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Constructing a Fuzzy Net Present Value Method to Evaluating the BOT Sport Facilities

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***Abstract*—**This paper is to develop a fuzzy net present value (FNPV) method by taking vague cash flow and imprecise required rate of return into account for evaluating the value of the Build-Operate-Transfer (BOT) sport facilities. In order to clearly manifest a more realistic capital budgeting model based on the classical net present value (NPV) method, some uncertain financial elements in NPV formula will be fuzzified as triangular fuzzy numbers. Through the conscientious manipulation of fuzzy set theory, we will find that the proposed FNPV model is a more explicit extension of classical (crisp) model and could be more practicable for the financial managers to capture the essence of capital budgeting of sport facilities than non-fuzzy model.

***Keywords*—**Fuzzy sets; Capital budgeting; Sport facility; Net present value (NPV); Build-Operate-Transfer (BOT) scheme

I. Introduction

RECENTLY public-private partnerships have been set up to finance major public assembly projects, particularly for sport facilities. There are three critical considerations for the private sector to participate in sport infrastructure projects (including building, financing, and operating). Firstly, the private sector possesses better mobility than the public sector. In other words, the private sector is not only contributive to promote the sport project quality in planning, design, construction and operation, but also to avoid the bureaucracy and to relieve the administrative load. Secondly, the private sector can provide the public sector with better service and establish a good public-private partnership so that the balance risk-return structure can be maintained. The third one is the government lacks the ability of raising massive funds for the large-scale sport infrastructure projects, but private participation can mitigate the government's financial burden [6]. In practice, as the argument of [15], the willingness of the private sector in developing infrastructure projects depends on the mature legal environment where the projects operate. Also since 2000, Taiwan's government has promulgated the Law for Promotion of Private Participation in Infrastructure Projects in order to attract the private sector to participate in infrastructure development and the main scope of this law merely prescribes for the types of private participation in infrastructure projects to suit to the use of Build-Operate-Transfer (BOT) type scheme.

For the BOT sport facility management, whether they are stadiums played in by professional teams, municipal facilities for public use, or a local recreational club, it is essential that the overall governance and construction of new sport facilities is underpinned by sound financial planning [20].

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Although each type of sport facilities has its special considerations during design, the most important requirement for the concessionaires to sustainably develop, operate and maintain the facilities is whether the positive present value can be correctly estimated or not. Once the profits are made they can be reinvested in the business to improve it and to keep ahead of the game. Since the huge amount of capital expenditures and the results of capital budgeting decisions would continually influence the facilities for many years, the facility managers should require even more financial skills such as capital budgeting appraisal and negotiating leverage to finance their operations.

The construction and operation of sport arenas, stadiums and multipurpose facilities are large capital projects for a concessionaire. Several mechanisms are used in structuring private sector participation in sport facility development, expansion, and renovation [24]. Capital budgeting decision is one of the most demanding responsibilities of top financial management that the evaluation of BOT-based projects is often accomplished through the discounted cash flow method. Classically, six key appraisal methods are used to rank projects and to decide whether they should be accepted in the capital budget or not, such like payback period method, discounted payback, net present value (NPV), internal rate of return (IRR), profitability index (PI), and real options. Indeed, most academics and professionals agree in considering the NPV rule as the most reliable criterion in ranking projects [16].

Bierman [2] surveyed the capital budgeting methods used by the Fortune 500 industrial companies and found every responding firm used some type of discounted cash flow (DCF) method and most firms preferred to use IRR and NPV methods. Graham and Harvey [11] also indicated that most of the companies still used IRR and NPV methods to evaluate their investment programs and they found that the different capital budgeting practices were employed in small firms (less than $1 billion in sales) and large firms (more than $1 billion in sales). The smaller firms are more likely to rely on the payback method, while the larger firms prefer to employ IRR and/or NPV method. The NPV rule is a pillar of modern finance theory and it is still so consolidated in the literature that we must admit that most financial concepts subsume it as a starting point for project's valuation, so the financial decision maker should not disregard the problem of the NPV [16]. Up to now, a great majority of financial managers have still regarded the NPV rule as one of the most important investment criteria [3, 4, 8, 18, 19], and most financial concepts are based on the notions of present value and opportunity cost of capital, which are just the bricks of the NPV building [16].

The NPV is frequently used for firm's capital budgeting and it has been disguised as what it is generally known as *Economic Value Added (EVA)*, which is only an algebraic transformation of the NPV [22]. Biddle, Bowen, and Wallace [1] addressed

