

Chattering-free Sliding Mode Control for an Active Magnetic Bearing System

Abdul Rashid Husain, Mohamad Noh Ahmad, and Abdul Halim Mohd Yatim

Abstract—In this paper, a few chattering-free Sliding Mode Controllers (SMC) are proposed to stabilize an Active Magnetic Bearing (AMB) system with gyroscopic effect that is proportional to the rotor speed. The improved switching terms of the controller inherited from the saturation-type function and boundary layer control technique is shown to be able to achieve bounded and asymptotic stability, respectively, while the chattering effect in the input is attenuated. This is proven to be advantageous for AMB system since minimization of chattering results in optimized control effort. The performance of each controller is demonstrated via result of simulation in which the measurement of the total consumed energy and maximum control magnitude of each controller illustrates the effectiveness of the proposed controllers.

Keywords—Active Magnetic Bearing (AMB), Sliding Mode Control (SMC), chattering-free SMC.

I. INTRODUCTION

SLIDING Mode Control (SMC) has been recognized as one of the efficient tools to design robust controllers for complex high-order nonlinear dynamic system operating under various conditions including the system with the presence of uncertainties and external disturbance [1][2]. Due to this advantageous feature, SMC has been applied to a wide range of dynamic system including linear, nonlinear, multi-input/multi-output, discrete-time and large scale systems such as robot manipulators [3][4], active suspension system [5] and magnetic bearings [6].

In SMC, the control system is designed such that the states are driven and then constrained to lie on a designed sliding surface or within a neighborhood of a suitable switching surface wherein as long as the system trajectories stay on the surface, the closed-loop dynamics are completely governed by the equations that define the surface [2][7]. However, in order for the sliding surface to be attractive, the reaching condition has to be met and this requires the use of discontinuous switching function which causes chattering of the control signals. This chattering control signal involves high control

activity which may excite neglected high-frequency dynamics in the system and also to cause possible damages to the actuators. However, eliminating the chattering also has other drawbacks in which in some application, the discontinuous switching input is necessary in order to maintain the characteristic of the systems. Many methods have been proposed such that a continuous control term is used to replace the discontinuous term such as the sat () function and boundary layer technique [7] [8] [9]. These methods are proven to be advantageous in eliminating, or at least minimizing chattering to certain acceptable degree, but in the expense of the robustness property of SMC. Besides, both of these methods require the use of thin boundary layer around the sliding surface in which magnitude of the boundary layer determines the control accuracy. The larger the boundary layer width, the smoother the control signal, however, it no longer drives the system to the origin, but to within the chosen boundary layer instead. These conflicting requirements to fulfill, which are the smoothness of the input signal and the control accuracy, have attracted my research works to propose methods that can overcome the chattering effect while retaining the property of the SMC, among others are the time-varying boundary layer [4][9], adaptive fuzzy based algorithm [10] and filtering technique [11]. Borrowing the idea from [4][8], an exponential-decaying boundary layer technique is proposed in this paper in which the chattering in the control signal is suppressed while the asymptotic stability is retained. The method is applied to a class of Active Magnetic Bearing (AMB) system where the model contains the gyroscopic effect in which the magnitude is proportional to the rotating speed of the rotor. Also, the linearized current-force factor of the AMB system is allowed to be varied by 20% of its nominal value to reflect the uncertainty present in the parameter. Other chattering elimination methods reported in the work included in the references are also used as a benchmark. The performances of each controller are measured in term of its total energy consumed and the maximum peak of the control signals to portray the effectiveness of each method.

The outline of this paper is as follows: In Section II, the preliminaries of the class of the system under proposed SMC is outlined based on [12]. The description of discontinuous terms of the SMC is also covered. Section III covers mathematical model of the AMB system where the model is transformed into the class of system using deterministic approach. Then, in Section IV, the performances of the controller are illustrated through simulation works and the total energy consumed and maximum peak of control input are measured to determine the effectiveness of the method.

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Finally, the conclusion in Section V summarizes the contribution of the work.

II. PRELIMINARIES: SLIDING MODE CONTROL DESIGN

In this section, the necessary theorems and lemma developed in [12] for the surface designs are outlined. Since the main emphasis of this work is on the chattering elimination method, the proofs are intentionally omitted and can be found in [12]. Then, from the derived controller, the discontinuous switching term is replaced with the proposed exponential-decaying boundary layer function.

