

# Improvements in Edge Detection Based on Mathematical Morphology and Wavelet Transform using Fuzzy Rules

Masrour Dowlatabadi and Jalil Shirazi

**Abstract**—In this paper, an improved edge detection algorithm based on fuzzy combination of mathematical morphology and wavelet transform is proposed. The combined method is proposed to overcome the limitation of wavelet based edge detection and mathematical morphology based edge detection in noisy images. Experimental results show superiority of the proposed method, as compared to the traditional Prewitt, wavelet based and morphology based edge detection methods. The proposed method is an effective edge detection method for noisy image and keeps clear and continuous edges.

**Keywords**—Edge detection, Wavelet transform, Mathematical morphology, Fuzzy logic.

## I. INTRODUCTION

ONE of the basic topics in image processing is edge detection. Edge is the most basic feature of images. If the edges in an image are identified accurately, some basic properties such as area, perimeter and shape can be measured. Edge detection can play a signification application in different fields such as computer vision, pattern recognition, image segmentation, remote sensing and medical image analysis.

Many classical edge detectors have been developed over time. Some of the well known edge detection operators based on the first derivative of the image are Roberts, Prewitt, Sobel [1]. The Laplacian methods like Marrs-ildreth do it finding the zeros of second derivative from the image [1]. However, classical edge detectors usually fail to handle images with strong noise. For improving the edge detection, fuzzy logic were used [2, 3, 4]. For example, the threshold value in edge detection was calculated based on fuzzy rules [2]. In order to overcome the effects of noises, different methods of edge detection based on wavelet transform was proposed [5]. Mallat has given the wavelet multi-resolution edge detection method based on singularity detection [6]. Xu *et al.* [7] proposed a spatially selected filtering technique by multiplying the adjacent DWT scales to enhance the significant structures. Zhang *et al.* [8] proposed edge detection base on multiplying the wavelet coefficients at two adjacent scales to magnify significant structures and suppress noise.

Masrour Dowlatabadi is with the Islamic Azad University, Gonabad Branch, Iran (e-mail: masror\_d@yahoo.com).

Jalil. Shirazi is with the Electrical Engineering Department, Islamic Azad University, Gonabad Branch, Iran (e-mail: J\_shirazi@iau-gonabad.ac.ir).

In addition, the morphological image processing has developed into an important research field of image processing like edge detection. Morphological filters were used in edge detection [9] and combination of the morphological method and wavelet was used [10].

While wavelet-based edge detection can suppresses most noises in noisy images, but detected edges are discontinuities. On the other hand, Mathematical morphology-based edge detection methods can obtains more completely continuous edges, but it is sensitive to noise.

In this paper, as a novel research, we proposed an edge detection method based on the fusion of the wavelet transform and the mathematical morphology using fuzzy rules. The experimental results show that the proposed method can suppress noises effectively and detect more edge details.

The paper is structured as follows. Wavelet transform, Mathematical morphology and fuzzy logic are described in section 2. The proposed method and experimental results are presented in section 3. The results are compared in section 4. The paper is concluded in section 5.

## II. WAVELET TRANSFORM, MATHEMATICAL MORPHOLOGY AND FUZZY LOGIC

In this section, a brief overview of Wavelet transform in edge detection and mathematical morphology is given.

### A. Edge Detection Based on Wavelet Transform

One-dimensional binary wavelet function is expressed as follows:

Suppose  $\theta(x)$  is a differentiable smooth function whose integral is 1 and converges to 0 at infinity, the  $\psi(x)$  is defined as the first-order derivative of  $\theta(x)$ :

$$\psi(x) = \frac{d\theta(x)}{dx} \quad (1)$$

Set:

$$\theta_{2^j}(x) = \frac{1}{2^j} \theta\left(\frac{x}{2^j}\right) \quad (2)$$

So, the wavelet transformation of function  $f(x)$  in the scale 2 is:

$$W_{2^j} f(x) = f * \psi_{2^j}(x) = f * \left(2^j \frac{d\theta_{2^j}}{dx}\right)(x) \quad (3)$$

Where \* denotes convolution operation. The wavelet used in this paper is the Mallat wavelet [6], whose  $\theta(x)$  is a cubic spline.