that the EVA model has gained increasing attention not only in the literature but also among practice which massively use this index, and is considered a reliable index for firms' evaluation or as a tool for rewarding managers. Besides, Magni [16] also made a criticism: the idea is misleading that the option pricing and dynamic programming are more refined tools for evaluating projects. Because it is commonly known that if an investment is not a real option (e.g., it is non-deferrable), option pricing and NPV give the same result. Dixit and Pindyck [9] clearly showed the evaluation of a real option by dynamic programming boils down to a comparison between two net present values, one of which is related to investing now, the other one to waiting till the next period. So, the essence of real option evaluation can also be seen as an appraising process based on the NPV criterion. The classical NPV method is the most basic frameworks of capital budgeting analysis described in detail in most financial management textbooks and it is taught in most introductory courses in financial management. In most theoretical models, the NPV method which relies on DCF technique is stated similar to [4, 21]. Although the classical NPV method plays a decisive role in capital budgeting, it does not take into account the uncertainties which may be inherent in these parameters used in practice. These parameters including the vague expected net cash inflow stream and the project's capital cost in the future. Especially in the uncertain financial environment, the capital costs of sport facilities should vary over time. In the classical NPV method, the financial managers tend to use point input prices, implicitly assuming that these prices are predictable, and they usually incorporate the uncertainty in the field of capital budgeting analysis based on intuitive method or probabilistic approach. However, these common methods still have the disadvantages of requiring the fulfillment of some assumptions for probabilistic distributions and relying on point estimation to obtain these uncertain parameters. Kahraman, Ruan, and Tolga [12] indicated that in an uncertain economic decision environment, an expert's knowledge about the cash flows and the capital costs consists of a lot of vagueness instead of randomness. In order to deal with the vagueness of human thought, Zadeh [27] first introduced the fuzzy set theory, which was based on the rationality of uncertainty due to imprecision of vagueness. Afterward, fuzzy set theory has become a powerful tool when sufficient objective data have not been obtained, and some developments in fuzzy-financial mathematics have been well applied to deal with such the financial problems. Several researchers have therefore proposed a series of excellent studies about the fuzzy techniques in order to assess the investment project. For example, Buckley [5] studied the fuzzy extension of the mathematics of finance to concentrate on the compound interest law. Then, Calzi [7] investigated a possible general setting by considering both compact fuzzy intervals and invertible fuzzy intervals for the fuzzy mathematics of finance. Kuchta [14] also generalized fuzzy equivalents for the methods of evaluating investment projects. Dourra and Siy [10] applied fuzzy information technologies to investments through technical analysis, and used it to examine various companies to achieve a substantial investment return. Furthermore, since the classical NPV method is subject to the assumption of a constant required rate of return throughout the project (i.e., discount cash flows with an equivalent-risk rate), it is not legitimate to compare money having different degrees of risk. Such as Magni's [16] argument: the NPV rule gives rise to a partial ordering among assets so the impossibility of comparing two assets with different risks must be coped. Also, Turner and Morrell [23] pointed out that discount rate estimates are variable, and clearly, companies' capital costs would shift over time. Thus, we generalize the classical NPV method as the net present value of expected future net cash inflows for period n which discounted at the different required rate of return k_t that is given by

$$N_n = C_O + \frac{C_1}{(1+k_1)^1} + \frac{C_2}{(1+k_2)^2} + \cdots + \frac{C_n}{(1+k_n)^n} \quad (1)$$

or shorter

$$N_n = C_O + \sum_{t=1}^{n} \frac{C_t}{(1+k_t)^t} \quad (2)$$

where

N_n: the net present value of the facility operation project for time period n (e.g. year), or an expected present value in an finite stream of expected future net cash inflow estimated by the facility financial managers.

C_O: the net cash outflow at the beginning of the project, which is treated as a certain negative value.

C_t: the expected net cash inflow of the facility operation project estimated by the facility financial mangers at t-th time period. All future net cash inflows are expected values, so the estimation values of C_t may differ among various types of facilities.

k_t: the required rate of return of the facility operation project estimated by the facility financial managers at t-th time period (i.e. the facility financial managers consider the returns available on other investments).

Eq. (2) is a normalized capital budgeting analysis in the sense that the time pattern of C_t should be a non-negative real number and may be rising, falling, constant, or fluctuating randomly. As we consider that a facility financial manager might be interesting to use NPV method to evaluate a BOT sport facility project in which the cash flows will be appraised prior to each time period, including all inflows and outflows, and then be discounted at a certain required rate of return. To sum up these discounted net cash inflows, the NPV of the BOT sport facility project will be obtained. If the NPV is positive, the BOT sport facility project will be accepted; on the contrary, it should be rejected. As to the mutually exclusive case, the one with the higher NPV should be chosen.

II. Methodology

A. Valuation the BOT sport facility project with Fuzzy net present value (FNPV) method

Before presenting the FNPV method based on the λ–signed distance approach, the following preliminary definitions are provided in advance with some relevant operations derived from [13].

Definition 1. A fuzzy set $[a,b;\alpha]$, $a<b$ defined on $\Re=(-\infty,\infty)$, which has the following membership function, is called a level α fuzzy interval.

$$\mu_{[a,b;\alpha]}(x)=\begin{cases}\alpha\ , & a\le x\le b,\\ 0\ , & otherwise.\end{cases}$$

Definition 2. By [17], fuzzy point $\tilde{a}$ is a fuzzy set is defined on $\Re$ with the following membership function.

$$\mu_{\tilde{a}}(x)=\begin{cases}1\ , & x=a,\\ 0\ , & x\ne a.\end{cases}$$

Definition 3. The triangular fuzzy number $\tilde{D}$ is defined on $\Re$ with the following membership function, and denoted by $\tilde{D}=(a,b,c)$, where $a<b<c$.

$$\mu_{\tilde{D}}(x)=\begin{cases}\dfrac{x-a}{b-a}, & a\le x\le b,\\ \dfrac{c-x}{c-b}, & b\le x\le c\\ 0, & otherwise\ .\end{cases}$$

Let F_s be the family of fuzzy sets defined on $\Re$, for each $\tilde{D}\in F_s$, the $\alpha-cut$ of $\tilde{D}$ is denoted by $D(\alpha)=\{x|\mu_{\tilde{D}}(x)\ge\alpha\}=[\tilde{D}_L(\alpha),\tilde{D}_U(\alpha)]$ $(0\le\alpha\le 1)$, and both $\tilde{D}_L(0)$ and $\tilde{D}_U(0)$ are finite values. For each $\alpha\in[0,1]$, the real numbers $\tilde{D}_L(\alpha)$, $\tilde{D}_U(\alpha)$ separately represent the left and right end points of $D(\alpha)$ and satisfy the conditions that both of $\tilde{D}_L(\alpha)$, $\tilde{D}_U(\alpha)$ exist in $\alpha\in[0,1]$ and are continuous over $[0,1]$.