Consider a class of uncertain system

$$\dot{x}(t) = (A + \Delta A(x, p, t))x(t) + (B + \Delta B(x, p, t))u(t) \quad (1)$$

where $x(t) \in \mathfrak{R}^n$ is the system states, $u(t) \in \mathfrak{R}^m$ is the control input and p is any time-varying scalar function. A and B are the system and input matrices, respectively, and B is of full rank. $\Delta A(x, p, t)$ and $\Delta B(x, p, t)$ represent the uncertainties in the system matrix and input matrices, respectively. To complete the description of the uncertain dynamical system, the following assumptions are introduced and assumed to be valid.

- A1) For existence purposes, $\Delta A(\cdot, \cdot, \cdot)$ and $\Delta B(\cdot, \cdot, \cdot)$ are continuous on their arguments.
 A2) There exist known positive scalars γ_1 and γ_2 and a function $h(x, p, t)$ such that

$$\begin{aligned} \|\Delta A(x, p, t)\| &\leq \gamma_1, \\ \Delta B(x, p, t) &= Bh(x, p, t), \\ \|h(x, p, t)\| &\leq \gamma_2 < 1. \end{aligned}$$

- A3) The pair (A, B) is controllable.

Define a linear switching surface as

$$\sigma = Sx = 0 \quad (2)$$

where $S \in \mathfrak{R}^{m \times n}$ is of full row rank m . The switching surface parameter matrix S should be selected such that

- E1) The matrix SB is nonsingular
 E2) The reduced $(n-m)$ order system of the system (1) restricted on the switching surface $Sx = 0$ is globally exponentially stable for all allowable uncertainties satisfying assumption A2).

A. Necessary Theorems and Lemma for Surface Design

Let $\Phi \in \mathfrak{R}^{n \times (n-m)}$ be any full column rank matrix such that $B^T \Phi = 0$ and $\Phi^T \Phi = I_{n-m}$. Then it can be established that

$$\begin{bmatrix} \Phi^T \\ (B^T B)^{-1} B^T \end{bmatrix} [\Phi \quad B] = \begin{bmatrix} I_{n-m} & 0 \\ 0 & I_m \end{bmatrix} \quad (3a)$$

or equivalently

$$\Phi^T \Phi + B(B^T B)^{-1} B^T = I_n. \quad (3b)$$

From this, it is obvious that the nonsingular transformation matrix can be defined as follows:

$$T = \begin{bmatrix} \Phi^T \\ (B^T B)^{-1} B^T \end{bmatrix} \quad (4)$$

Substituting (4) into system (1) gives

$$\begin{aligned} \dot{x}(t) &= (A + \Phi \Phi^T \Delta A(x, p, t))x(t) + B\{[I + h(x, p, t)]u(t) \\ &\quad + (B^T B)^{-1} B^T (\Delta A(x, p, t)x)\} \end{aligned} \quad (5)$$

This new system representation has mapped the uncertainties into the matched and mismatched components in which it is obvious that $\Delta A(x, p, t)$ and $\Delta B(x, p, t)$ belong to the mismatched and matched components, respectively.

Theorem 1. Consider the system (1) and suppose assumption (A1)-(A3) hold. Then there exist a matrix S satisfying (E1) and (E2), if there is a matrix $P > 0$ and positive scalar δ such that

$$\begin{aligned} &\Phi^T A^T \Phi P + P \Phi^T A \Phi + \delta^{-1} P^2 \\ &- (\delta \gamma_1)^{-1} P \Phi^T A B (B^T B)^{-1} B^T A^T \Phi P + (\delta \gamma_1^2) I_{n-m} < 0 \end{aligned} \quad (6)$$

where γ_1 is given in assumption (A2). Moreover, the matrix S can be proposed to be as follows:

$$S = (B^T B)^{-1} B^T (I_n + (\delta \gamma_1^2)^{-1} A^T \Phi P \Phi^T) \quad (7)$$

Proof See [12] □

Theorem 1 guarantees that the surface parameter S exists if the condition outlined is met and asymptotic stability can be achieved even with the mismatched uncertainty present. However, the solution can be improved by casting the Riccati equation (7) into a Linear Matrix Inequality (LMI) problem as stated in the following Lemma.

