World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:6, No:4, 2012

New exact three-wave solutions for the (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov system

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***Abstract*—New exact three-wave solutions including periodic two-solitary solutions and doubly periodic solitary solutions for the (2+1)–dimensional asymmetric Nizhnik–Novikov–Veselov (ANNV) system are obtained using Hirota's bilinear form and generalized three-wave type of ansatz approach. It is shown that the generalized three-wave method, with the help of symbolic computation, provides an effective and powerful mathematical tool for solving high dimensional nonlinear evolution equations in mathematical physics.

***Keywords*—Soliton Solution; Hirota Bilinear Method; ANNV System.

I. INTRODUCTION

IN last decades, increasing attention has been paid to the study of the solution theory in many natural sciences particularly in almost all branches of physics like the fluid dynamics, plasma physics, field theory, nonlinear optics and condensed matter physics.

Integrable models play a prominent role in theoretical physics. The reason is not only the direct phenomenological interest of some of them, but also the fact that they often provide some deep insights into the mathematical structure of theory in which they arise. Up to now, the (1+1)-dimensional integrable models are well understood due to many systematic methods such as the inverse scattering transformation [9], the Darboux transformation, Hirota's bilinear method [3], [10], [11], [12], [13], [15], Bäcklund transformation method [18], Painlevé expansion method [8], tanh function method [16], homogenous balance method [19], variable separation approach [6] and the three-wave approach [2], [5]. However, studies of the (2+1)-dimensional cases are fewer in number and such systems are being actively investigated from different viewpoints.

In this paper, we consider the following (2+1)–dimensional asymmetric Nizhnik–Novikov–Veselov (ANNV) equation:

$$u_t + u_{xxx} + 3\left[u\left(\int u_x dy\right)\right]_x = 0. \qquad (1)$$

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Boiti et al. [4] was first derived Eq. (1) by using the weak Lax pair. It may be considered as a model for an incompressible fluid where u and v are the components of the (dimensionless) velocity [8]. The spectral transformation for this system has been investigated in [1], [4]. Moreover, Hu et al. used the Darboux transformations to find the variable separation solutions of this equation in [14], Wazwaz used the Hirota bilinear method to derive N-soliton solutions of this completely integrable equation [17], Dai and Wang used the exp-function method for a new general variable separation solutions [7] and the Bäcklund transformation for the ANNV equation was derived by using the extended homogeneous balance method by Zhang in [20].

II. ANNV SYSTEM

The ANNV equation can be written as

$$\begin{cases} u_t + u_{xxx} + 3\,[uv]_x = 0 \\ u_x = v_y. \end{cases} \qquad (2)$$

To solve the ANNV system (2), we substitute the dependent variable transformation

$$u = 2\,(\ln f)_{xy}\,, \qquad v = 2\,(\ln f)_{xx}\,, \qquad (3)$$

where $f(x, y, t)$ is an unknown real function. Substituting (3) into system (2), we can get

$$f\left(\frac{\partial^2 f}{\partial y \partial t} + \frac{\partial^4 f}{\partial y \partial x^3}\right) - \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial t} + \frac{\partial^3 f}{\partial x^3}\right) + 3\left(\frac{\partial^2 f}{\partial y \partial x}\frac{\partial^2 f}{\partial x^2} - \frac{\partial f}{\partial x}\frac{\partial^3 f}{\partial y \partial x^2}\right) = 0$$

or equivalently in bilinear form

$$D_y\left(D_t + D_x^3\right) f \cdot f = 0, \qquad (4)$$

where the Hirota D-operator is defined by [10]

$$D_x^m D_t^n f \cdot g = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n f(x,t) g(x',t')_{|x'=x,t'=t}.$$

We propose a novel test function of extended three-soliton method

$$f(x, y, t) = e^{\xi_1} + d_1 \cos(\xi_2) + d_2 \cosh(\xi_3) + d_3 e^{-\xi_1} \qquad (5)$$

where

$$\xi_i = a_i x + b_i y + c_i t, \qquad i = 1, 2, 3 \qquad (6)$$

and a_i, b_i, c_i, d_i are some constants to be determined later.

Substituting (5) into Eq. (4), and equating all the coefficients of $\sin(a_2x+b_2y+c_2t), \cos(a_2x+b_2y+c_2t), \sinh(a_3x+b_3y+$