Subsequently, we define $\lambda-$signed distance approach and provide the following properties asserted by [25, 26].

Definition 4. (a) For each $\tilde{D}\in F_s$ and each $\lambda\in(0,1)$, the $\lambda-$signed distance from $\tilde{D}$ to $\tilde{0}$ is defined by

$$d(\tilde{D},\tilde{0};\lambda)=\int_0^1\left[\lambda\tilde{D}_L(\alpha)+(1-\lambda)\tilde{D}_U(\alpha)\right]d\alpha \qquad (3)$$

(b) When $\tilde{D}=(a,a,a)=\tilde{a}$ is a fuzzy point at a and for all $\alpha\in[0,1]$, $\tilde{D}_L(\alpha)=\tilde{D}_U(\alpha)=a$, then by (a) yields: $d(\tilde{a},\tilde{0};\lambda)=a$ for all $\lambda\in(0,1)$.

Definition 5. Let $\tilde{A},\ \tilde{B}\in F_s$ and for each $\lambda\in(0,1)$, define the metric ρ_λ by

$$\rho_\lambda(\tilde{A},\tilde{B})=\left|d(\tilde{A},\tilde{0};\lambda)-d(\tilde{B},\tilde{0};\lambda)\right|$$

Definition 6. For $\tilde{A},\ \tilde{B}\in F_s$ and each $\lambda\in(0,1)$, relations "$\prec$, $\approx$" on F_s are

$\tilde{A}\prec\tilde{B}$ *iff* $d(\tilde{A},\tilde{0};\lambda)<d(\tilde{B},\tilde{0};\lambda)$;

$\tilde{A}\approx\tilde{B}$ *iff* $d(\tilde{A},\tilde{0};\lambda)=d(\tilde{B},\tilde{0};\lambda)$.

In real economic environment, the operations of concessionaires often go through life cycle and have the following pattern with regard to the economic cycle: during the early part of their lives, the growth rates of the concessionaires are higher than that of the economy; then match the economy's growth; and finally maintain a steady growth or lower than the economy's growth. Similarly, for their investment projects, the cash inflows and the required rates of returns will vary with the shifting economy. Based on this viewpoint, when the facility financial managers use the classical (crisp) NPV method to evaluate their BOT projects, it is necessary to make several assumptions for the net cash inflows and the required rates of returns and further to estimate them by using educated guesses or other statistical skills because of the difficulties of precisely predicting these parameters in the future operation periods. Nonetheless, the real values of these two parameters won't be necessarily equal to the former estimations exactly. Especially regarding the estimations of the required rates of returns (k_t) in different time periods, they usually could be derived from using the capital asset pricing model (CAPM) in which the risk factor, the market expected rate of return, and the expectations about the risk-free rate are embodied [21]. Since either k_t or the other financial data in CAPM is uncertain, these magnitudes should be more suitable to be considered as fuzzy numbers. Thus it will more fit in with real situation to predict the future cash flows and required rates of returns in different time periods by taking possible intervals such as $[C_t-\varepsilon_t,C_t+\beta_t]$ and $[k_t-\theta_t,k_t+\omega_t]$ in each period t (e.g. $t=1,\ 2,\ 3,\ \cdots,\ n$) instead of point estimation. In such the closed intervals, ε_t, β_t, θ_t and ω_t may be appropriately determined by the facility financial managers satisfying the conditions of $0<\varepsilon_t<C_t$, $0<\theta_t<k_t$, $0<\beta_t$, and $0<\omega_t$.

Furthermore, since both the intervals $[C_t-\varepsilon_t,C_t+\beta_t]$ and $[k_t-\theta_t,k_t+\omega_t]$ are not definite values, the facility financial managers must respectively estimate a certain value from such the intervals for the calculation of the project's NPV. When they takes the estimation values of net cash inflow and required rate of return by C_t and k_t as the same as the former expected C_t and k_t, the estimation errors would be zero. Hence we can link the statistical concept of confidence level with membership grade in fuzzy set theory and hereby set the maximum confidence level as "*1.*" According to the essence of confidence level, if the estimation values of CF_t and k_t respectively determined during the intervals $[C_t-\varepsilon_t)$ or $(C_t+\beta_t]$, $[k_t-\theta_t)$ or $(k_t+\omega_t]$ are more far away from the expected C_t and k_t, then the confidence level would be smaller. Similarly, the right and left end points (i.e. $C_t-\varepsilon_t$, $C_t+\beta_t$, $k_t-\theta_t$ and $k_t+\omega_t$) have the same minimum confidence levels set to be "*0.*"

Therefore, corresponding to the intervals of C_t and k_t (i.e. $[C_t-\varepsilon_t,C_t+\beta_t]$ and $[k_t-\theta_t,k_t+\omega_t]$), the fuzzy intervals of C_t and k_t can be expressed as the following triangular fuzzy numbers.

$$\tilde{C}_t=(C_t-\varepsilon_t,C_t,C_t+\beta_t),\ t=1,\ 2,\ 3,\ \cdots,\ n; \qquad (4)$$

$$\tilde{k}_t=(k_t-\theta_t,k_t,k_t+\omega_t),\ t=1,\ 2,\ 3,\ \cdots,\ n. \qquad (5)$$

where ε_t, β_t, θ_t and ω_t may be appropriately determined by the facility financial managers satisfying the following conditions.

