

Optimal Route Policy in Air Traffic Control with Competing Airlines

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Abstract—This work proposes a novel market-based air traffic flow control model considering competitive airlines in air traffic network. In the flow model, an agent based framework for resources (link/time pair) pricing is described. Resource agent and auctioneer for groups of resources are also introduced to simulate the flow management in Air Traffic Control (ATC). Secondly, the distributed group pricing algorithm is introduced, which efficiently reflect the competitive nature of the airline industry. Resources in the system are grouped according to the degree of interaction, and each auctioneer adjusts the price of one group of resources respectively until the excess demand of resources becomes zero when the demand and supply of resources of the system changes. Numerical simulation results show the feasibility of solving the air traffic flow control problem using market mechanism and pricing algorithms on the air traffic network.

Keywords—Air traffic control, Nonlinear programming, Market mechanism, Route policy.

I. INTRODUCTION

IN the research of ATC, much work has been reported recently for the air traffic management. Flow scheduling is proved quite a difficult task for air traffic control based on sector occupancy, which is presented in [1]. A continuous model based on airway links is proposed in [2]. Partial differential equation is also applied in ATC [3-4]. From these works, how to realize flow allocation properly is proved the main objection in air traffic network. From the point of resource allocation, flow allocation in proper is also equal to finding optimal route policies for all flights by keeping their schedule. Airlines can make the flight plan properly with the route policy. Rerouting strategies are employed to solve the problem. Hu [5] considers the restriction of multiple airport and airspace capacity as a multiple unit ground holding problem. Nilim [6] has applied Markov Decision Process and stochastic dynamic programming to multi-flight rerouting. There are some drawbacks among works outlined:

(1) A key aspect that has not been addressed by these works is to incorporate airline preferences in air traffic flow optimization methodologies. In the real world, it is the airlines that are in the best position to accurately determine the relative costs of delay for conflicting on constrained routes. They will compete with other airlines to minimize their travel cost. As a result, airline preferences should be concerned in designing optimal routing policies.

(2) In these works, demand and capacities are treated as deterministic. However, the highly stochastic nature of weather

and capacity constraints, as major causes of delay, are poorly addressed by a deterministic framework. Therefore, research on dynamic link travel time is very necessary in ATC.

(3) The research airspace in these works is only confined to single airport or airspace, while flights delays will transfer to other hub airport or airspace nearby. For the reasons, we should extend our research scope to an air traffic network.

In this paper, we detail a method that accounts for bidding in competition for link/time resources. Moreover, it derives dynamic route policies based on market mechanism. On basis of the method, a novel air traffic control model is proposed to solve the problem of finding optimal route policy for each flight in ATC.

This paper is organized as follows. The following section is the problem formulation, then, section 3 presents optimal route policy in ATC. In section 4, numerical examples on small-size network are carried out to demonstrate the applications of the proposed model and algorithms. The final section then concludes the paper.

II. PROBLEM FORMULATION

A. Network initialization

The air traffic network is defined as a directed graph $G(N, A)$, with N being the set of nodes and A being the set of links, respectively. It treats airports or waypoints as nodes and airways as links. The link capacity fluctuates with the weather.

The network is discretized with time steps indexed by $t \in T$. Each airline, $i \in I$, can plan flights from origin nodes $n_o \in N$ to destination nodes $n_d \in N$. $\{o, d\} \in OD$ is origin-destination pair. Each flight can be started at any point in time T , and it means the ground holding strategy is allowed in the ATC. Each link-time pair $j = (l, t) \in J$, is defined as a resource. Resources (R^J) are mainly constrained by link capacity. The link capacity satisfies the vector $C^e \in R^J$. The path, $p \in P$ is defined as the resources consumed by the flights at the special time. Each resource is assigned to the path at the time of entry onto a link. The path-resource matrix is $A \in (p, j)^{P \times J}$.