$c_3t), \cosh(a_3x+b_3y+c_3t)$ and any product of them to zero, we get the set of algebraic equations for a_i, b_i, c_i, d_i $(i = 1, 2, 3)$

$$\begin{gathered}
c_2b_1 - b_1a_2^3 + b_2c_1 + 3b_1a_2a_1^2 - 3b_2a_1a_2^2 + b_2a_1^3 = 0,\\
c_1b_1 - c_2b_2 - 3b_2a_2a_1^2 + b_2a_2^3 - 3b_1a_1a_2^2 + b_1a_1^3 = 0,\\
c_3b_2 + c_2b_3 - 3b_2a_3a_2^2 - b_3a_2^3 + 3b_3a_2a_3^2 + b_2a_3^3 = 0,\\
c_3b_3 - c_2b_2 - 3b_3a_3a_2^2 + b_2a_2^3 - 3b_2a_2a_3^2 + b_3a_3^3 = 0,\\
c_1b_3 + c_3b_1 + 3b_1a_3a_1^2 + b_3a_1^3 + 3b_3a_1a_3^2 + b_1a_3^3 = 0,\\
c_1b_1 + c_3b_3 + 3b_3a_3a_1^2 + b_1a_1^3 + 3b_1a_1a_3^2 + b_3a_3^3 = 0,\\
4c_1b_1d_3 - c_2b_2d_1^2 + 4b_2d_1^2a_2^3 + c_3b_3d_2^2+\\
16b_1d_3a_1^3 + 4b_3a_3^3d_2^2 = 0
\end{gathered}.$$

Solving the resulting equations simultaneously with the help of symbolic computation, we obtain the following:

Case 1:

$$a_1 = a_2 = c_1 = c_2 = b_3 = 0, \qquad c_3 = -a_3^3, \tag{7}$$

where a_3, b_1, b_2, c_3, d_i $(i = 1, 2, 3)$ are some free constants.

Substituting (5) into Eq. (3) with (7) and (6), we obtain the periodic two-solitary solution as follows

$$u(x,y,t) = 2a_3d_2\sinh\left(a_3x - a_3^3t\right) \times \frac{\left(d_1b_2\sin(b_2y) - 2b_1\sqrt{d_3}\sinh(b_1y-\theta)\right)}{\left(2\sqrt{d_3}\cosh(b_1y-\theta) + d_2\cosh\left(a_3x-a_3^3t\right) + d_1\cos(b_2y)\right)^2} \tag{8}$$

$$v(x,y,t) = 2a_3^2d_2 \times \frac{d_2 + d_1\cos(b_2y)\cosh\left(a_3x-a_3^3t\right) + 2\sqrt{d_3}\cosh(b_1y-\theta)\cosh\left(a_3x-a_3^3t\right)}{\left(2\sqrt{d_3}\cosh(b_1y-\theta) + d_2\cosh\left(a_3x-a_3^3t\right) + d_1\cos(b_2y)\right)^2} \tag{9}$$

where $\theta = \frac{1}{2}\ln d_3$ and $d_3 > 0$. Moreover, when $\theta = \frac{1}{2}\ln(-d_3)$ and $d_3 < 0$

$$u(x,y,t) = 2a_3d_2\sinh\left(a_3x - a_3^3t\right) \times \frac{\left(d_1b_2\sin(b_2y) - 2b_1\sqrt{-d_3}\cosh(b_1y-\theta)\right)}{\left(2\sqrt{-d_3}\sinh(b_1y-\theta) + d_2\cosh\left(a_3x-a_3^3t\right) + d_1\cos(b_2y)\right)^2} \tag{10}$$

$$v(x,y,t) = 2a_3^2d_2 \times \frac{\left(d_2 + d_1\cos(b_2y)\cosh\left(a_3x-a_3^3t\right) + 2\sqrt{-d_3}\sinh(b_1y-\theta)\cosh\left(a_3x-a_3^3t\right)\right)}{\left(2\sqrt{-d_3}\sinh(b_1y-\theta) + d_2\cosh\left(a_3x-a_3^3t\right) + d_1\cos(b_2y)\right)^2} \tag{11}$$

Case 2:

$$b_1 = b_2 = c_3 = a_3 = 0, \qquad c_1 = -a_1^3, \qquad c_2 = -a_2^3, \tag{12}$$

where a_1, a_2, b_3, d_i $(i = 1, 2, 3)$ are some free constants.

Substituting (5) into Eq. (3) with (12) and (6), we obtain the periodic two-solitary solution

$$u(x,y,t) = 2b_3d_2\sinh(b_3y) \times \frac{\left(a_2d_1\sin\left(a_2x+a_2^3t\right) - 2a_1\sqrt{d_3}\sinh\left(a_1x-a_1^3t-\theta\right)\right)}{\left(2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cosh(b_3y)\right)^2} \tag{13}$$

$$\begin{aligned}
v(x,y,t) = &-\frac{2\left(d_1^2a_2^2 - 4d_3a_1^2\right) + 4d_1\sqrt{d_3}\left(a_2^2-a_1^2\right)\cosh\left(a_1x-a_1^3t-\theta\right)\cos\left(a_2x+a_2^3t\right)}{\left(2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cosh(b_3y)\right)^2} +\\
&\frac{2d_2\left(2a_1^2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) - d_1a_2^2\cos\left(a_2x+a_2^3t\right)\right)\cosh(b_3y)}{\left(2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cosh(b_3y)\right)^2} +\\
&\frac{8a_1a_2d_1\sqrt{d_3}\sinh\left(a_1x-a_1^3t-\theta\right)\sin\left(a_2x+a_2^3t\right)}{\left(2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cosh(b_3y)\right)^2}
\end{aligned} \tag{14}$$

