

Estimation of Load Impedance in Presence of Harmonics

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Abstract—This paper presents a fast and efficient on-line technique for estimating impedance of unbalanced loads in power systems. The proposed technique is an application of a discrete time-dynamic filter based on stochastic estimation theory which is suitable for estimating parameters in noisy environment. The algorithm uses sets of digital samples of the distorted voltage and current waveforms of the non-linear load to estimate the harmonic contents of these two signals. The non-linear load impedance is then calculated from these contents. The method is tested using practical data. Results are reported and compared with those obtained using the conventional least error squares technique. In addition to the very accurate results obtained, the method can detect and reject bad measurements. This can be considered as a very important advantage over the conventional static estimation methods such as the least error square method.

Keywords—Estimation, identification, Harmonics, Dynamic Filter.

I. INTRODUCTION

HARMONIC impedance of a non-linear load characterizes the frequency response characteristics of the system at different power system buses. It is important data for verification of harmonic limit compliance and thus designing effective harmonic filters. A number of non-linear loads estimation methods have been developed for this purpose. These methods can be classified into two types: transients based methods and steady state based methods [1],[2]. The main problems associated with the application of the first type is the need for high speed data acquisition system and the source of the transient disturbance which will be injected into the system to perform the simulation. Steady state methods are much preferable for their ease to implement [3]. Most of recent techniques are digital bases. In these kind of techniques, voltage and current data are collected in digital forms. After collecting voltage and currents data, in either transient or steady state mode, there is always a need for an adequate and fast identification technique in order to extract the non-linear load model in the form of admittance or impedance. Several identification techniques have been proposed to perform this job in either frequency or time domain. Some of these techniques are based on artificial intelligent methods such as artificial neural networks [4] and Genetic algorithms [5]. Other methods are based on Fast

Fourier transforms (FFT) and state estimation techniques. Shun et-al [6] described a method based on FFT for harmonic power measurement due to harmonic loads. In this reference the FFT technique is used to analyze the digitized voltage and current waveforms. Although FFT is a very effective tool in signal analysis; it has some drawbacks such as leakage effect. Heydt [7] described a state estimation formulation for estimating the current injection spectrum due to non-linear loads. The method based on the conservation of active power at each frequency. Soliman et-al [8] proposed the application of the least error square estimation technique for frequency domain modelling of the non-linear loads in admittance form. The least error squares (LES) is an accurate estimation technique. However, in case of the data set is contaminated with bad measurements the (LES) solution would never be accurate unless extra filters are used to eliminate the bad data points.

This paper presents a new method based on recursive algorithm, which can be used for digital identification of the harmonic impedance of non-linear loads. The algorithm is a dynamic estimator based on stochastic estimation theorem, which is applicable for estimating and tracking of non-stationary signals in noisy environments [9]. The estimator has been applied recently to many complicated estimation problems in power system successfully [10]. Authors of this paper presented an alternative way of identification of the non-linear load admittance using the proposed filter [11]. Unlike Kalman filter, which minimizes the error square, the proposed estimator gain matrix is derived in such way to minimize the absolute error in the estimation process. The method allows a very fast determination of the harmonic impedance of the non-linear loads even when the harmonic contents are varied with time.

II. MATHEMATICAL MODELING

Assuming that the voltage and current signals of a non-linear load are in the form of harmonic series as:

$$v(t) = \sum_{j=1}^n \sqrt{2} V_j \sin(w_j t + \theta_{vj}) \quad (1)$$

$$i(t) = \sum_{j=1}^n \sqrt{2} I_j \sin(w_j t + \theta_{ij}) \quad (2)$$

where

- j equals 1 for fundamental and equals 2,3,.. for harmonics
- n is the maximum order of harmonic considered
- V_j and I_j are the j^{th} rms values of the voltage and current component