The airport departure and arrival flow can be defined by summing all OD pairs that arrive at or depart from a specific airport. Let $A^o \in \{0, 1\}^{P \times J}$, $A^d \in \{0, 1\}^{P \times J}$ be the corresponding matrices. The airport capacities are constrained by

$$\begin{cases} A^o f^o \leq C^o \\ A^d f^d \leq C^d \\ f^o \leq S f \end{cases} \quad (1)$$

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where $C^o \in R^{T \times n_o}$, $C^d \in R^{T \times n_d}$, are the constraints on safe operation, and f is the allocated flow and S is the scheduled flow in requirement. From equation (1), the initial flow allocation of the network is deduced as

$$\begin{aligned} \min & \|S^d - F^d\|^2 \\ \text{s.t.} & Af \leq C \\ & f \geq 0 \end{aligned} \quad (2)$$

Here S^d is the cumulative scheduled arrival flow, F^d is the arrival flow allocated in practice, A is the set of A^o , A^d , A^e , and C is the set of C^o , C^d , C^e . Through the computation, the initial global information about the network is formed, which can be used in preparation for subsequent research on optimal route policy.

B. Market-based resource allocation mechanism

In view of the initial global information of the network, we can build a market-based resource allocation mechanism, which can efficiently match resource supply and demand. According to the definition of the network, it is easily concluded that the link/time is the resource in ATC, while the flight plans are defined as the demand in market theory. Therefore, the optimal flow scheduling is formulated as finding equilibrium between supply and demand in competing with the resources. Moreover, it is equal to finding optimal route policy under time-dependent environment. From the perspective of economics, the market mechanism ideally suits for the problem of resource allocation, and is also proved a distributed self decision mechanism in economic activities, namely, the changes between supply and demand reflects the pricing in the market [7]. The optimal resource allocation is achieved as the supply meets the demand .

The market-based resource allocation mechanism in ATC is divided into two steps.

- 1) Establish the equilibrium price of all resources through central pricing management.
- 2) Develop resource allocation and optimal route policy with resource pricing and allocation strategies applied into bidding local resource.

The structure of the market-based resource allocation mechanism is depicted in Fig 1.

In Fig 1, the bottom rectangle is defined the resource, that is link/time pair in air traffic network. Upon on it, the rectangle written with "Group1" means the group of the resources. In this paper, the group is defined as Resource Group (RG), namely,

$$X = \{X_i, i = 1, 2, 3, \dots, n\} \quad (3)$$

where $X_i = (l_i, t_i)$, $(l, t) \in J$, i is the number of links in air traffic network. The aim of the clustering method is to find a reasonable distribution of G , which is the cluster of all resources (link/time pair), $G = \{G_1, G_2, \dots, G_m\}$, $G_m = \{(l_i, t_i)\}$. According to the definition of cluster, the classification of the group can be decomposed into three parts

$$\begin{aligned} X &= \bigcup_{l=1}^m G_l \\ G_i &\neq \Phi, (i = 1, 2, 3, \dots, m) \\ G_i \cap G_j &(i, j = 1, 2, 3, \dots, m; i \neq j) \end{aligned} \quad (4)$$

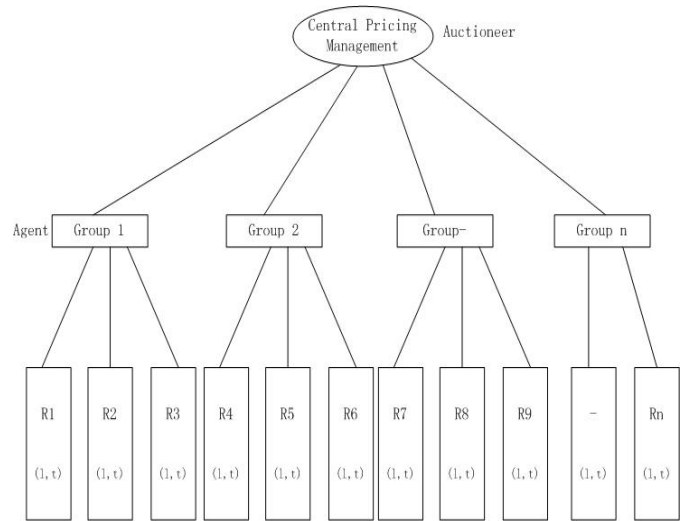


Fig. 1. Procedure of Market-based resource allocation mechanism

The subscript of variable G_i denotes the sequence of resources. Therefore, the object function of resource group is to minimize the total dispersion of clusters