Lemma 1. The existence solution (14) is equivalent to solving the following LMI set which is:

$$\begin{bmatrix} \bar{A}^T P + P \bar{A} + \bar{C} & P \Phi^T \\ \Phi P & V^{-1} \end{bmatrix} < 0 \quad (8)$$

$$P > 0 \quad (9)$$

where

$$\begin{aligned} \bar{A} &= \Phi^T A \Phi, \\ \bar{C} &= \delta \gamma_1^2 I_{n-m}, \\ V &= \delta \gamma_1^2 A B (B^T B)^{-1} B^T A^T - \delta^{-1}. \end{aligned}$$

Proof See [12]. □

B. Sliding Mode Control Law

To find the control law such the trajectories of system (1) are driven to the designed sliding surface and to remain on the surface for the subsequent time, the following reachability condition is used [1][2]

$$\sigma^T \dot{\sigma} \leq -\beta \|\sigma\| \quad (10)$$

where β is a positive scalar and σ is the sliding surface defined by (2). With this reaching condition the following theorem can be introduced.

Theorem 2. Consider system (1) with the assumption A1) and A2). If the LMI set (8) and (9) has a solution and the sliding surface S is solvable given by (7), the reachability condition (10) is satisfied by employing the control law given below:

$$u(t) = -SAx - \frac{1}{(1-\gamma_2)} \{[\gamma_1 \|S\| + \gamma_2 \|SA\| \|x\| + \beta\} f_{sw} \quad (11)$$

where S is given by (15), $f_{sw} = \frac{\sigma}{\|\sigma\|} = \text{sgn}(\sigma)$ and β is a designed positive scalar.

Proof See [12]. □

The discontinuous control term in (11), $f_{sw} = \text{sgn}(\sigma)$ is defined as

$$\text{sgn}(\sigma) = \begin{cases} +1 & \text{if } \sigma \geq 1 \\ -1 & \text{if } \sigma \leq -1 \end{cases} \quad (12)$$

This introduces high chattering effect which is undesirable in any dynamic system due to its infinite switching frequency. The natural attempt to overcome the chattering is to smooth out the signal with a continuous function that closely approximate the $\text{sgn}(\sigma)$ term especially around the neighbourhood of the sliding surface. The introduction of a small boundary layer near the surface is usually adapted which correspond to the substitution of $\text{sgn}(\sigma)$ with $\text{sat}(\cdot)$ function defined as follows:

$$f_{sw} = \text{sat}\left(\frac{\sigma}{\Pi}\right) = \begin{cases} \frac{\sigma}{\Pi} & \text{if } |\sigma| < \Pi \\ \text{sgn}(\sigma) & \text{otherwise} \end{cases} \quad (13)$$

where Π is the thickness of the boundary layer. It can be seen that when the system state is outside the boundary layer, the discontinuous $\text{sgn}(\sigma)$ is in place which ensuring the robustness property of the controller and the states remain attractive to the bound of the surface. When the system states enter the boundary layer, the continuous term takes place which guarantees smooth control signal. The boundary layer control which is manifested by $\text{sat}(\cdot)$ is shown in Fig. 1, where U_{fsw} is the magnitude of the controller (11) that contains the f_{sw} term.

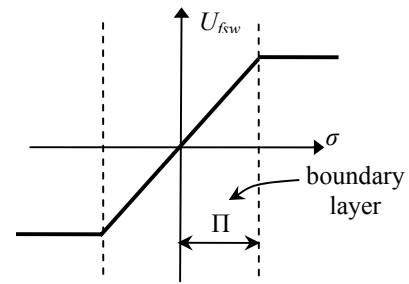


Fig. 1 Boundary layer for sliding mode

It is obvious that as the boundary layer thickness is approaching zero, the $\text{sgn}(\sigma)$ (12) is restored and likewise, as the boundary layer is increased the control signal becomes smoother but it introduces steady-state error that is proportional to the boundary thickness [8].

To overcome this limitation, in this work an exponentially-decaying boundary layer is proposed such that as time continuous the boundary thickness is decreasing and approaching zero which thus guarantees asymptotic stability of the system. This is proposed in the following lemma.

Lemma 2. The system (1) under the controller (11) will achieve asymptotic stability with the switching term is given below:

$$f_{sw} = \frac{\sigma}{|\sigma| + \varepsilon e^{-\pi}} \quad (14)$$

where $\varepsilon > 0$ and $\pi > 0$.