$0 < \varepsilon_t < C_t$, $0 < \theta_t < k_t$, $0 < \beta_t$, and $0 < \omega_t$. (6)

Note that if we set the membership grade of $\tilde{C}_t$ at C_t (or $\tilde{k}_t$ at k_t) as "*1*," then the larger distance from the right and left end points, $C_t - \varepsilon_t$ and $C_t + \beta_t$, (or $k_t - \theta_t$ and $k_t + \omega_t$) to C_t (or k_t) is, the smaller membership grade would be. Namely, both the membership grades on the end points are "*0*." Obviously, there are similar characteristics between membership grade and confidence level.

The $\alpha - cut$ of $\tilde{C}_t$ and $\tilde{k}_t$ can be denoted by $C_t(\alpha) = \left[\tilde{C}_{tL}(\alpha), \tilde{C}_{tU}(\alpha)\right]$ and $k_t(\alpha) = \left[\tilde{k}_{tL}(\alpha), \tilde{k}_{tU}(\alpha)\right]$, respectively, $\alpha \in [0,1]$, where

$$\tilde{C}_{tL}(\alpha) = C_t - (1-\alpha)\varepsilon_t = (C_t - \varepsilon_t) + \alpha\varepsilon_t > 0$$
$$\tilde{C}_{tU}(\alpha) = C_t + (1-\alpha)\beta_t > 0 \quad (7)$$

Subsequently, employing $\lambda-$ signed distance approach to defuzzify $\tilde{C}_t$ and $\tilde{k}_t$, then for each $\lambda \in (0,1)$, we have

$$C^*_{t,\lambda} \equiv d(\tilde{C}_t, \tilde{0}; \lambda) = C_t + \frac{1}{2}[(1-\lambda)\beta_t - \lambda\varepsilon_t] > 0; \quad (8)$$

$$k^*_{t,\lambda} \equiv d(\tilde{k}_t, \tilde{0}; \lambda) = k_t + \frac{1}{2}[(1-\lambda)\theta_t - \lambda\omega_t] > 0. \quad (9)$$

By (8) and (9), we denote $C^*_{t,\lambda}$ and $k^*_{t,\lambda}$ as the estimations of net cash inflow and required rate of return in the fuzzy sense based on $\lambda-$ signed distance, where $C^*_{t,\lambda} > 0$, $C^*_{t,\lambda} \in [C_t - \varepsilon_t, C_t + \beta_t]$ and $k^*_{t,\lambda} > 0$, $k^*_{t,\lambda} \in [k_t - \theta_t, k_t + \omega_t]$. The relation between λ and $C^*_{t,\lambda}$, $k^*_{t,\lambda}$ will be further discussed.

According to the fuzzy operations, let $\sum_{t=0}^{n} \tilde{A}_t$ be represented as $\tilde{A}_1(+)\tilde{A}_2(+)\cdots(+)\tilde{A}_n$ and $\tilde{M}^t = \tilde{M}(\times)\tilde{M}(\times)\cdots(\times)\tilde{M}$ (t times). Using (4) and (5) to fuzzify (2), then we have the project's NPV for n period in the fuzzy sense expressed by

$$\tilde{N}_n = \tilde{C}_O(+)\sum_{t=1}^{n} \tilde{C}_t(\div)\left[\tilde{1}(+)\tilde{k}_t\right]^t \quad (10)$$

where both $\tilde{C}_O$ and $\tilde{1}$ are fuzzy points at C_O and *1*, respectively. In (10), the left and right end points of $\alpha - cut$ of $\left[\tilde{1}(+)\tilde{k}_t\right]^t$ are

$$\left(\left[\tilde{1}(+)\tilde{k}_t\right]^t\right)_L(\alpha) = \left[1 + k_t - (1-\alpha)\theta_t\right]^t \ (>0); \quad (11)$$

$$\left(\left[\tilde{1}(+)\tilde{k}_t\right]^t\right)_U(\alpha) = \left[1 + k_t + (1-\alpha)\omega_t\right]^t \ (>0). \quad (12)$$

Meanwhile, we can also conduct the right and left end points of $\alpha - cut$ of $\tilde{C}_t(\div)\left[\tilde{1}(+)\tilde{k}_t\right]^t$ shown below.

$$\left(\tilde{C}_t(\div)\left[\tilde{1}(+)\tilde{k}_t\right]^t\right)_L(\alpha) = \frac{C_t - (1-\alpha)\varepsilon_t}{\left[1 + k_t + (1-\alpha)\omega_t\right]^t} \ (>0), \ t = 1, 2, 3, \cdots, n; \quad (13)$$

$$\left(\tilde{C}_t(\div)\left[\tilde{1}(+)\tilde{k}_t\right]^t\right)_U(\alpha) = \frac{C_t + (1-\alpha)\beta_t}{\left[1 + k_t - (1-\alpha)\theta_t\right]^t} \ (>0), \ t = 1, 2, 3, \cdots, n. \quad (14)$$

According to the decomposition theory, we can obtain the following Theorem 1.