For image processing, one-dimensional wavelet transform can be extended to two-dimensional wavelet transform by using the tensor product of two one dimensional analyses, one in the horizontal and the other in the vertical direction.

Set two-dimension smoothing function  $\theta(x, y)$ , its first-order partial derivative in horizontal and vertical directions will act as two basic wavelets, respectively. The convolution of scale transformed wavelets of these two basic wavelets and the image function  $f(x, y)$  are defined as the horizontal and vertical component of wavelet transformation. The two-dimension wavelet function is defined as:

$$\psi_s^1(x, y) = \frac{\partial \theta(x, y)}{\partial x} \quad \psi_s^2(x, y) = \frac{\partial \theta(x, y)}{\partial y} \quad (4)$$

The wavelet transform of image function  $f(x, y)$  of wavelet  $\psi_s^1(x, y)$  and  $\psi_s^2(x, y)$  on scale  $s$  are defined as:

$$w_s^1 f(x, y) = f(x, y) * \psi_s^1(x, y) \quad (5)$$

$$w_s^2 f(x, y) = f(x, y) * \psi_s^2(x, y) \quad (6)$$

For  $s = 2^j$ , these two wavelet transform become binary wavelet transform is termed as two-dimensional binary wavelet transformation of the image  $f(x, y)$ .

The two dimensional binary wavelet transforms are the first-order derivative of the smoothing image along horizontal and vertical direction respectively and can be viewed as two components of the gray gradient vector of smoothed image  $f(x, y)$ . The modulus of gray gradient vector is defined as follows :

$$M_s f(x, y) = \sqrt{((w_s^1 f(x, y))^2 + (w_s^2 f(x, y))^2)} \quad (7)$$

### B. Edge Detection Based on Mathematical Morphology

Mathematical morphology theory is developed from set theory. It was introduced by Matheron as a technique for analyzing geometric structure of metallic and geologic samples [9]. It was extended to image analysis by Serra [9]. Based on set theory, mathematical morphology is established by introducing fundamental operators applied to two sets. One set is said to be processed by another which is known as structuring element. The aim of this transformation is to search the special set structure of original set.

The basic mathematical morphological operators are dilation and erosion, and the other morphological operations are derived from these two basic operations. Suppose,  $F(x, y)$  indicate a grey-scale two dimensional image and  $B$  indicate structuring element. Dilation of a grey-scale image  $F(x, y)$  by a grey-scale structuring element  $B(s, t)$  is indicated by

$$(F \oplus B)(x, y) = \max \{F(x-s, y-t) + B(s, t)\} \quad (8)$$

Erosion of a grey-scale image  $F(x, y)$  by a grey-scale structuring element  $B(s, t)$  is indicated by

$$(F \ominus B)(x, y) = \min \{F(x+s, y+t) - B(s, t)\} \quad (9)$$

Opening and closing of grey-scale image  $F(x, y)$  by grey-scale structuring element  $B(s, t)$  are indicated respectively by

$$F \circ B = (F \ominus B) \oplus B \quad (10)$$

$$F \bullet B = (F \oplus B) \ominus B \quad (11)$$

Erosion decreases the grey-scale value of the image, while dilation increases it. Both of them are sensitive to the image edge whose grey-scale value changes apparently. Opening is erosion followed by dilation and closing is dilation followed by erosion. To detect fine edges in low-frequency of the image (10) and (11) are applied.

### III. PROPOSED METHOD AND EXPERIMENTS

Fig. 1 is the diagram of our proposed method based on fusion of the wavelet transform and the mathematical morphology using fuzzy rules. A Mamdani fuzzy inferences system was implemented using Type-1 fuzzy Logic, with three inputs, one output and 18 rules. We used Mallat wavelet transform [7].

It was shown that using the adjacent scale multiplication achieves better results than using other combination of two scales [7]. We proposed a novel method that yields a higher performance in the edge detection. We used two images from details coefficients of scale 2 and 3 wavelet transform as first and second fuzzy system inputs. As the third input, mathematical morphology on smoothed image from wavelet transform was used.