Proof The proof is quite similar in line with the proof shown in [4][7] and is intentionally omitted. □

From this lemma, the ε and π are designed parameter in which ε determines the initial thickness of the layer while π gives the rate the exponential converges to zero. If $\pi = 0$, (14) becomes a fix boundary layer term that is very similar to the $\text{sat}(\cdot)$, except that the term now is continuous either inside or outside the boundary layer, which implies that although the a chattering-free continuous signal is produced, the robustness during the reaching phase is no longer guarantee, especially if with bigger ε value.

The main advantage of this method, however, is its simplification of its term while the stability of the system can still be assured when $\pi > 0$. This is very much favoured for many practical applications that require rather accurate tracking capability. In this work, this chattering elimination method is applied to an AMB system as shown in the next section.

III. ACTIVE MAGNETIC BEARING SYSTEM MODEL

Based on the AMB model for turbo molecular pump [6], the system dynamical equations are restated such that the AMB system is transformed into a class of the system (1) that is suitable for the application of the proposed controller. Referring to Fig. 2, the dynamical equation for rotor-magnetic

bearing, net rotor force and permanent magnet force equations are given by (15), (16) and (17), respectively.

$$\left. \begin{aligned} m\ddot{x}_g &= f_{x_u} + f_{x_b} \\ J_r\ddot{\beta} &= -J_a\omega_z\dot{\alpha} + L_u f_{x_u} - L_b f_{x_b} \\ m\ddot{y}_g &= f_{y_u} + f_{y_b} \\ J_r\ddot{\alpha} &= J_a\omega_z\dot{\beta} - L_u f_{y_u} + L_b f_{y_b} \end{aligned} \right\} \quad (15)$$

$$\left. \begin{aligned} f_{x_u} &= 2K_{du}x_g + 2L_uK_{du}\beta + 2K_{iu}I_x \\ f_{y_u} &= 2K_{du}y_g - 2L_uK_{du}\alpha + 2K_{iu}I_y \end{aligned} \right\} \quad (16)$$

$$\left. \begin{aligned} f_{x_b} &= -2C_b\dot{x}_g + 2C_bL_b\dot{\beta} - 2K_bx_g + 2K_bL_b\beta \\ f_{y_b} &= -2C_b\dot{y}_g - 2C_bL_b\dot{\alpha} - 2K_by_g - 2K_bL_b\alpha \end{aligned} \right\} \quad (17)$$

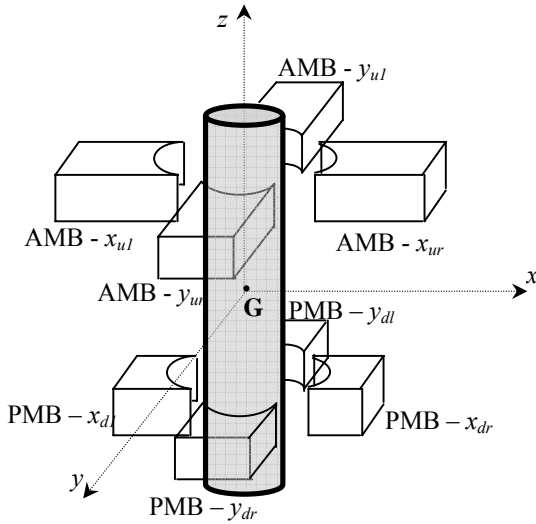


Fig. 2 Vertical Active Magnetic Bearing System

The detail of the variables and parameters are shown in Table II in the appendix. It can be noticed that the gyroscopic effect in (15) is represented by the terms $-J_a\omega_z\dot{\alpha}$ and $J_a\omega_z\dot{\beta}$, causes the coupling between the axes of motions to be proportional to the speed of the rotor, ω . Equations (15), (16) and (17) can be integrated to produce the AMB model in the following form:

$$\dot{X}(t) = A(\omega)X(t) + BU(t) \quad (18)$$

where $X = [x_g, \beta, y_g, \alpha, \dot{x}_g, \dot{\beta}, \dot{y}_g, \dot{\alpha}]^T$ are the states of the system, $A(\omega) \in \mathfrak{R}^{8 \times 8}$ is the system matrix, $B \in \mathfrak{R}^{8 \times 2}$ is the input matrix, $U(t) = [I_x, I_y]^T$ the input currents. The nonzero elements of the matrices are also shown in the appendix. Based on [12][13], the AMB system model can be treated as uncertain system and the result derived in Section II can be used to control the model. Thus, with the range of rotor rotational speed to be as follows:

$$0 \text{ rpm} \leq \omega \leq 20,000 \text{ rpm} \quad (19)$$

the uncertain AMB model can be derived as

$$\dot{X}(t) = [A + \Delta A(\omega)]X(t) + [B + \Delta B(K_{ix})]U(t) \quad (20)$$

where $\Delta A(\omega)$ and $\Delta B(K_{ix})$ represent the uncertainties due to gyroscopic effect and K_{ix} parameter variation, respectively. The values of these uncertainties can be easily calculated based on the deterministic approach proposed in [13].