Theorem 1. By (4), (5) and fuzzify C_O, C_t and k_t shown in (2), the project's FNPV can be expressed as

$$\tilde{N}_n = \tilde{C}_O(+)\sum_{t=1}^{n} \tilde{C}_t(\div)\left[\tilde{1}(+)\tilde{k}_t\right]^t$$
$$= \bigcup_{0\le\alpha\le1}\left[C_O + \sum_{t=1}^{n}\frac{C_t - (1-\alpha)\varepsilon_t}{\left[1 + k_t + (1-\alpha)\omega_t\right]^t}, C_O + \sum_{t=1}^{n}\frac{C_t + (1-\alpha)\beta_t}{\left[1 + k_t - (1-\alpha)\theta_t\right]^t}; \alpha\right] \quad (15)$$

B. Defuzzification by using the $\lambda-$signed distance approach

By Definition 4, for each $\lambda \in (0,1)$, we defuzzify (15) by the $\lambda-$signed distance approach to yield

$$N^*_{n,\lambda} = d(\tilde{N}_n, \tilde{0}; \lambda)$$
$$= C_O + \sum_{t=1}^{n}\int_0^1 \left\{\frac{\lambda[C_t - (1-\alpha)\varepsilon_t]}{\left[1 + k_t + (1-\alpha)\omega_t\right]^t} + \frac{(1-\lambda)[C_t + (1-\alpha)\beta_t]}{\left[1 + k_t - (1-\alpha)\theta_t\right]^t}\right\} d\alpha \quad (16)$$

Theorem 2. In Theorem 1, for each $\lambda \in (0,1)$, using $\lambda-$signed distance approach to defuzzify the fuzzy sets $\tilde{N}_n = \tilde{C}_O(+)\sum_{t=1}^{n}\tilde{C}_t(\div)\left[\tilde{1}(+)\tilde{k}_t\right]^t$, the estimate value of the project's NPV for n period ($N^*_{n,\lambda}$) in the fuzzy sense corresponding to (2) can be expressed as the following three general forms:

(a) For $n = 1$,

$$N^*_{1,\lambda} = C_O + \lambda\left[\left(\frac{C_1\omega_1 + \varepsilon_1(1+k_1)}{\omega_1^2}\right)\left(\ln\frac{1+k_1+\omega_1}{1+k_1}\right) - \frac{\varepsilon_1}{\omega_1}\right]$$
$$+ (1-\lambda)\left[\left(\frac{C_1\theta_1 + \beta_1(1+k_1)}{\theta_1^2}\right)\left(\ln\frac{1+k_1}{1+k_1-\theta_1}\right) - \frac{\beta_1}{\theta_1}\right]$$

(b) For $n = 2$,

$$N^*_{2,\lambda} = N^*_{1,\lambda} + \lambda\left[\left(\frac{\varepsilon_2}{\omega_2^2}\right)\left(\ln\frac{1+k_2}{1+k_2+\omega_2}\right) + \frac{\varepsilon_2(1+k_2) + C_2\omega_2}{\omega_2(1+k_2)(1+k_2+\omega_2)}\right]$$
$$+ (1-\lambda)\left[\left(\frac{\beta_2}{\theta_2^2}\right)\left(\ln\frac{1+k_2-\theta_2}{1+k_2}\right) + \frac{\beta_2(1+k_2) + C_2\theta_2}{\theta_2(1+k_2)(1+k_2-\theta_2)}\right]$$

(c) For $n \ge 3$,

$$N^*_{n,\lambda} = N^*_{1,\lambda} + N^*_{2,\lambda} + \sum_{t=3}^{n}\left\{\lambda\left[(-1)^t\left(\frac{\varepsilon_t(1+k_t) - C_t\omega_t(t-2)}{\omega_t^2(t-1)(t-2)(-1-k_t)^{t-1}}\right)\right.\right.$$
$$\left.- (-1)^t\left(\frac{\varepsilon_t(1+k_t) - C_t\omega_t(t-2) + \varepsilon_t\omega_t(t-1)}{\omega_t^2(t-1)(t-2)(-1-k_t-\omega_t)^{t-1}}\right)\right]$$
$$+ (1-\lambda)\left[\frac{\beta_t(1+k_t) - C_t\theta_t(t-2)}{\theta_t^2(t-1)(t-2)(1+k_t)^{t-1}} - \frac{\beta_t(1+k_t) - C_t\theta_t(t-2) - \beta_t\theta_t(t-1)}{\theta_t^2(t-1)(t-2)(1+k_t-\theta_t)^{t-1}}\right]\Bigg\}.$$

III. Case Study and Numerical Simulations

In this section, we will illustrate the methodology given in the previous sections to evaluate the FNPV of a sport facility project with the different λ levels ($\lambda = 0.2, 0.5, 0.9$).

A. The application of FNPV method: The BOT project of Kaohsiung Modern Multipurpose Dome Stadium

Kaohsiung City Government signed the contract of Kaohsiung Modern Multipurpose Dome Stadium with the concessionaire to encourage private investment in sport

facilities in 2004. The NT$7.8 billion dome stadium project is developed on BOT basis. In this BOT project, Hanwei Dome Development Co., Ltd. is given 50 years concession period from Kaohsiung City Government for the development, operation and maintenance of the dome stadium.