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θ_{ij} is the phase angle of the j^{th} current component
 θ_{vj} is the phase angle of the j^{th} voltage component
 Equations 1 and 2 can be expanded as:

$$v(t) = \sqrt{2} V_1 \cos(\theta_{v1}) \sin(\omega_1 t) + \sqrt{2} V_1 \sin(\theta_{v1}) \cos(\omega_1 t) + \sqrt{2} V_2 \cos(\theta_{v2}) \sin(\omega_2 t) + \sqrt{2} V_2 \sin(\theta_{v2}) \cos(\omega_2 t) + \dots \quad (3)$$

$$i(t) = \sqrt{2} I_1 \cos(\theta_{i1}) \sin(\omega_1 t) + \sqrt{2} I_1 \sin(\theta_{i1}) \cos(\omega_1 t) + \sqrt{2} I_2 \cos(\theta_{i2}) \sin(\omega_2 t) + \sqrt{2} I_2 \sin(\theta_{i2}) \cos(\omega_2 t) + \dots \quad (4)$$

$$v(t) = \sin(\omega_1 t) X_{v1} + \cos(\omega_1 t) X_{v2} + \sin(\omega_3 t) X_{v3} + \cos(\omega_3 t) X_{v4} + \sin(\omega_5 t) X_{v5} + \cos(\omega_5 t) X_{v6} + \sin(\omega_7 t) X_{v7} + \cos(\omega_7 t) X_{v8} \quad (5)$$

$$i(t) = \sin(\omega_1 t) X_{i1} + \cos(\omega_1 t) X_{i2} + \sin(\omega_3 t) X_{i3} + \cos(\omega_3 t) X_{i4} + \sin(\omega_5 t) X_{i5} + \cos(\omega_5 t) X_{i6} + \sin(\omega_7 t) X_{i7} + \cos(\omega_7 t) X_{i8} \quad (6)$$

where:

$$X_{vk} = \sqrt{2} V_k \cos(\theta_{vk}), \quad k = 1, 2, \dots, 8 \quad (7)$$

$$X_{ik} = \sqrt{2} I_k \cos(\theta_{ik}), \quad k = 1, 2, \dots, 8 \quad (8)$$

This analysis assumes that the power system has no fundamental frequency neutral offset, that is no zero sequence voltage or current other than the triplex harmonics 3, 9, 15, etc.

If both voltage and current signals are sampled at a pre-selected rate, ΔT , then m samples, for each of the voltage and current would be obtained at t_1, t_2, \dots, t_m .

In a compact matrix form, equations 5 and 6 can be rewritten in state space form as,

$$\begin{aligned} A_v(k) &= H_v(k) X_v(k) + e_v(k) \\ A_i(k) &= H_i(k) X_i(k) + e_i(k) \end{aligned} \quad (9)$$

where

$A(k)$ is $m \times 1$ measurement vector of voltage or current samples

$H(k)$ is $m \times 8$ measurement matrix

$X(k)$ is 8×1 state vector to be estimated

$e(k)$ is $m \times 1$ measurement error vector, to be minimized. It is assumed to be white sequence with known covariance $R(k)$.

Subscript (i) is used for current and (v) for the voltage.

The state transition equations will be in the form:

$$X_v(k+1) = \Phi_v(k) X_v(k) + \varpi_v(k) \quad (10)$$

$$X_i(k+1) = \Phi_i(k) X_i(k) + \varpi_i(k) \quad (11)$$

where

$\Phi(k)$ 8×8 state transition matrix given as a unit diagonal matrix (assuming rotating reference)

$\varpi(k)$ 8×1 error vector of the state assumed to be a white (uncorrelated) sequence with known covariance matrix $Q(k)$.

Once the state vectors for the voltage and current waveforms are identified, the RMS values of fundamental and harmonic voltages, currents and their phase angles can be calculated as:

$$V_k = \sqrt{X_{vk}^2 + X_{v(k+1)}^2} \quad (12)$$

$$\theta_{vk} = \tan^{-1}(X_{v(k+1)} / X_{vk}) \quad (13)$$

where k equals 1 for fundamental and 3, 5, 7 for the other harmonics.