$$D = \sum_{k=1}^m \sum_{X_i \in C_k} d(X_i, Z_k) \quad (5)$$

where Z_k is the k th clustering center, and $d(X_i, Z_k)$ is the distance from X_i to the clustering center (Z_k) associated with the samples. The objection of (5) is to compute Euclidean distance of X_i and Z_k , namely,

$$\begin{aligned} d(X_i, Z_k) &= \sum_{l=i}^k (X_l - \overline{X_{ik}}) \\ \overline{X_{ik}} &= \frac{1}{k-i+1} \sum_{l=i}^k X_l \end{aligned} \quad (6)$$

Therefore, equation (5-6) can efficiently depict distribution of resource. We can obtain the cluster G_i and clustering center Z_k by using K-means clustering algorithm[8].

It is obvious that the procedure of finding groups has done the same work as the preprocess in pursuing the optimal route policy. The problem's dimensions tends to be confined into a single group. Thus, the computation cost of finding the optimal route policy is obviously decreased.

III. OPTIMAL ROUTE POLICY BASED ON MARKET IN AIR TRAFFIC CONTROL

The assumption of airline cooperation is irrational due to the competitive nature of the airline industry. The market mechanism is a method of resource allocation based on distributed self-decision. That is, each participant makes decisions on its preference in accordance with market prices. Simultaneously, the variation of the prices reflect the dynamic of the demand and supply. According to the market mechanism, the central flow optimization can be formulated as a distributed market optimization. The procedure of the transformation consists of two parts: local optimizations performed by the airlines that

trade off the cost of deviating from the scheduled flow with the cost of purchasing network resources. The other part is, a central pricing mechanism that enforces constraint satisfaction by the airlines [9].

The central pricing is formed with Lagrange multipliers $\mu \in R^{nc}$ that are referred to as resource prices, and the central flow allocation problem is

$$P(\mu) = \sum_{i \in I} Fl_i(f_i) + \mu^T(Af - C) \quad (7)$$

here S^d is the cumulative scheduled arrival flow, f^d is the arrival flow allocated in practice. μ is price vector for resources, $Fl_i(f_i) = \|\max(0, S_i^d - f_i^d)\|^2$, $f_i \in R^K$, where K is set of OD/time pairs, and resource $(l, t) \in R$, f_i denotes allocation flow of specific path. A resource payment, $Pm_i \in R$, is defined for each airline,

$$Pm_i = \mu^T A(f_i - f_i^s) \quad (8)$$

where i denotes i th airline. f_i^s is the scheduled flow. The equation charges each airline for any additional resources consumed, and rewards the airline for any resources surplus.

According to Fig.1, the auctioneers in central pricing management publish equilibrium price. Given a current price vector, μ and payment rule Pm , the airline seeks to minimize its price-taking cost function during the procedure of local flow allocation optimization.

$$\begin{aligned} \min [Fl_i(f_i) + Pm_i] \\ \text{s.t. } f_i \geq 0 \end{aligned} \quad (9)$$

After finish the operation, the agent will sent flow request to the auctioneer. As a result, the distributed market-based flow allocation algorithm is developed as

TABLE I

DISTRIBUTED MARKET-BASED FLOW ALLOCATION ALGORITHM FOR ATC

Algorithm 1 Distributed market-based algorithm for air traffic

flow control

- 1: Set prices to zero, $\mu = 0$
- 2: **While** $f_i \geq \xi$ **and** $t < \max Num$ **do**
- 3: Central auctioneer publishes price, μ^t , and price variation tags, $f_i(\mu)$.
- 4: Each airline applies local flow allocation optimization with equation (9).
- 5: Airlines return flow requests, f_i^t .
- 6: Central auctioneer updates prices, μ^{t+1} .
- 7: **End while**

As far as auctioneer is concerned, they publish the equilibrium price μ . The procedure of price update is described as follows.