We used relation 7 for giving the first and second fuzzy system inputs:

We tested relation 7 for different wavelet scales and get the best results for scale 2 and scale3. Fig. 2 shows a not noisy original image and the corresponding detected images using different scales wavelet transform.

We used fuzzy rules on three images pixels that are fuzzy system inputs images. The membership functions for the first and second fuzzy system input is shown in Fig. 3 (a). The membership functions for the second input, third input and fuzzy system output are shown in Fig. 3 (b), Fig. 3 (c) and Fig. 3 (d) respectively.

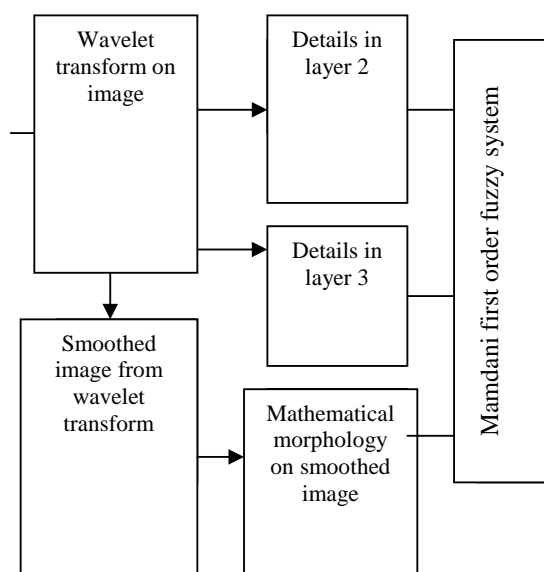


Fig. 1 Block diagram of our proposed method

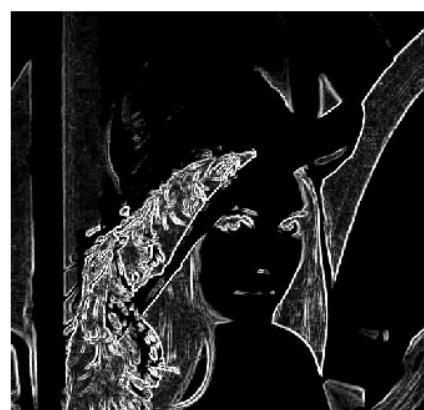
For the purpose of evaluating the performance of proposed edge detection method, we conducted the edge detection experiments using Prewitt, wavelet transform based, mathematical morphology based and proposed algorithm edge detector respectively. Fig.4 (a) and Fig.5 (a) show a noisy image. We added 5% salt and pepper noise to original images. Fig. 4 (b-e) and Fig. 5 (b-e) show the corresponding detected images using the Prewitt, wavelet transform based, mathematical morphology based and proposed algorithm edge detector respectively.

#### IV. RESULTS AND DISCUSSION

Fig. 4 (b) and Fig. 5 (b) indicate that Prewitt method is noise sensitive and produces many false edges in detected image. Fig. 4 (c) and Fig. 5 (c) show that wavelet-based method is noise resistant, but produces many discontinuous edges, and loses some information. Fig. 4 (d) and Fig. 5 (d) indicate that morphology based method is noise sensitive and produces many false edges in detected image. Fig. 4 (e) and Fig. 5 (e) indicate that proposed method is not only noise immune but also produces more edges detail.



(a)

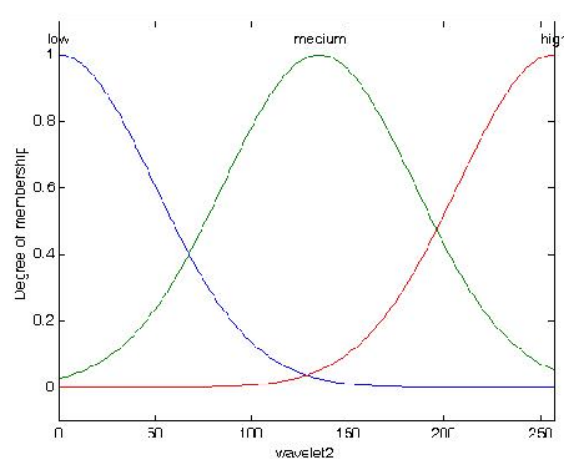


(b)