IV. SIMULATION ON AMB SYSTEM AND DISCUSSION

The simulation work is performed by using MATLAB[®] and Simulink[®]. The initial values of the states are set at $[x_g, \beta, y_g, \alpha, \dot{x}_g, \dot{\beta}, \dot{y}_g, \dot{\alpha}]^T = [2 \times 10^{-4} \text{ m}, -1.5 \times 10^{-5} \text{ rad}, 1 \times 10^{-4} \text{ m}, 0, 0, 0, 0]^T$. All the three controller structures are simulated and the performances of each controller are also discussed. For the controller with sat (\cdot) function (13), the boundary layer is varied to see to reflect how the performance changes with the boundary layer variation. In addition, for the exponentially-decaying boundary layer (14), a few combinations of parameters ε and π are also chosen to similarly see performance of the controller.

In Fig. 3, the X and Y trajectories of the system under the switching term (12) are illustrated in which it is obviously shown that the system can achieve asymptotic stability and this is verified by the theoretical result.

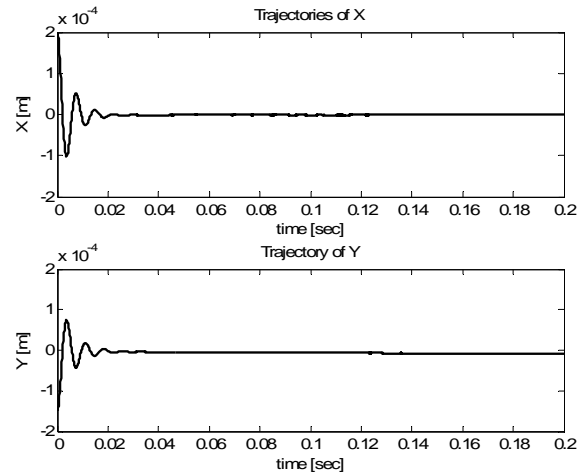


Fig. 3 X (top) and Y (bottom) trajectories for $f_{sw} = \text{sgn}(\sigma)$

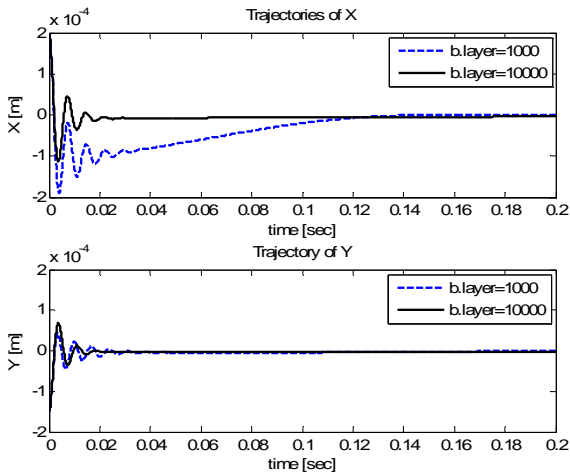


Fig. 4 X (top) and Y (bottom) trajectories for $f_{sw} = \text{sat} \left(\frac{\sigma}{\Pi} \right)$

When the continuous term (13) is used, the performance of the system experiences some variation under different boundary layer thickness. In Fig. 4, when two boundary layer are chosen, which are $\Pi = 1000$, and $\Pi = 10000$, the trajectories of both X and Y states are approaching zero, however, with $\Pi = 1000$, the response time is slower compared to $\Pi = 10000$. Similar to (13), the continuous function (14) is used with a few combinations of design parameters, ε and π are selected. In Fig. 5, the system response for two combination of the parameters are shown, which are called exponential boundary type i - thick boundary and type ii - thin boundary. The actual values for these parameters can be found in Table 1. In Fig. 5, the response of the system with thick boundary layer (type i) is slower compared to the response with the thin boundary layer (type ii) which is consistent with the theoretical result. This is due to the fact that the thicker the boundary layer is, the less robust the system becomes. Although all the states X and Y trajectories are approaching zero or around the close neighbourhood of acceptable values for all type of controllers when proper selection of boundary layer thickness are chosen, the control