After the 50-year fixed concession period, i.e. when the contract terminates, Hanwei Dome Development Co., Ltd. has to return the ownership of the dome stadium to Kaohsiung City Government.

B. Simulation results

To specify the capital budgeting analysis, we employ (2) and Theorem 2 to compute the NPV and FNPV of the BOT sport facility constructing project with the following simplified scenarios:

1. Total building input: NT$ 7.8 billion.
2. Required rate of return: about 5 %.
3. Estimated net cash inflow for operating this sport facility: about NT$480 million per year (i.e. the estimation of C_t amounts to about NT$ 480 million per year and k_t is about 5 %).

As to the variations of ε_t, β_t, θ_t and ω_t, they may be appropriately determined by the facility financial managers according to their professional considerations. For simplicity, we only provide the case of $\varepsilon_t = \beta_t$ and $\theta_t = \omega_t$, other scenarios could be on the reason by analogy. Furthermore, in order to more clearly compare the fuzzy cases ($N^*_{50,\lambda}$) with crisp case (N_{50}),the simulation results are shown in Figs. 1 to 4.

According to the outcome computed by EXCEL software, the crisp NPV of this BOT project amounts to “about” NT$ 962.84 million, and a significant positive FNPV of this BOT project can be received by the concessionaire. Therefore, it implies that the concessionaire is worthy to participate in the BOT sport facility project.

IV. Discussions

As we are interested in applying the FNPV method to solve the problem of capital budgeting for BOT sport facility project in which the net cash inflows and the required rates of returns are uncertain, it allows us to employ triangular fuzzy numbers to explicitly analyze and provide insights into how the NPV in the fuzzy sense could be impacted by the variations of these two vague parameters. This theoretical conduction has provided the following aspects for the FNPV method.

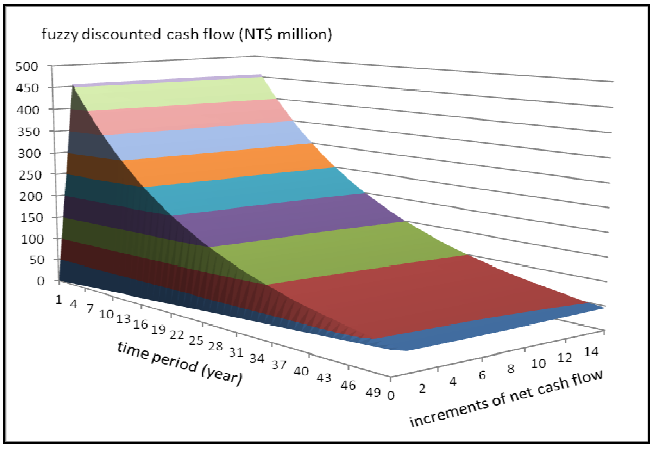

Fig. 1 The fuzzy discounted cash flows along with different variations of net cash flow and required rate of return by estimating NT$480 millions/year net cash inflow at 5% interest rate level ($\lambda = 0.5$, $\varepsilon_t = \beta_t$, $\theta_t = \omega_t$)

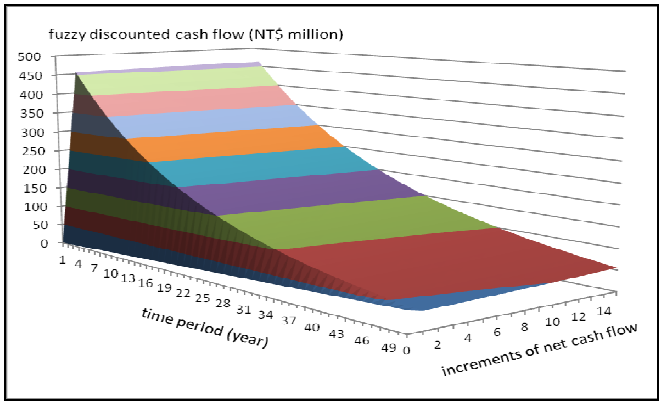

Fig. 2 The fuzzy discounted cash flows along with different variations of net cash flow and required rate of return by estimating NT$480 millions/year net cash inflow at 5% interest rate level ($\lambda = 0.2$, $\varepsilon_t = \beta_t$, $\theta_t = \omega_t$)

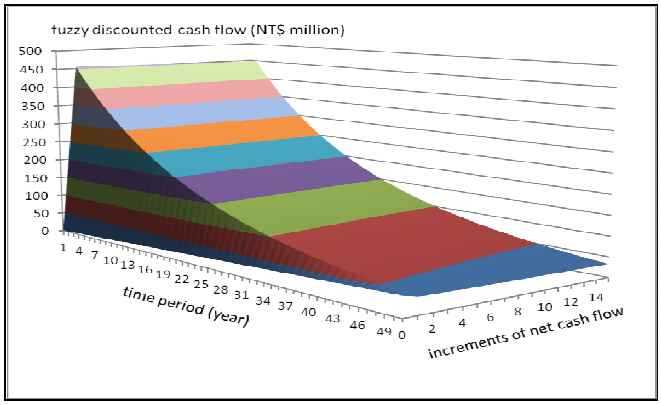

Fig. 3 The fuzzy discounted cash flows along with different variations of net cash flow and required rate of return by estimating NT$480 millions/year net cash inflow at 5% interest rate level ($\lambda = 0.9$, $\varepsilon_t = \beta_t$, $\theta_t = \omega_t$)