In the same manner the fundamental and harmonics RMS currents and phase angles can be obtained from the current state vector of equations 8.

$$I_k = \sqrt{X_{ik}^2 + X_{i(k+1)}^2} \quad (15)$$

$$\theta_{ik} = \tan^{-1}(X_{i(k+1)} / X_{ik}) \quad (16)$$

Accordingly, estimation of harmonic impedance parameters can be done through voltage – current estimation. The load impedance at any frequency can be easily found as:

$$\bar{Z}(\omega) = \frac{\bar{V}(\omega)}{\bar{I}(\omega)} \quad (17)$$

After this formulation, the problem now is how to estimate the parameter vectors of equations 7 and 8 from which the harmonic impedance can be found using equation 13. The detailed application of this equation is presented in the next section. In this section, the proposed estimation algorithm is presented.

III. DESCRIPTION OF THE PROPOSED ALGORITHM

The on-line estimation process of the admittance parameters is performed using the discrete least absolute value filtering algorithm (DLAVF). The complete derivation of the proposed filter equations is beyond the scope of this paper and is given in reference [10]. The dynamic filter works on the discrete state space model described by the measurement equation and the state transition equation in the following form.

$$A(k) = H(k) X(k) + e(k) \quad (18)$$

$$X(k+1) = \Phi(k) X(k) + \varpi(k) \quad (19)$$

As mentioned before the measurement error vector $e(k)$ is assumed to be white sequence with known covariance as,

$$E\{e(k)e(j)^T\} = \begin{cases} 0 & ; j \neq k \\ R(k) & ; j = k \end{cases} \quad (20)$$

The initial condition of $X(0)$ is a Gaussian random vector with the following statistics,

$$E\{X(0)\} = \bar{X}(0) \quad (21)$$

$$E\{[X(0) - \bar{X}(0)][X(0) - \bar{X}(0)]^T\} = \bar{P}(0)$$

where $\bar{P}(0)$ is $m \times m$ initial error covariance matrix of the states. The covariance of the error at any step (k) can be obtained by replacing $X(0)$ with $X(k)$ in equation (21b). The covariance matrix for $\varpi(k)$ is given as:

$$E\{\varpi(k)\varpi(j)^T\} = \begin{cases} 0 & ; j \neq k \\ Q(k) & ; j = k \end{cases} \quad (22)$$

The algorithm starts with an initial estimate for the system parameter vector $\bar{X}(0)$ and its error covariance matrix ($\bar{P}(0)$) at some point $k=0$. These estimates are denoted as \bar{X}, \bar{P} , where (-) means that these are the best estimations at this point, prior to assimilating the measurement at instant k . With such initial values, of both parameters and error covariances, filter gain matrix $K(k)$ at this step is calculated as follows,

$$K(k) = [H(k) + R(k)Ly^T \bar{P}^{-1}(k)]^{-1} \quad (23)$$

assuming that the state vector dimension is $u \times 1$, the vectors L

and y are defined as: L is $u \times 1$ column vector $(1, 1, \dots, 1)^T$; and y^T is $1 \times u$ row vector $(1, 1)$ [9]. Using the filter gains, estimates are updated with measurements $A(k)$ through equation (2), and error covariances for update estimates are computed from equation (19).

$$\hat{X}(k) = \bar{X}(k) + K(k)[Z(k) - H(k)\bar{X}(k)] \quad (24)$$

$$P(k) = [I - K(k)H(k)]P(k)[I - K(k)H(k)]^T + K(k)R(k)K^T(k) \quad (25)$$

Finally, error covariances and estimates are projected ahead to repeat with $k=2$.

$$\bar{P}(k+1) = \Phi(k)P(k)\Phi^T(k) + Q(k) \quad (26)$$

$$\bar{X}(k+1) = \Phi(k)\hat{X}(k) + R(k) \quad (27)$$

The process is repeated until the last sample is reached. It

is assumed that the co-variances and the transition matrices are known. It is also assumed that a good initialization of the filter is obtained using the results of static method such as least squares error or least absolute value. From the test examples, we will show that good initialization is not necessary to satisfy the required accuracy in this application of the filter.

