy BG-subalgebra of Y.

Proof. Assume that A is an i-v fuzzy BG-subalgebra of X, then $A = [\mu_A^L, \mu_A^U]$ is an i-v fuzzy BG-subalgebra of X if and only if μ_B^L and μ_B^U are fuzzy BG-subalgebra of X. By Proposition 2.5, $f[\mu_A^L]$ and $f[\mu_A^U]$ are fuzzy BG-subalgebra of Y, by Lemma 3.13, and Theorem 3.5, we can conclude that $f[A] = [f[\mu_A^L], f[\mu_A^U]]$ is an i-v fuzzy BG-subalgebra of Y.

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