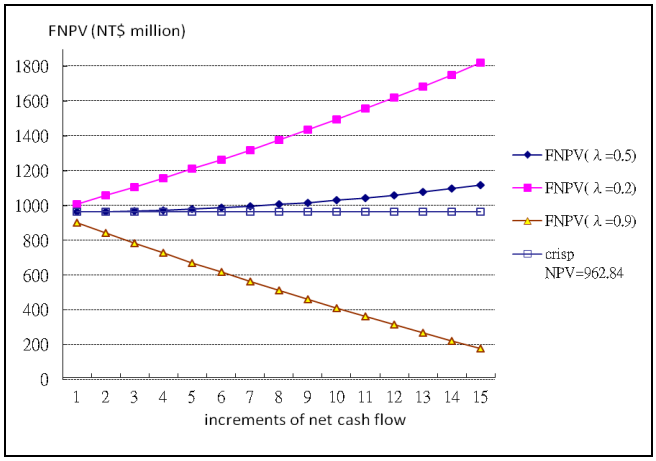

Fig. 4 The variations of the FNPV ($N^*_{50,\lambda}$) and crisp NPV (N_{50}) along with different λ levels and variations of net cash inflow ($\varepsilon_t = \beta_t$) and required rate of return ($\theta_t = \omega_t$)

A. The relationship between Theorem 2 and crisp case

(a) In Theorem 2, let $\varepsilon_t = \beta_t = \theta_t = \omega_t = 0$, then both (4) and (5) become fuzzy points respectively (i.e., $\tilde{C}_t = (C_t, C_t, C_t)$ at C_t and $\tilde{k}_t = (k_t, k_t, k_t)$ at k_t ,). Hence (15) becomes

$$\tilde{N}_n = \tilde{C}_O(+)\sum_{t=1}^{n}\tilde{C}_t(\div)\left[\tilde{1}(+)\tilde{k}_t\right]^t$$

$$= \bigcup_{0\le\alpha\le1}\left[C_O + \sum_{t=1}^{n}\frac{C_t}{[1+k_t]^t}, C_O + \sum_{t=1}^{n}\frac{C_t}{[1+k_t]^t}; \alpha\right]$$

Using the λ–signed distance approach to defuzzify (15), we have $d(\tilde{N}_n, \tilde{0}; \lambda) = N_n$ that is the same as (2).

(b) Let $\varepsilon_t = \beta_t = \theta_t = \omega_t \equiv \Delta_t$, by Theorem 2, for each $\lambda \in (0,1)$, we have

$$N^*_{n,\lambda} = C_O + \sum_{t=1}^{n}\int_0^1\left\{\frac{\lambda[C_t - (1-\alpha)\Delta_t]}{[1+k_t+(1-\alpha)\Delta_t]^t} + \frac{(1-\lambda)[C_t + (1-\alpha)\Delta_t]}{[1+k_t-(1-\alpha)\Delta_t]^t}\right\} d\alpha.$$

For $n = 1$,

$$N^*_{1,\lambda} = C_O + \lambda\left[\left(\frac{C_1\Delta_1 + \Delta_1(1+k_1)}{\Delta_1^2}\right)\left(\ln\frac{1+k_1+\Delta_1}{1+k_1}\right) - \frac{\Delta_1}{\Delta_1}\right]$$

$$+ (1-\lambda)\left[\left(\frac{C_1\Delta_1 + \Delta_1(1+k_1)}{\Delta_1^2}\right)\left(\ln\frac{1+k_1}{1+k_1-\Delta_1}\right) - \frac{\Delta_1}{\Delta_1}\right].$$

For $n = 2$,

$$N^*_{2,\lambda} = N^*_{1,\lambda} + \lambda\left[\left(\frac{\Delta_2}{\Delta_2^2}\right)\left(\ln\frac{1+k_2}{1+k_2+\Delta_2}\right) + \frac{\Delta_2(1+k_2)+C_2\Delta_2}{\Delta_2(1+k_2)(1+k_2+\Delta_2)}\right]$$

$$+ (1-\lambda)\left[\left(\frac{\Delta_2}{\Delta_2^2}\right)\left(\ln\frac{1+k_2-\Delta_2}{1+k_2}\right) + \frac{\Delta_2(1+k_2)+C_2\Delta_2}{\Delta_2(1+k_2)(1+k_2-\Delta_2)}\right].$$

For $n \ge 3$,

$$N^*_{n,\lambda} = N^*_{1,\lambda} + N^*_{2,\lambda} + \sum_{t=3}^{n}\left\{\lambda\left[(-1)^t\left(\frac{\Delta_t(1+k_t) - C_t\Delta_t(t-2)}{\Delta_t^2(t-1)(t-2)(-1-k_t)^{t-1}}\right)\right.\right.$$

$$\left. - (-1)^t\left(\frac{\Delta_t(1+k_t) - C_t\Delta_t(t-2) + \Delta_t^2(t-1)}{\Delta_t^2(t-1)(t-2)(-1-k_t-\Delta_t)^{t-1}}\right)\right]$$

$$+ (1-\lambda)\left[\frac{\Delta_t(1+k_t) - C_t\Delta_t(t-2)}{\Delta_t^2(t-1)(t-2)(1+k_t)^{t-1}}\right.$$

$$\left.\left. - \frac{\Delta_t(1+k_t) - C_t\Delta_t(t-2) - \Delta_t^2(t-1)}{\Delta_t^2(t-1)(t-2)(1+k_t-\Delta_t)^{t-1}}\right]\right\}.$$

When $\Delta_t \to 0$,

$$\lim_{\Delta_t\to0} N^*_{n,\lambda}$$

$$= \lim_{\Delta_t\to0}\left\{C_O + \sum_{t=1}^{n}\int_0^1\left\{\frac{\lambda[C_t - (1-\alpha)\Delta_t]}{[1+k_t+(1-\alpha)\Delta_t]^t} + \frac{(1-\lambda)[C_t + (1-\alpha)\Delta_t]}{[1+k_t-(1-\alpha)\Delta_t]^t}\right\} d\alpha\right\}$$

$= N_n$, the result is the same as (2).