IV. TESTING OF ALGORITHM

A. Balanced Load

In order to check the validity and applicability of the proposed method for on-line impedance measurements, it is tested on actual recorded data of a nonlinear load. The load is a large induction motor rated 1250 HP connected to 44 KV. The motor is driven by a 6-pulse inverter [8],[11]. The inverter is considered as a harmonic source injecting harmonics into the system. The voltage and current wave forms are shown in Fig. 1.

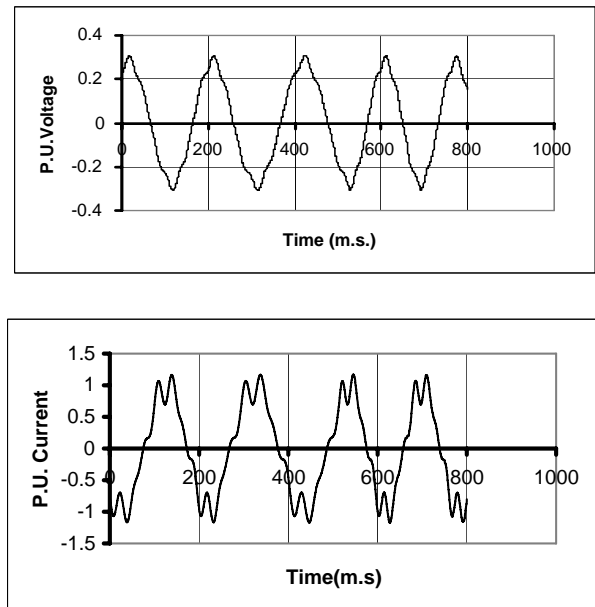


Fig. 1 The recorded waveforms

To investigate the performance of the proposed method, the voltage and current signals are sampled at different sampling rates. The generated data are categorized in different groups to study the effects of sampling rate, number of samples and the data window size. The effects of initial conditions and bad data points on the filter performance are also studied.

In group 1, the data window size is chosen to be constant at 1 cycle and the number of samples per cycle is varied as 10, 50 100, 150, 200, 300, 400 and 500. Figs. 2 and 3 show the effects of varying the number of samples at constant window size of 1 cycle on the estimation process. In these figures the steady state values of the estimated harmonic impedance and their phase angles are reported. In Fig. 2 the fundamental impedance is shown as well. It can be noticed that changing the number of samples does not effect the results. Samples of

the results are tabulated in Table I. Results obtained using the conventional least error squares technique are also shown in this table for comparison. In all cases initial conditions are assumed zeros.

In group 2 the effect of varying the data window size is examined. In this group different data window sizes are considered with different numbers of samples. The data window size is varied from 1 to 5 cycles in step of 1. For each window size the number of samples is varied as 10, 50, 100, 200, 300 and 400. Samples of the results obtained are shown in Table II. In this table, the data window size is varied between 1 and 5 cycles and the number of samples is held constant at 50 samples per cycle. It is clear that increasing the data window size does not effect the solution accuracy. This would be expected, since both the current and voltage signals in these applications are stationary. It is thus concluded that one cycle is sufficient to give very high estimation accuracy. Increasing the data window size would increase the calculation time considerably without changing the accuracy.