where $\theta = \frac{1}{2}\ln d_3$ and $d_3 > 0$. In the case when $\theta = \frac{1}{2}\ln(-d_3)$ and $d_3 < 0$, we get that

$$u(x,y,t) = 2b_3d_2\sinh(b_3y) \times \frac{\left(a_2d_1\sin\left(a_2x+a_2^3t\right) - 2a_1\sqrt{-d_3}\cosh\left(a_1x-a_1^3t-\theta\right)\right)}{\left(2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cosh(b_3y)\right)^2} \tag{15}$$

$$\begin{aligned}
v(x,y,t) = &-\frac{2\left(d_1^2a_2^2 - 4d_3a_1^2\right) + 4d_1\sqrt{-d_3}\left(a_2^2-a_1^2\right)\sinh\left(a_1x-a_1^3t-\theta\right)\cos\left(a_2x+a_2^3t\right)}{\left(2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cosh(b_3y)\right)^2} +\\
&\frac{2d_2\left(2a_1^2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) - d_1a_2^2\cos\left(a_2x+a_2^3t\right)\right)\cosh(b_3y)}{\left(2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cosh(b_3y)\right)^2} +\\
&\frac{+8a_1a_2d_1\sqrt{-d_3}\cosh\left(a_1x-a_1^3t-\theta\right)\sin\left(a_2x+a_2^3t\right)}{\left(2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cosh(b_3y)\right)^2}
\end{aligned} \tag{16}$$

Case 3: Using the transformation $b_3 = iB_3$, where B_3 is real, in Eqs. (13)-(14) and Eqs. (15)-(16), we get the doubly periodic solitary solutions

$$u(x,y,t) = 2B_3d_2\sin(B_3y) \times \frac{\left(2a_1\sqrt{d_3}\sinh\left(a_1x-a_1^3t-\theta\right) - a_2d_1\sin\left(a_2x+a_2^3t\right)\right)}{\left(2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cos(B_3y)\right)^2} \tag{17}$$

$$\begin{aligned}
v(x,y,t) = &-\frac{2\left(d_1^2a_2^2 - 4d_3a_1^2\right) + 4d_1\sqrt{d_3}\left(a_2^2-a_1^2\right)\cosh\left(a_1x-a_1^3t-\theta\right)\cos\left(a_2x+a_2^3t\right)}{\left(2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cos(B_3y)\right)^2} +\\
&\frac{2d_2\left(2a_1^2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) - d_1a_2^2\cos\left(a_2x+a_2^3t\right)\right)\cos(B_3y)}{\left(2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cos(B_3y)\right)^2} +\\
&\frac{8a_1a_2d_1\sqrt{d_3}\sinh\left(a_1x-a_1^3t-\theta\right)\sin\left(a_2x+a_2^3t\right)}{\left(2\sqrt{d_3}\cosh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cos(B_3y)\right)^2}
\end{aligned} \tag{18}$$

when $d_3 > 0$ and when $d_3 < 0$

$$u(x,y,t) = 2B_3d_2\sin(B_3y) \times \frac{\left(2a_1\sqrt{-d_3}\cosh\left(a_1x-a_1^3t-\theta\right) - a_2d_1\sin\left(a_2x+a_2^3t\right)\right)}{\left(2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cos(B_3y)\right)^2} \tag{19}$$

$$\begin{aligned}
v(x,y,t) = &-\frac{2\left(d_1^2a_2^2 - 4d_3a_1^2\right) + 4d_1\sqrt{-d_3}\left(a_2^2-a_1^2\right)\sinh\left(a_1x-a_1^3t-\theta\right)\cos\left(a_2x+a_2^3t\right)}{\left(2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cos(B_3y)\right)^2} +\\
&\frac{2d_2\left(2a_1^2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) - d_1a_2^2\cos\left(a_2x+a_2^3t\right)\right)\cos(B_3y)}{\left(2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cos(B_3y)\right)^2} +\\
&\frac{+8a_1a_2d_1\sqrt{-d_3}\cosh\left(a_1x-a_1^3t-\theta\right)\sin\left(a_2x+a_2^3t\right)}{\left(2\sqrt{-d_3}\sinh\left(a_1x-a_1^3t-\theta\right) + d_1\cos\left(a_2x+a_2^3t\right) + d_2\cos(B_3y)\right)^2}
\end{aligned} \tag{20}$$

To our knowledge, these solutions (8)–(20) have not been reported in other literatures. All the solutions of the (2+1)-dimensional ANNV system obtained in this paper include three independent variables. In these solutions the arbitrary constants imply that (1) has abundant local physical structures.

III. Conclusion

By choosing different ansatz of extended three-soliton type, we obtain a new type of three-wave solution, periodic type of three-wave solutions, including periodic two-solitary-wave, doubly periodic solitary-wave of (2+1)-dimensional ANNV system. Actually, our present short paper investigates different mechanical features of these wave solutions. It is worth noting that this is merely a beginning work, and we can obtain richer exact solutions by a more general ansatz of extended three-soliton type.

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