Obviously, according to the above discussions, we can easily verify that the FNPV method is one extension of the crisp NPV methods.

B. The relationship of the estimated net cash inflow $C^*_{t,\lambda}$ (cf. (8)) and required rate of return $k^*_{t,\lambda}$ (cf. (9)) in the fuzzy sense with different λ levels Theorem 2 and crisp case

(a) When $\lambda < 0.5$, $\lambda < 0.5 < (1-\lambda)$, for each $\alpha \in [0,1]$, the point $\lambda\tilde{C}_{tL}(\alpha) + (1-\lambda)\tilde{C}_{tU}(\alpha)$ in $\left[\tilde{C}_{tL}(\alpha), \tilde{C}_{tU}(\alpha)\right]$ will be closer to the right-end point $\tilde{C}_{tU}(\alpha)$. Obviously, because $\tilde{0} < \tilde{C}_{tL} < \tilde{C}_{tU}(\alpha)$ for all $\alpha \in [0,1]$, we have

$\lambda\tilde{C}_{tL}(\alpha) + (1-\lambda)\tilde{C}_{tU}(\alpha) > \tilde{C}_{tU}(\alpha) - 0.5\left[\tilde{C}_{tU}(\alpha) - \tilde{C}_{tL}(\alpha)\right]$

$= 0.5\tilde{D}_L(\alpha) + 0.5\tilde{D}_U(\alpha)$. By (4.4), accordingly we have

$$C^*_{t,\lambda} = d(\tilde{C}_t, \tilde{0}; \lambda) = \int_0^1\left[\lambda\tilde{C}_{tL}(\alpha) + (1-\lambda)\tilde{C}_{tU}(\alpha)\right] d\alpha$$

$$> \int_0^1\left[0.5\tilde{C}_{tL}(\alpha) + 0.5\tilde{C}_{tU}(\alpha)\right] d\alpha$$

Similarly, when $\lambda > 0.5$, for each $\alpha \in [0,1]$, then $C^*_{t,\lambda} < C^*_{t,0.5}$. Based on the derivation, we can also obtain the same relations with respect to $\tilde{k}_t$. That is, when $\lambda < 0.5$, then $k^*_{t,\lambda} > k^*_{t,0.5}$; when $\lambda > 0.5$, then $k^*_{t,\lambda} < k^*_{t,0.5}$. The above-mentioned relations may refer to Figs. 1 to 4.

From the analytic results, we may conclude that the use of λ level can be regarded as a simple concept of describing the facility financial manager's attitude to risk. That is, if $\lambda < 0.5$, then we may denote that such the manager is optimist in estimating the values of fuzzy net cash inflow ($\tilde{C}_t$) and fuzzy required rate of return ($\tilde{k}_t$); if $\lambda > 0.5$, then he or she is pessimist in estimating them. Also, if $\lambda = 0.5$, then he or she is a neutral to risk.

V. Conclusion

The financial arrangements of the BOT project are often the foundation for a successful facility. Through the conscientious mathematical derivation, this paper has proposed a practicable notion that the FNPV technique could offer the potential for flexibility beyond its classical interpretation. In practice, since the parameters in classical NPV formula such like real net cash

inflows and required rate of return in each year may vary with the shifting economy, the real NPV estimated by the facility financial managers might not be exactly equal to their former expectations. Therefore, the FNPV will more fit in with real situation to capture these uncertain parameters by estimating a possible interval instead of point estimation. Also, we have emphasized the suitability of using fuzzy model and pointed out the disadvantages of using classical NPV method.

With the proposed perspective of FNPV method, we demonstrate the variability of overall returns for the BOT sport facility project and provide insight into capital budgeting decisions unavailable through classical NPV analysis. In order to benefit from the technique, the financial managers of sport facilities need very clear definitions of the elements of analysis-capital costs and net cash flows to reveal the potential of the FNVP method. Practitioners can reason in terms of uncertain financial variables to yield a complete picture of the BOT sport facility project.

Although some potential limitations may still exist in the FNPV method which requires further research and elucidation before it will be widely applied in practice, The FNPV method can still help the sport financial managers efficiently grasp the imprecision in financial environment before executing their capital budgeting decisions since the estimated NPV shows only an approximate value. In other words, when the financial managers of sport facilities are interested in applying the fuzzy logic to substitute their rough and arbitrary estimates with more appropriate fuzzy formulations in order to deal with the imprecise cash flows and required rate of return, the conscientious FNPV method may well be the feasible models than the traditional NPV method for evaluating the capital budgeting of BOT-type sport facilities. In conclusion, this paper has successfully extended the classical NPV method by constructing an easy-to-understand and more realistic fuzzy FNPV method without losing the essence of original capital budgeting decision.

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