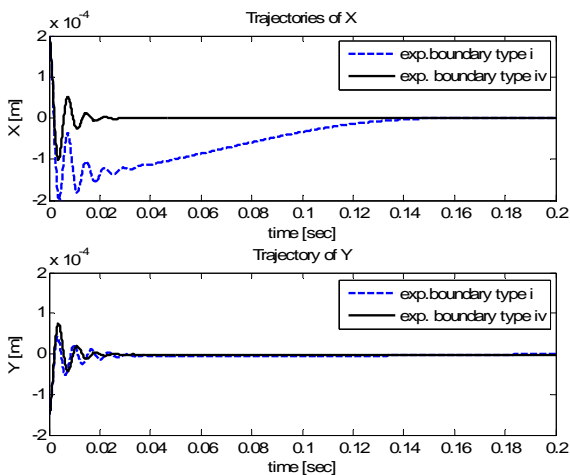


Fig. 5 X (top) and Y (bottom) trajectories for $f_{sw} = \frac{\sigma}{|\sigma| + e^{-\pi}}$

effort needed to attain the response varies quite noticeably. As shown in Fig. 6, the input currents I_x and I_y under $\text{sgn}(\sigma)$ term have a very high chattering effect. For the system under the controller term (13), the current input I_x under both boundary layer thicknesses is shown in Fig. 7 where no chattering effect presents at all. Also, for controller term (14), the I_x response is shown in Fig. 8 in which both type i and type ii show no chattering. To quantify the effectiveness of each controller in removing the chatter, the total energy consumed and maximum peak value of the control input are measured and the values are shown in Table I. From the measured values, it is evident the continuous type control term, either (13) or (14), the have smaller total consumed energy compared to (12) due to high chattering in the scheme. Also, it is worth noted that the total energy consumed also related to the thickness of the boundary layer. Generally, the energy is less for higher thickness layer which reflect the smoothness of the signal, however, if the thickness is chosen too large, the energy consumption is higher due to the fact that large boundary

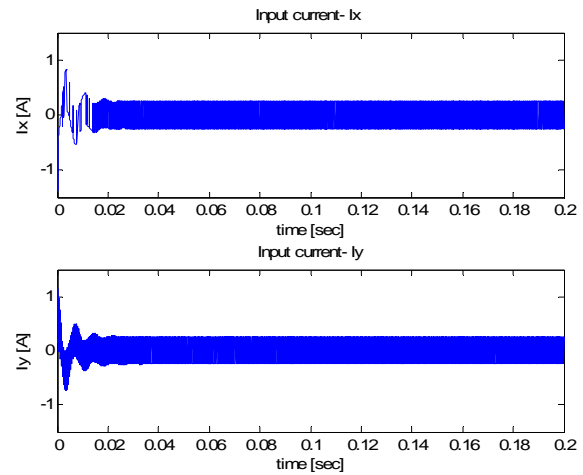


Fig. 6 I_x (top) and I_y (bottom) for $f_{sw} = \text{sgn}(\sigma)$

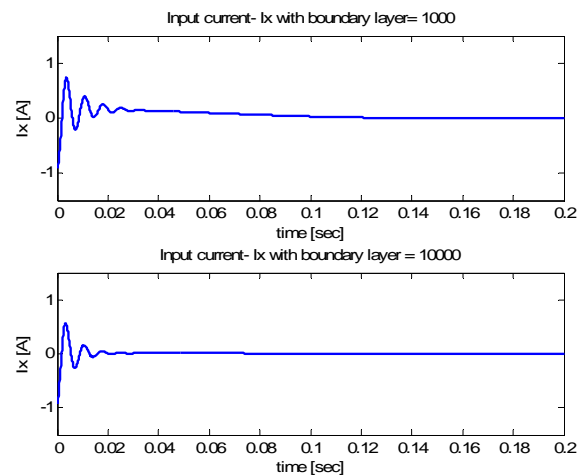


Fig. 7 I_x for $f_{sw} = \text{sat} \left(\frac{\sigma}{\Pi} \right)$ with $\Pi = 1000$ (top) and $\Pi = 10000$ (bottom)

layer reduces the ability of the controller to reach the sliding regime which results the controller to pump extra energy. For the maximum peak value of the control signal, the magnitudes are relatively the same but noticeably higher for controller (12). This value is important to measure since it indicates the maximum value that the power supply should be able to supply to the system. If there is limited power source available, it will force the input to saturate and anti-windup algorithm is needed to ensure system stability.