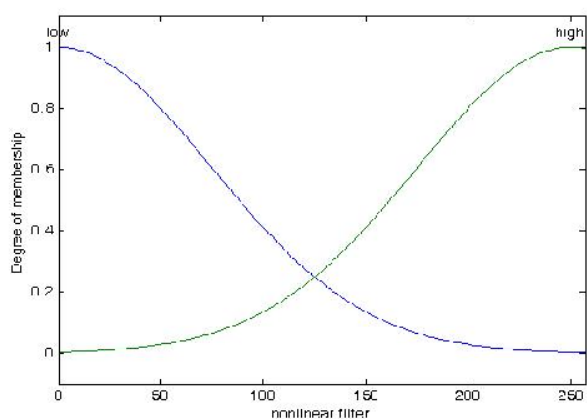


(c)

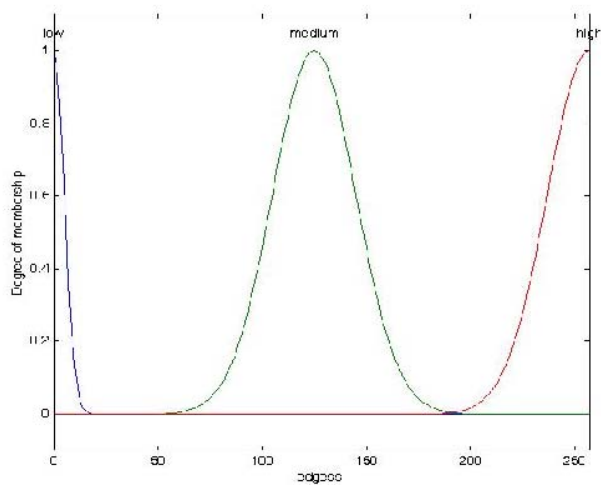
Fig. 2 (a) Not noisy original image, (b) Edge detection based on scale 2, (c) Edge detection based on scale 3



(a)



(b)

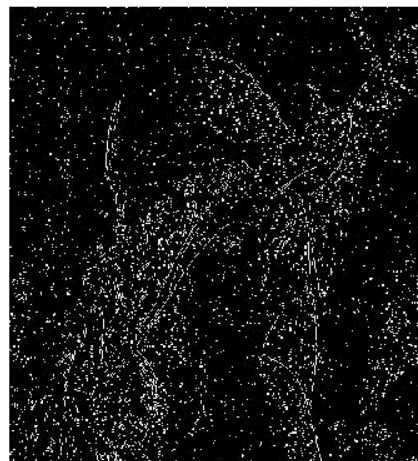


(c)

Fig. 3 (a) The membership function for the first and second fuzzy system inputs, (b) The membership function for third input, (c) The membership function for output.



(a)



(b)



(c)



(d)



(e)



(c)

Fig. 4 (a) Noisy image, (b) Edge detection based on Prewitt, (c) Edge detection based on wavelet, (d) Edge detection based on morphology, (e) Edge detection based on proposed method



(a)



(d)



(b)



(e)

Fig. 5 (a) Noisy image, (b) Edge detection based on Prewitt, (c) Edge detection based on wavelet, (d) Edge detection based on morphology, (e) Edge detection based on proposed method

## V.CONCLUSION

In this paper, we have proposed an edge detection approach based on fusion of wavelet transform and mathematical morphology using fuzzy rules. The proposed method combines the merits wavelet, fuzzy logic and morphology respectively. We compared proposed method in edge detection with Prewitt, wavelet based and mathematical morphology edge detectors. Experimental results show that the proposed algorithm not only suppresses noise effectively, but also keeps clear and continuous edges. We have shown that the proposed method outperform other compared edge detectors that are widely used in the edge detection. An extension to this work would be to using fuzzy logic type-2 or neuro-fuzzy techniques.

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