1) Evaluate the initial price μ_k^0 , which comes from the previous equilibrium price, where k denotes the k th resource group (RG).

2) Receive the flow requests $P(\mu_k)$ from M agents.

3) Solve the new resource price μ_k^* iteratively based on the flow request.

An approximation formula of $P(\mu_k)$ based on the Taylor series expansion on μ_k^0 is formulated as follows

$$P(\mu_k) \approx P(\mu_k^0) + P'(\mu_k^0)(\mu_k - \mu_k^0) = 0 \quad (10)$$

It is deduced that the expression of new price with (10), that is

$$\mu_k = \mu_k^0 - [P'(\mu_k^0)]^{-1}P(\mu_k^0) \quad (11)$$

Here $P'(\mu_k^0)$ is the first order derivative of vector function $P(\mu_k)$. According to (11), we obtain the new price μ_k , then, substitute the temporary price μ_k^0 with μ_k , repeat the procedure until it meets the converge criterion, $P(\mu_k) < \xi$. The final new price μ_k^* is computed at that time.

4) Compute scope of prices variation as follows,

$$V = \|\mu_k^* - \mu_k^0\| \quad (12)$$

In accordance with price threshold δ , we can evaluate the principle of price variation.

$$v_k^p = \begin{cases} 0, & V < \delta \\ 1, & V \geq \delta \end{cases} \quad (13)$$

5) Send new price vector μ_k^* and price variation symbol v_k^p to all agents.

The procedure of price update is

TABLE II

THE PROCEDURE OF PRICE UPDATE IN MARKET-BASED FLOW ALLOCATION

Algorithm 2 The procedure of price update

- 1: **For** ($k = 1; k < K; k++$)
- 2: Evaluate the group j 's price vector μ , $j = 1, 2, \dots, K, j \neq k$.
- 3: Compute the additional flow request function Pm_i and new price in group j based on price μ_k with (10), (11).
- 4: Send μ_k to auctioneer k .
- 5: **End while**

Definition 1 (Route policy): The route policy is what link k to take next at each decision epoch, based on the state space $S = \{s | s = (i, t, In_i)\}$, where i denotes link i , t is the discrete time step, and $In_i = (l_i, tra_i)$ denotes information about travel time of link i , tra_i and capacity of link i , l_i under current weather.

In the example of Figure 2, the decision at Airport1 can then be described as: when the state is $(1, t, (2, 13m))$, take link 3 next; when the state is $(1, t, (1, 15m))$, take link 1 next, for all t . Note that information In_i is one component of a state and refers to link travel time realizations based on which the current decision is made, while a reference to information alone is in the general sense. A routing policy $RP(x)$ is defined as a mapping from states to decisions (next links), $\pi : RP(x) \rightarrow A$ [10]. For a given current state and a given decision, probabilities of all possible next states can be evaluated from the network probabilistic description P . A statement of the optimal routing policy problem in an air traffic network is to find

$$\pi^* = \arg \min_{\pi \in \{\pi^1, \pi^2, \dots, \pi^n\}} E(t_{x_s} - t_{x_0}) \quad (14)$$

here t_{x_s} denotes path travel time at the destination state, and t_{x_0} denotes path travel time at the start state, and $E(t_{x_s} - t_{x_0})$ is the expected value of the path travel time. Notice $f_i \in R^K$, where K is set of OD/time pairs, and resource $(l, t) \in R$. Therefore, the optimal routing policy can be formulated as

$$f_i = \arg \min_{f_i \in \{f_1, f_2, \dots, f_n\}} \{Fl_i(f_i) + \rho_i\} \quad (15)$$

$$\pi^* = F(f_1, \dots, f_i)$$

where $F(\cdot)$ is the mapping function between f_i and π .

IV. NUMERICAL EXPERIMENTS AND ANALYSIS OF RESULTS

The purposes of the numerical examples are to illustrate: the effects of optimal routing policy based on market theory, and the cost comparison and resource payment for all airlines. A number of sensitivity tests and investigations will be carried out as follows for these purposes.