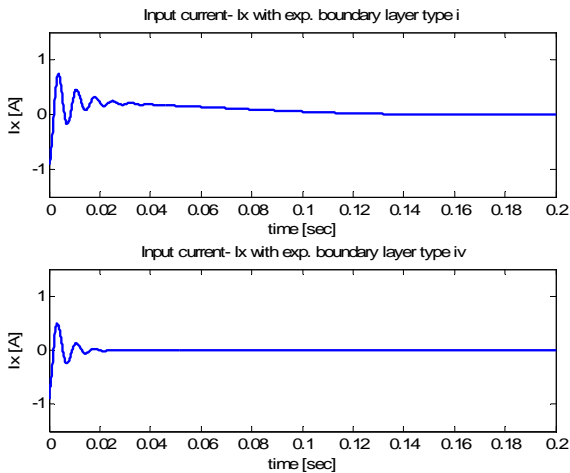


Fig.8. I_x for $f_{sw} = \frac{\sigma}{|\sigma| + \varepsilon e^{-\pi}}$ with $\varepsilon = 10^{-3}$ (top) and $\varepsilon = 10^{-5}$ (bottom)

TABLE I
 PERFORMANCE OF SLIDING MODE CONTROLLERS WITH VARIOUS SWITCHING TERMS

Controllers with switching terms: $u(t) = u_{eq} + u_{non} \times f_{sw}$	Total consumed energy: $\int^r \mathbf{u} \mathbf{u}$	Maximum Control Magnitude: $\ \mathbf{u}\ _{\infty}$
$f_{sw} = \text{sgn}(\sigma)$	0.1018	1.387
$f_{sw} = \text{sat}\left(\frac{\sigma}{\Pi}\right)$	i. $\Pi = 1000$	0.003781
	ii. $\Pi = 10000$	0.001777
$f_{sw} = \frac{\sigma}{ \sigma + \varepsilon_0 e^{-\alpha t}}$	i. $\pi = 0.1$ $\varepsilon_0 = 0.001$,	0.004934
	ii. $\pi = 0.01$ $\varepsilon_0 = 0.001$,	0.004934
	iii. $\pi = 0.1$ $\varepsilon_0 = 0.0001$,	0.001936
	iv. $\pi = 0.1$ $\varepsilon_0 = 0.00001$,	0.001534

V. CONCLUSION

In this work, new chattering-free SMC controller is proposed for stabilization of a five-DOF AMB system. Three type of switching function are discussed and a new

exponentially-decaying function is introduced to replace the discontinuous $\text{sgn}(\sigma)$ to alleviate chattering while retaining asymptotic stability. Various simulation works are performed and the total consumed energy and maximum peak value of the input are measured to demonstrate the performance of the method.

APPENDIX

The nonzero elements of matrix $A(\omega, t)$, B where i and j indicate the i -th and j -th entry of each element.

$$\begin{aligned}
 a_{51} &= \frac{2(K_{du} - K_b)}{m}, a_{52} = \frac{2(L_u K_{du} - L_b K_b)}{m}, a_{55} = \frac{2C_b}{m}, a_{56} = \frac{2C_b L_b}{m}, \\
 a_{61} &= \frac{2(L_u K_{du} - L_b K_b)}{J_r}, a_{62} = \frac{2(L_u^2 K_{du} - L_b^2 K_b)}{J_r}, a_{65} = \frac{2L_b C_b}{J_r}, \\
 a_{66} &= \frac{2L_b^2 C_b}{J_r}, a_{68} = \frac{J_a}{J_r} p, a_{73} = \frac{2(K_{du} - K_b)}{m}, a_{74} = \frac{2(L_u K_{du} + L_b K_b)}{m}, \\
 a_{77} &= \frac{2C_b}{m}, a_{78} = \frac{2C_b L_b}{m}, a_{83} = \frac{2(L_u K_{du} + L_b K_b)}{J_r}, a_{84} = \frac{2(L_u^2 K_{du} - L_b^2 K_b)}{J_r}, \\
 a_{86} &= \frac{J_a}{J_r} p, a_{87} = \frac{2L_b C_b}{J_r}, a_{88} = \frac{2L_b^2 C_b}{J_r}, b_{51} = b_{72} = \frac{2K_{iu}}{m}, b_{61} = -b_{82} = \frac{2L_u K_{iu}}{J_r}.
 \end{aligned}$$