The effects of Initial conditions are studied by changing the initial state values. Ten samples per cycle are used to perform this study within a data window size of 10 cycles. The LS solution and a flat starting (zeros) are used as an initial solution to extract a conclusion. It is found that same steady state solutions are obtained in all cases. The only difference is that, using the least error squares solution as an initial start accelerates the solution only little. Fig. 4 shows the obtained Z_1 magnitude using zero initial conditions while Fig. 5 shows the obtained magnitude of Z_1 using the LS solution as an initial start. It is clear that both solutions have the same steady state value of 0.2824 p.u. The only difference is that the solution in Fig. 5 reaches steady state earlier.

Finally, in order to check the capability of the proposed filter of handling bad data, the sign of some simulated data of an arbitrary case (50 samples, 6 cycle) is deliberately reversed after the filter solution reaches the steady state, at steps 100, 101. Table III shows the effect of bad data on the solution obtained using the proposed method and the LS method. The filter passed the test extremely well within 20 to 30 iterations after supplying the bad data. The least error square method failed in obtaining the correct solution. This gives the filter the advantage of rejecting bad data inherently without needing extra filter as in the case of least squares method.

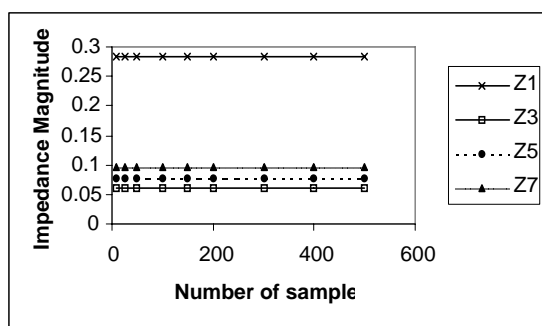


Fig. 2 Impedance magnitudes

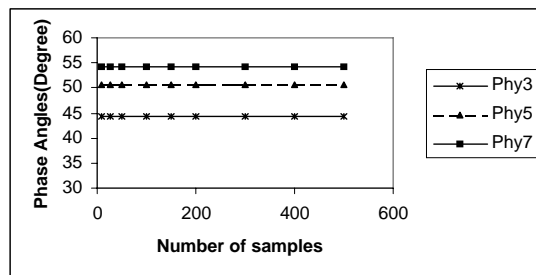


Fig. 3 Impedance phase angles

TABLE I
 EFFECTS OF VARYING THE NUMBER OF SAMPLES

N/c	Method	Z_1	Z_3	Z_5	Z_7
10	Filter	0.282486	0.059259	0.076277	0.095511
	LS	0.282486	0.059259	0.076295	0.095511
50	Filter	0.282486	0.059880	0.076277	0.096805
	LS	0.282486	0.059259	0.076295	0.095511
100	Filter	0.282486	0.060642	0.076277	0.09775
	LS	0.282486	0.059259	0.076277	0.095511
150	Filter	0.282486	0.060901	0.076288	0.095147
	LS	0.282486	0.059259	0.076277	0.095511
200	Filter	0.282486	0.061425	0.076982	0.098425
	LS	0.282486	0.059259	0.076219	0.095511
300	Filter	0.282486	0.059276	0.076288	0.097180
	LS	0.282486	0.059259	0.076277	0.095511
400	Filter	0.282486	0.059347	0.076452	0.092850
	LS	0.282486	0.059259	0.076395	0.095238

TABLE II
 EFFECTS OF VARYING THE DATA WINDOW SIZE

Window	Z_1	Z_3	Z_5	Z_7
1	0.282486	0.05988	0.076277	0.096805
2	0.282486	0.05987	0.076277	0.096805
3	0.282486	0.05987	0.076277	0.096805
4	0.282486	0.05987	0.076277	0.096805
5	0.282486	0.05987	0.076277	0.096805

TABLE III
 BAD DATA EFFECTS

	Filter	LS
Z_1	0.282486	0.28650
Z_3	0.059276	0.03849
Z_5	0.076290	0.08077
Z_7	0.095510	0.11010