A small-size network of seven nodes, ten links, two O-D pairs and twelve paths, as shown in Figure 2, is used to illustrate the application of the proposed model. In Figure 2, node airport1 and airport2 are original airports, and node airport3 and airport4 are destinations while others are waypoints. There are 30 aircrafts take off from each OD pair during the time T . The research time is divided into 10 time steps, such as t_1, t_2, \dots, t_{10} .

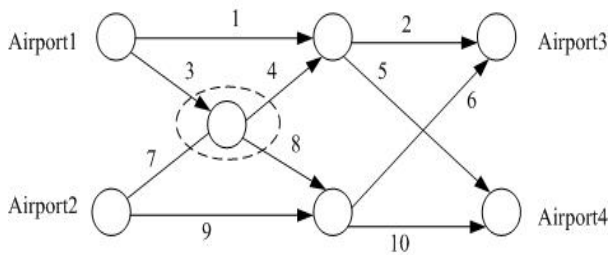


Fig. 2. Test network with seven nodes and ten links

Figure 2 shows the aggregate flow allocation over the network for all airlines. The elliptic region means the airspace influenced by weather, which happens at time step t_5, t_6, t_7 . It is assumed the capacity of link 7, 8, 3, 4 decrease by 30% under adverse weather. Notice tra_i denotes link travel time of link i on uncongested condition. For convenience, we assume that $tra_1 = 25m$, $tra_2 = 23m$, $tra_3 = 15m$, $tra_4 = 11m$, $tra_5 = 20m$, $tra_6 = 18m$, $tra_7 = 12m$, $tra_8 = 13m$, $tra_9 = 25m$, $tra_{10} = 23m$, and m is the time unit. The link travel time follows normal distribution.

The following table presents the simulated results with the proposed market-based method.

According to Table 3, we conclude the congested links will pay more than other links, and the routing policy evolves with the links' situation dynamically. The arrival flow at the destination airport being reduced during the weather disturbance, and then recover once the disturbance has passed. Note that the price vectors for the central ideal solution and the market-based algorithm are very close, as are the resulting flow allocations. The high cost airlines (7 - 4 - 2),

TABLE III
 ROUTING POLICY IN THE TEST NETWORK

Departure Time	O-D	Route Choice	Payment
t1	Airport1-Airport3	1-2	21.2
	Airport2-Airport4	9-10	23.8
t2	Airport1-Airport3	3-4-2	22.8
	Airport2-Airport4	7-8-6	30.1
t3	Airport1-Airport3	1-2	24.4
	Airport2-Airport4	7-8-10	23.2
t4	Airport1-Airport3	3-8-6	19.7
	Airport2-Airport4	9-10	11.3
t5	Airport1-Airport3	1-2	32.1
	Airport2-Airport4	9-10	36.2
t6	Airport1-Airport3	1-2	35.2
	Airport2-Airport4	9-10	36.4
t7	Airport1-Airport3	1-2	40.2
	Airport2-Airport4	9-10	43.2
t8	Airport1-Airport3	3-8-6	18.3
	Airport2-Airport4	7-8-10	17.6
t9	Airport1-Airport3	1-2	19.6
	Airport2-Airport4	9-10	23.2
t10	Airport1-Airport3	1-2	24.1
	Airport2-Airport4	9-10	21.8

(7 - 4 - 5), (1 - 2), (9 - 10) reduce their deviations in the resource allocation, whereas the low cost airlines (7 - 8 - 10), (3 - 4 - 2) increase their delays. The difference occurs when airport capacity becomes available after the weather passes; the high cost airlines immediately begin to recover while the low cost airlines either stay flat or increase their deviation.

According to the optimal flow allocation cited above, it is easily deduce the optimal route policy in view of minimizing the total travel cost.

V. CONCLUSION

This study presents a novel air traffic control model based on market theory in air traffic network. A resource bidding mechanism is formulated to simulate the competition nature of each airline in pursuit of the maximum profits under resource sharing environment. Furthermore, the algorithms are designed such that optimal routing policies are realized efficiently in ATC. The proposed method provides us a efficient way to do the flow control in ATC. The drawback of the algorithm is revealed in the price update, which required a large number of iterations to converge.

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