TABLE II
 PARAMETER FOR AMB SYSTEM [8]

Symbol	Parameter	Value	Unit
m	Mass of Rotor	1.595	kg
J_a	Moment of Inertia about rotational axis	1.61×10^{-3}	kg.m^2
J_r	Moment of Inertia about radial axis	3.83×10^{-3}	kg.m^2
L_u	Distance of upper AMB to G	0.0128	m
L_b	Distance of lower PMB to G	0.0843	m
K_{iu}	Linearized force/current factor	200	N/A
K_{du}	Linearized force/displ. factor	2.8×10^5	N/m
K_b	Stiffness coefficient of PMB	1.0×10^5	N/m
C_b	Damping coefficient of PMB	48	kg/s
m_{un}	Static imbalance	0.6×10^{-3}	m
l	Distance of unbalance mass from G	0.02	m
ω	Rotor rotational speed	0 – 1047 (0 – 10000)	rad/sec (rpm)

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REFERENCES

- [1] J. Y. Hung, W. B. Gao, and J. C. Hung, "Variable Structure control: A Survey," *IEEE Trans. Industrial. Electronics*, vol. 40, no. 1, pp. 2-22., 1993.
- [2] S. Spurgeon, and C. Edwards, *Sliding Mode Control: Theory and Applications*. London: Taylor and Francis, 1998.
- [3] M. N. Ahmad, J. H. S. Osman, and M. R. A. Ghani, "Proportional-Integral Sliding Mode Tracking Controller with Application to a Robot Manipulator," in *7th Conf. on Contr., Auto., Rob and Sys., ICARCV*, Singapore, 2003, pp. 863-868.
- [4] Y. W. Liang, S. D. Xu and T. C. Chu, "Robust Control of the Robot Manipulator via an Improved Sliding Mode Scheme," *In. Proc. of IEEE In. Conf. on . Mechatronics and Auto.*, Harbin China, Aug. 5-8, 2007, pp. 1593-1598.
- [5] Y. M. Sam, and J. H. S. Osman, "Modeling and Control of Active Suspension System Using Proportional Integral Sliding Mode Approach," *Asian Jour. Contr.*, vol. 7, no. 2, pp. 91-98, 2005.
- [6] S. Sivrioglu, and K. Nonami, "Sliding Mode Control With Time-Varying Hyperplane for AMB Systems," *IEEE/ASME Trans. on Mechatronics*, vol. 3, no. 1, pp. 51-59, 1998.
- [7] M. S. Chen, Y. R. Hwang and M. Tomizuka, "A State-Dependant Boundary Layer Design for Sliding mode Control," *IEEE Trans. on Auto. Contrl.*, vol. 47, no. 10, pp. 1677-1681, 2002.
- [8] D. Q. Zhang, and S. K. Panda, "Chattering-free and fast-response sliding mode controller," *IEE Proc. -Contr. Theory Appl.*, vol. 146, no. 2, pp. 171-177, March 1999.
- [9] H. Lee and V. Utkin, "Chattering suppression methods in sliding mode control system," *Annual Review in Control*, vol. 31, pp. 179-188, 2007.
- [10] C. W. Tao, M. -L. Chan and T. -T. Lee, "Adaptive fuzzy sliding mode controller for linear systems with mismatched time-varying uncertainties," *IEEE Trans. Sys., Man, and Cybernatics -Part B*, vol. 33, no. 2, pp. 283-294, 2003.
- [11] E. Z. Taha, G.S. Happawana, and Y. Hurmuzlu, "Quantitative feedback theory (QFT) for chattering reduction and improved tracking in sliding mode control (SMC)," *ASME Journal of Dynamic Systems, Meas. And Contrl.*, vol. 125, no. 4, pp. 665-669, 2003.
- [12] A. R. Husain, M. N. Ahmad and A. H. M. Yatim, " Asymptotic Stabilization of an Active Magnetic Bearing System using LMI-based Sliding Mode Control," *Int. Jour. of Mechanical Sys. Sci. and Eng.*, vol 2, no. 1, pp. 9-16, 2008.
- [13] J. H. S. Osman, and P.D. Roberts, "Two Level Control Strategy for Robot Manipulators," *Int. Jour. of Control*, vol. 61, no. 6, pp. 1201 – 1222, 1995.