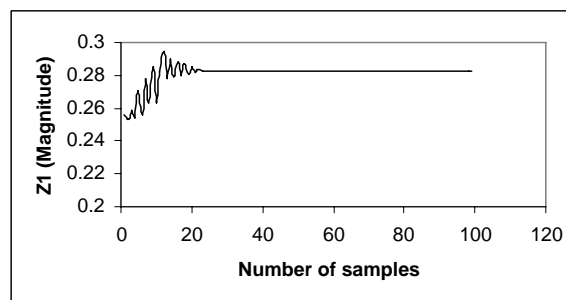


Fig. 4 Z_1 Magnitude (zero Initial conditions)

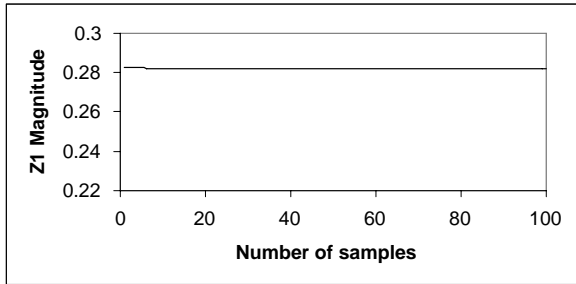


Fig. 5 Z_1 Magnitude (LS Initial conditions)

B. Distorted Unbalanced Load

In this case, the algorithm is tested using simulated data based on unbalanced three phase harmonic contaminated current given in Fig. 6 [13]. This test is used to show the ability of the algorithm for tracking the harmonic loads even in unbalanced loading conditions. These currents are sampled at 3000 Hz. The obtained samples are fed to the algorithm. The three fundamental currents are filtered first using the algorithm described earlier. The extracted fundamental signals are then used to estimate the magnitudes and phase angles of the harmonic impedance using the voltage signal as before. Here only estimation of the current contents is presented. Examination of Table IV reveals that the results obtained are very accurate. It is very easy to find the impedance as mentioned before.

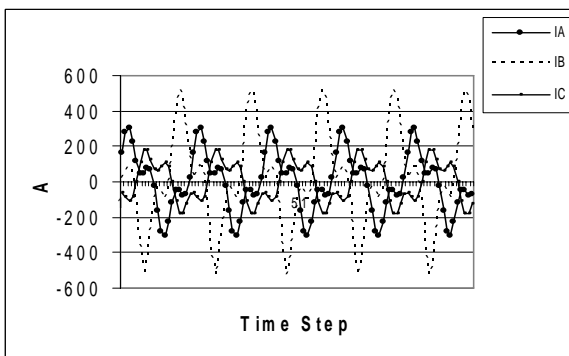


Fig. 6 The generated unbalanced waveforms

TABLE IV
 FUNDAMENTAL & HARMONIC CONTENTS OF LOAD CURRENT

Fundamental component Magnitudes		Fundamental component Phase angles (degree)	
Estimated	Exact	Estimated	Exact
212.128	212.132	44.98	45
353.547	353.55	150.06	150
141.420	141.42	299.91	300
harmonic Third magnitudes		Third harmonic Phase angles(degree)	
Estimated	Exact	Estimated	Exact
106.00	106.067	106.00	106.067
176.77	176.78	176.77	176.78
70.70	70.71	70.70	70.71

V. CONCLUSION

In conclusions, a new application of the discrete least absolute value filter for harmonic impedance measurements is introduced in this work. The effect of filter parameters has been studied and showed that the filter performance is highly accurate and fast. It has been shown that the filter can detect and reject bad data points and gives very accurate solutions within 12 to 13 steps. The filter does not need any pre knowledge about the parameters, where it did estimate the parameters accurately starting from any initial guess. A minimum number of only 10 samples per cycle is needed. This makes the implementation of the algorithm very easy. The model is based on stochastic estimation theorem; therefore it is suitable for on line measurements of non-stationary parameters.

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